## Explanation of the mechanism for acousto-optically induced unidirectional operation of a ring laser

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A mechanism is proposed for the nonreciprocal behavior of a traveling-wave acousto-optic Q switch, whose use can provide an effective way to enforce unidirectional and hence single-frequency operation of a ring laser. Simple energy and momentum conservation considerations show that the Bragg condition is satisfied at different angles of incidence for the two counterpropagating beams, which leads to a difference in their diffraction losses. The loss difference measured directly for a diode-pumped Nd:YAG ring laser, yielded values in close agreement with calculated values. Implications of these results for both single-frequency cw and Q-switched operation of diode-pumped solid-state ring lasers are discussed.

The use of an intracavity acousto-optic (A-O) Q switch to enforce unidirectional operation of a ring laser was first demonstrated for Nd:YAG (lamp-pumped), Ti:sapphire, and dye ring lasers. More recently the technique has proved to be an extremely effective, low-loss means to achieving reliable single-frequency output from diode-pumped Nd:YAG and YLF ring lasers while also providing a means for Q switching the laser. A particular advantage of this technique over the usual Faraday isolator techniques is that it does not rely on polarization discrimination and is therefore suited to resonators containing birefringent components, such as laser media and frequency doublers.

So far, to our knowledge, an adequate explanation for the nonreciprocal behavior has not been reported. A knowledge of the mechanism is nevertheless important for further optimization to be possible. Here we explain the origin of the nonreciprocal behavior of the A-O Q switch. Our model allows calculation of both magnitude and sign of the loss difference in any given situation, thus providing a strategy for the design of efficient and reliable single-frequency lasers, cw or Q switched. As proof, we present experimental results, including measurements of the loss difference in a diodepumped Nd:YAG ring laser, which agree well with our predictions.

The nonreciprocity of a traveling-wave A-O Q switch is a consequence of the fact that when light is reflected from a moving surface, the angles of incidence and reflection are no longer identical. Light incident upon the Q switch experiences partial reflection from the moving index grating associated with the acoustic wave. When angles of incidence and reflection are such that reflected contributions are in phase, then the Bragg condition is satisfied, and the reflection is at a maximum. However, a consequence of the traveling grating is that the Bragg condition is satisfied at different angles of incidence for the two counterpropagating beams. As a result they generally experience different diffrac-

tion losses. Although this angular difference is extremely small, we show that it can lead to significantly different diffraction losses.

To obtain an accurate estimate of the loss difference it is necessary to know the difference in the incidence angles for the two counterpropagating beams that satisfy the Bragg condition. We refer to these angles as the Bragg incidence angles, shown in Figs. 1 and 2 as  $\theta_B^+$  for the beam propagating in the forward (+) direction (i.e., for the beam with the wave vector component in the direction of the acoustic wave) and  $\theta_B^-$  for the counterpropagating beam in the (-) direction. It is important to distinguish these angles from the Bragg angle  $\theta_B$ . A more general statement of the Bragg condition may be written as

$$\sin(\theta_B^{\pm} + \theta_d^{\pm}) = \lambda/n\lambda_s = \sin(2\theta_B), \tag{1}$$

where  $\theta_d^+$  and  $\theta_d^-$  are the diffraction angles for the two counterpropagating beams as shown in Figs. 1 and 2,  $\lambda$  is the vacuum wavelength of light,  $\lambda_s$  is the wavelength of sound in the acoustic medium, and n is its refractive index. The difference  $\Delta\theta_B$  between Bragg incidence angles for the two counterpropagating beams can be calculated by considering energy and momentum conservation. Figures 2(a) and 2(b) are the wave vector diagrams showing conservation of momentum for the forward (+) and counterpropagating (-) beams, respectively, at the Bragg condition. Resolving into components perpendicular to  $k_s$ , we obtain, using Eq. (1),

$$k_i \cos(\theta_B^{\pm}) = k_d^{\pm} \cos(2\theta_B - \theta_B^{\pm}), \qquad (2)$$

where  $k_i = k_i^+ = k_i^-$ . Since  $|\Delta \theta_B| \ll 1$  and  $|\theta_B^{\pm}| \ll 1$ , then from Eq. (2) we obtain the following relation for  $\Delta \theta_B$ :

$$\Delta\theta_B \equiv \theta_B^+ - \theta_B^- \approx \frac{k_i(k_d^- - k_d^+)}{k_d^+ k_d^- \sin 2\theta_B}.$$
 (3)

In order to satisfy energy conservation,

$$k_i - k_d^{\pm} = \pm 2\pi n \nu_s/c, \qquad (4)$$

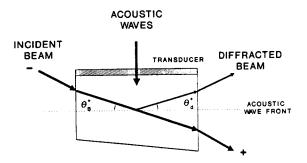


Fig. 1. Bragg diffraction in a traveling-wave A-O Q switch.

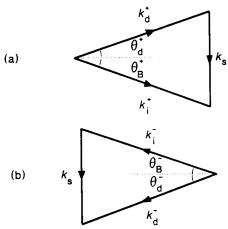


Fig. 2. (a), (b) Wave vector diagrams showing the Bragg condition for the (+) and (-) counterpropagation directions, respectively.  $k_s$  is the wave vector for the acoustic wave, and  $k_i^\pm$  and  $k_d^\pm$  are the wave vectors for the incident and diffracted beams, respectively.

where  $\nu_s$  is the acoustic frequency. By substituting Eqs. (1) and (4) into relation (3) and using the approximation  $k_i^2 \approx k_d^+ k_d^-$ , we obtain a simplified expression for  $\Delta\theta_B$ , namely,

$$\Delta\theta_B \approx 2nv_s/c$$
, (5)

where  $v_s$  is the sound velocity. For the purpose of further discussion all the angles quoted are those measured in air, since these can be compared directly with experimental results. In this case the value for  $\Delta\theta_B$  in relation (5) is multiplied by an extra factor n.

To verify that the Bragg condition is indeed satisfied at different incidence angles for counterpropagating beams, we have made direct measurements of the Bragg incidence angles for a lead molybdate Q switch. To ensure that the counterpropagating beams were exactly antiparallel, the measurements were made in a standing-wave laser. A diodepumped Nd:YAG laser was used with a folded-cavity design,7 the A-O Q switch being placed in the collimated arm of the resonator. In order to measure  $\Delta\theta_B$ , the diffracted outputs in both the (+) and (-) directions were monitored independently, and an optical lever system was used to measure small angular tilts of the A-O Q switch. It was found that the diffraction maxima did indeed occur for slightly different orientations of the Q switch, the measured value for  $\Delta\theta_B$  (in air) being  $0.009^{\circ} \pm 0.002^{\circ}$ , in good agreement with the value of 0.0076° predicted by relation (5).

In order to calculate the value for the difference in the diffraction losses for the two counterpropagating beams at a given incidence angle, it is necessary to know the diffraction loss as a function of incidence angle. The shape of this curve depends on a number of parameters, including lasing wavelength, beam size, beam divergence, acoustic wavelength, and length of the Q switch. Since the exact analysis is complicated, we have determined the form of the diffraction-loss-versus-angle curve experimentally. The loss difference can now be calculated by simply considering two diffraction loss curves offset by angle  $\Delta\theta_B$ . The results shown in Fig. 3 are for a lead molybdate Q switch of length 20 mm with  $\nu_s=80$  MHz,  $\lambda=1.06~\mu m,$  and a lasing mode radius of 380  $\mu$ m. Figure 3(a) shows that when  $\theta_i \neq \theta_B$  the diffraction loss is different for opposite directions of propagation. For  $\theta_i < \theta_B$  the loss is lowest for the (+) direction, and lasing occurs preferentially in this direction for a ring laser, while for  $\theta_i > \theta_B$  the (-) direction is preferred. In Fig. 3(b) the predicted value for loss difference as a fraction of the peak diffraction loss is plotted as a function of the angular deviation  $(\theta_i - \theta_B)$  from the Bragg angle. In this particular case it can be seen that the loss difference  $\Delta L$  is a maximum near  $|\theta_i - \theta_B| = 0.15^{\circ}$  and has a value of  $\sim 3.2\%$  of the maximum diffraction loss (at  $\theta_i = \theta_B$ ) and 5.0% of the actual diffraction loss at that angle. A potentially better situation for cw unidirectional operation is to tilt the Q switch so that  $|\theta_i - \theta_B| \approx 0.3^\circ$ . Here the loss difference is decreased slightly but is

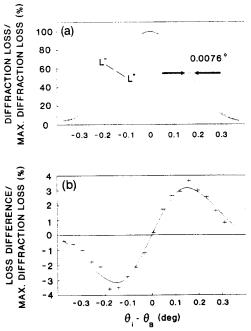


Fig. 3. (a) Diffraction losses  $L^+$  and  $L^-$  for the counterpropagation directions as a function of the incidence angle (in air). (b) Loss difference  $\Delta L = L^+ - L^-$  as a function of the incidence angle. The solid curve is the predicted curve based on the diffraction loss curves shown above, and the crosses are measured values for loss difference in a ring laser.

now  $\sim$ 14% of the diffraction loss. The net result is that for a particular value of loss difference, the diffraction loss and hence the effective insertion loss for the Q switch is now reduced. Clearly for unidirectional operation to be realized in any particular laser, the loss difference must be sufficiently large to overcome any coupling between the two lasing directions. For many miniature solid-state lasers this can be very small  $(\sim$ 0.01%) and would involve an increase in cavity loss of only  $\sim$ 0.07%, which can be considered negligible in most cases.

To verify these predictions a number of measurements were made on a diode-pumped Nd:YAG ring laser incorporating the lead molybdate A-O Q switch. Care was taken to use a resonator design such that the beam size in the Q switch exactly matched that used previously in the determination of the diffraction loss versus angle curves. The triangular ring resonator used had a long perimeter (~100 cm) to ensure that any diffracted beams completely left the resonator and did not interfere with the operation of the Q switch. When a low-power ( $\sim 0.01$  W) rf signal was applied, and the Q switch was orientated close to the Bragg condition, but with  $\theta_i < \theta_B$ , unidirectional lasing was observed in the (+) direction, as expected. When tilted slightly so that  $\theta_i > \theta_B$ , the direction of lasing reversed, also as predicted [see Fig. 3(b)]. To provide further confirmation of the proposed mechanism, the loss difference was determined from observations of the transient behavior of the ring laser. The procedure involves Q switching the laser by using a second A-O Q switch. Since the loss difference is generally quite small compared with the laser gain, it takes many round trips of the resonator before lasing is suppressed in the higher-loss direction. This does not usually occur on the time scale of a Q-switched pulse; hence when building up from noise, Q-switched pulses are obtained for both lasing directions. These pulses do however have different buildup times as a consequence of their different overall gains. By measuring the ratio  $\gamma$  of the output powers, near the beginning of the pulses (so that the inversion has not been depleted significantly and the gain is effectively constant), the loss difference  $\Delta L$  can be calculated from

$$\Delta L = 1 - \gamma^{1/q}, \tag{6}$$

where q is the number of round trips. The results obtained, shown in Fig. 3(b), are seen to be in good agreement with those predicted by the model. When the rf power was increased, for a particular orientation of the Q switch, we observed a linear in-

crease in loss difference with diffraction loss. This was expected since there was no observable change in shape of the diffraction loss curve with rf power.

Further enhancement in the ratio of loss difference to diffraction loss can be achieved by reducing the diffraction bandwidth, e.g., by using a longer Q switch, operating at a higher acoustic frequency, or using a larger laser beam with a smaller divergence. This ratio can also be increased by using an A-O medium with a higher speed of sound. Note that the requirements for efficient cw unidirectional operation are not necessarily the same as for unidirectional Q-switched operation. For cw operation it is desirable to have a high loss difference but a low diffraction loss. For unidirectional Q-switched operation, however, it is necessary to use the technique of prelase Q switching, which requires both a high loss difference and a high diffraction loss. Clearly the choice of acoustic medium and orientation of the Q switch will depend on the desired mode of operation. However, since the loss differences required are typically extremely small, a compromise can often be made, which permits either unidirectional cw or Q-switched operation to be achieved without adjusting the Q switch.

In conclusion, we have described the mechanism responsible for the nonreciprocal behavior of a traveling-wave A-O Q switch. By using the model developed it should now be possible to design Q switches optimized for reliable and efficient unidirectional, single-frequency, cw or Q-switched operation in a wide variety of lasers.

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