Circular birefringence in helical-core fibre

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Circular birefringence in an isotropic helical fibre is studied using coupled-mode analysis. The formulas derived here can be used to explain the existing paradox in calculating circular birefringence.

Introduction: The fact that the polarisation state of light is rotated if the light advances along a helical fibre path has been successfully applied to making circularly birefringent fibres. However, the formulas for the rotation (angle) of the plane of polarisation per turn of the helix in References 2 and 3 are different. The angle difference between the two formulas is 2x, which is usually not detectable in measurements, but it causes difficulty in calculating the birefringence.

The difference arises in determining the rotation of polarisation, which in Reference 3 is relative to the helix axis while in Reference 2 it is relative to the fibre axis.

In this letter we show that if we use the helix axis as the reference axis, i.e. we consider the whole helix as a waveguide, then the formula of Varnham et al. is valid. On the other hand, if the helical fibre axis is the reference axis, in this case the helical fibre is itself considered as the waveguide, and the Ross formula is valid provided that the pitch angle is less than 45° and a slight change to the formula of Varnham et al. applies for pitch angles between 45° and 90°.

Our theory puts the formula in Reference 3, which seems to have been intuitively derived, on a rigorous theoretical basis. The theory can now be used for helical fibre design.

Formulation: We start from the following coupled-mode equations describing the coupling caused by the geometric torsion in a helical fibre:

\[
\frac{dA_a}{ds} = -j\beta_a A_a + \tau A_b
\]
\[
\frac{dA_b}{ds} = -\tau A_a - j\beta_b A_b
\]

where \(A_a\) and \(A_b\) are the amplitudes of the two orthogonally linearly polarised modes, and \(\tau\) is the torsion which is positive in the case of a right-handed helical system and is given by

\[
\tau = \frac{2\pi P}{P^2 + (2\pi R_0)^2} = \frac{2\pi P}{S^2}
\]

where \(P\) is the pitch, \(R_0\) is the radius of the helix and \(S\) is the arc length for one turn. These equations ignore the coupling caused by other properties of the waveguide structure. The co-ordinate system \((n, b, s)\) is the Serret–Frenet frame; the \(n\), \(b\) and \(s\) are the principal normal, binormal and tangent co-ordinates, respectively. These co-ordinates are shown for the helix in Fig. 1a.

To simplify the analysis, we assume no linear birefringence in the fibre. Then the propagation constants of the two linearly polarised modes are equal; namely

\[
\beta_n = \beta_b = \beta
\]

To transform the two linearly polarised modes to two circularly polarised modes, we use

\[
A_r = \frac{1}{\sqrt{2}}(A_a - jA_b)
\]
\[
A_i = \frac{1}{\sqrt{2}}(A_a + jA_b)
\]

where \(A_r\) and \(A_i\) are the amplitudes of the right and left circularly polarised modes, the electric field vectors of which rotate around the \(s\)-axis according to the right-hand and left-hand rules, respectively.

Substituting \(A_r\) and \(A_i\) and \(A_b\) in eqn. 1 by using eqn. 1, it can be shown that

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\[
\frac{dA_t}{ds} = -j(\beta - \tau)A_t
\]

These equations imply that the two circularly polarised modes are eigenmodes on an ideal isotropic helical fibre, and that their propagation constants are \((\beta - \tau)\) and \((\beta + \tau)\) in the \((n, b, s)\) system.

Circular birefringence: When the pitch angle \(\sigma = \sin^{-1} \frac{P}{S}\) is greater than 45°, it is obvious from Fig. 1b that the \((n, b)\) co-ordinates are rotated in the plane transverse to the \(s\)-axis and around the \(s\)-axis with the rotation rate \(\alpha = 2\pi S/P\) per unit length of the helical fibre. Relative to the fixed co-ordinate system in which the input and output straight fibres are located, the propagation constants \((\beta \mp \tau)\) for both circularly polarised modes are therefore modified. As we are here considering a right-handed system, the right circularly polarised mode propagates faster, i.e. its propagation constant is \((\beta - \tau + \alpha)\), and the left one advances slower, i.e. its propagation constant is \((\beta + \tau - \alpha)\), than in the original system.

When the pitch angle is less than 45°, then from Fig. 1c we find that the co-ordinates \((n, b)\) do not rotate around the \(s\)-axis but still rotate around the helix axis, i.e. the \(z\)-axis in Fig. 1. Consequently, the propagation constants are still \((\beta \mp \tau \mp \alpha)\) and are \((\beta \mp \tau)\) if the fibre axis is the reference axis. Since \(\alpha\) is always greater than \(\tau\), there is an abrupt change in the sign of the propagation constant difference at \(\sigma = 45°\) for the case where the fibre axis is the reference axis. Thus, for any pitch angle, the circular birefringence of the waveguide formed by the helical fibre is

\[
B = \left(\frac{S}{P}\right) \left(\frac{1}{\pi}\right) (\alpha - \tau) = \frac{2\alpha}{P} \left(1 - \frac{P}{S}\right)
\]

The factor \(S/P\) in eqn. 6 is due to the fact that the helix axis is taken as the reference axis. From eqn. 6 we have the beat length

\[
L_p = \frac{PS}{2(S - P)}
\]

This is the same equation as mentioned in Reference 3.

However, relative to the fibre axis, the circular birefringences of the helically wound fibres are different for different ranges of \(\sigma\). When \(\sigma > 45°\), then

\[
L_p = \frac{S^2}{2(S - P)}
\]

which can be derived from eqn. 7 by deleting the factor \(S/P\).

When \(\sigma < 45°\), then

\[
L_p = S/2
\]

Eqns. 7, 8 and 9 are plotted in Fig. 2; the experimental points are cited from References 2, 3 and 6. Even though the \(L_p/S\) is small in the region \(45° > \sigma > 0\), it may not have practical use for circularly birefringence in fibres unless the large bend loss can be reduced.

**Fig. 2** Beat length of helical fibre against pitch angle \(\sigma\)

Solid line corresponds to using the helix axis as reference and broken line corresponds to using the fibre axis as reference. Points are calculated by \(L_p = \pi P/\theta\), where \(\theta\) is the rotation angle of the plane of polarisation per unit and the value of the \(\theta\) are from Ross. Crossed points are from Varnham et al.

**Conclusion:** The inconsistency of the formulas for the rotation angle in helical fibres, as presented in previous papers, arises from using a different co-ordinate reference frame, and has been resolved here using coupled-mode analysis. Different formulas are shown to apply for the different reference axes. The formulas also provide a theoretical base for helical fibre design.

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**References**


