# CIRCULAR BIREFRINGENCE IN HELICAL-CORE FIBRE

*Indexing terms: Optical fibres, Polarisation, Birefringence*

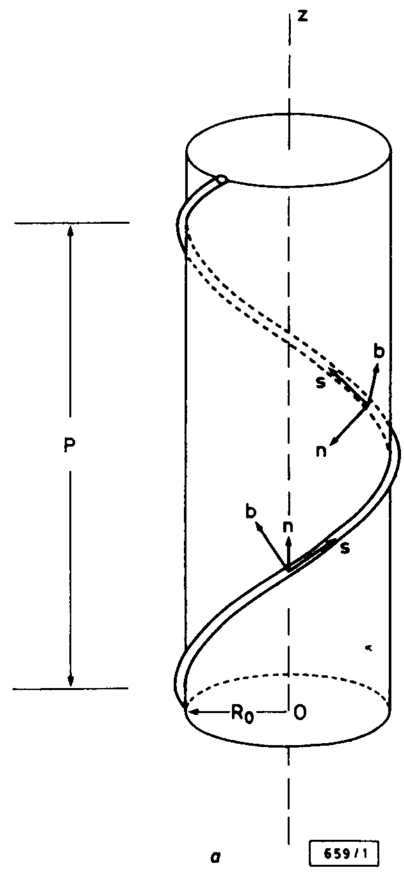
Circular birefringence in an isotropic helical fibre is studied using coupled-mode analysis. The formulas derived here can be used to explain the existing paradox in calculating circular birefringence.

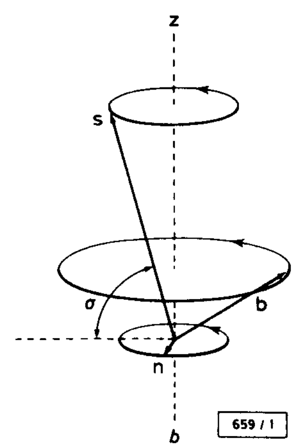
*Introduction:* The fact that the polarisation state of light is rotated if the light advances along a helical fibre path[1,2] has been successfully applied to making circularly birefringent fibres.[3] However, the formulas for the rotation (angle) of the plane of polarisation per turn of the helix in References 2 and 3 are different. The angle difference between the two formulas is $2\pi$, which is usually not detectable in measurements, but it causes difficulty in calculating the birefringence.

The difference arises in determining the rotation of polarisation, which in Reference 3 is relative to the helix axis while in Reference 2 it is relative to the fibre axis.

In this letter we show that if we use the helix axis as the reference axis, i.e. we consider the whole helix as a waveguide, then the formula of Varnham *et al.*[3] is valid. On the other hand, if the helical fibre axis is the reference axis, in this case the helical fibre is itself considered as the waveguide, and the Ross formula[2] is valid provided that the pitch angle is less than 45° and a slight change to the formula of Varnham *et al.*[3] applies for pitch angles between 45° and 90°.

Our theory puts the formula in Reference 3, which seems to have been intuitively derived, on a rigorous theoretical basis. The theory can now be used for helical fibre design.



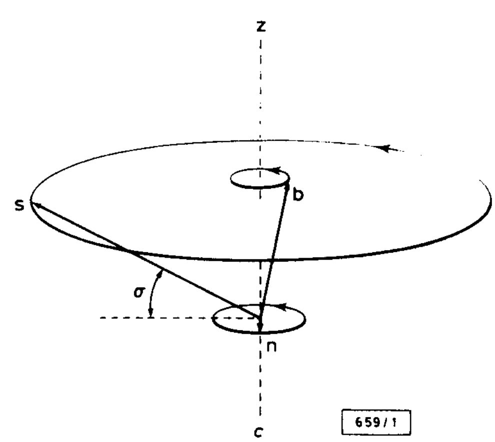




**Fig. 1** *Helical fibre (a) showing (n, b, s) co-ordinate system and vibration of directions of three co-ordinates as the point (n, b, s) moves along the helix (b) when 45° < σ < 90° and (c) when 0 < σ < 45°*

*Formulation:* We start from the following coupled-mode equations[2,4] describing the coupling caused by the geometric torsion in a helical fibre:

$$\begin{aligned} \frac{dA_n}{ds} &= -j\beta_n A_n + \tau A_b \\ \frac{dA_b}{ds} &= -\tau A_n - j\beta_b A_b \end{aligned} \tag{1}$$

where $A_n$ and $A_b$ are the amplitudes of the two orthogonally linearly polarised modes, and $\tau$ is the torsion which is positive in the case of a right-handed helical system and is given by

$$\tau = \frac{2\pi P}{P^2 + (2\pi R_0)^2} = \frac{2\pi P}{S^2} \tag{2}$$

where $P$ is the pitch, $R_0$ is the radius of the helix and $S$ is the arc length for one turn. These equations ignore the coupling caused by other properties of the waveguide structure.[5] The co-ordinate system $(n, b, s)$ is the Serret–Frenet frame;[4] the $n$, $b$ and $s$ are the principal normal, binormal and tangent co-ordinates, respectively. These co-ordinates are shown for the helix in Fig. 1*a*.

To simplify the analysis, we assume no linear birefringence in the fibre. Then the propagation constants of the two linearly polarised modes are equal; namely

$$\beta_n = \beta_b = \beta \tag{3}$$

To transform the two linearly polarised modes to two circularly polarised modes, we use

$$\begin{aligned} A_r &= \frac{1}{\sqrt{2}}(A_n - jA_b) \\ A_l &= \frac{1}{\sqrt{2}}(A_n + jA_b) \end{aligned} \tag{4}$$

where $A_r$ and $A_l$ are the amplitudes of the right and left circularly polarised modes, the electric field vectors of which rotate around the $s$-axis according to the right-hand and left-hand rules, respectively.

Substituting $A_r$ and $A_l$ and $A_b$ in eqn. 1 by using eqn. 1, it can be shown that

$$\frac{dA_r}{ds} = -j(\beta - \tau)A_r$$
$$\frac{dA_l}{ds} = -j(\beta + \tau)A_l \qquad (5)$$

These equations imply that the two circularly polarised modes are eigenmodes on an ideal isotropic helical fibre, and that their propagation constants are $(\beta - \tau)$ and $(\beta + \tau)$ in the $(n, b, s)$ system.

*Circular birefringence:* When the pitch angle $\sigma$ ($\sigma = \sin^{-1} P/S$) is greater than 45°, it is obvious from Fig. 1*b* that the $(n, b)$ co-ordinates are rotated in the plane transverse to the $s$-axis and around the $s$-axis with the rotation rate $\alpha$ ($\alpha = 2\pi/S$) per unit length of the helical fibre. Relative to the fixed co-ordinate system in which the input and output straight fibres are located, the propagation constants $(\beta \mp \tau)$ for both circularly polarised modes are therefore modified. As we are here considering a right-handed system, the right circularly polarised mode propagates faster, i.e. its propagation constant is $(\beta - \tau + \alpha)$, and the left one advances slower, i.e. its propagation constant is $(\beta + \tau - \alpha)$, than in the original system.

When the pitch angle is less than 45°, then from Fig. 1*c* we find that the co-ordinates $(n, b)$ do not rotate around the $s$-axis but still rotate around the helix axis, i.e. the $z$-axis in Fig. 1. Consequently, the propagation constants are still $(\beta \mp \tau \mp \alpha)$ if we take the helix axis as the reference axis, and they are $(\beta \mp \tau)$ if the fibre axis is the reference axis. Since $\alpha$ is always greater than $\tau$, there is an abrupt change in the sign of the propagation constant difference at $\sigma = 45°$ for the case where the fibre axis is the reference axis. Thus, for any pitch angle, the circular birefringence of the waveguide formed by the helical fibre is

$$B = \left(\frac{S}{P}\right)\left(\frac{\lambda}{\pi}\right)(\alpha - \tau) = \frac{2\lambda}{P}\left(1 - \frac{P}{S}\right) \qquad (6)$$

The factor $(S/P)$ in eqn. 6 is due to the fact that the helix axis is taken as the reference axis. From eqn. 6 we have the beat length

$$L_p = \frac{PS}{2(S - P)} \qquad (7)$$

This is the same equation as mentioned in Reference 3.

However, relative to the fibre axis, the circular birefringences of the helically wound fibres are different for different ranges of $\sigma$. When $\sigma > 45°$, then

$$L_p = \frac{S^2}{2(S - P)} \qquad (8)$$

which can be derived from eqn. 7 by deleting the factor $S/P$.

When $\sigma < 45°$, then

$$L_p = S/2 \qquad (9)$$

Eqns. 7, 8 and 9 are plotted in Fig. 2; the experimental points are cited from References 2, 3 and 6. Even though the $L_p/S$ is small in the region $45° > \sigma > 0$, it may not have practical use for circularly birefringence in fibres unless the large bend loss can be reduced.

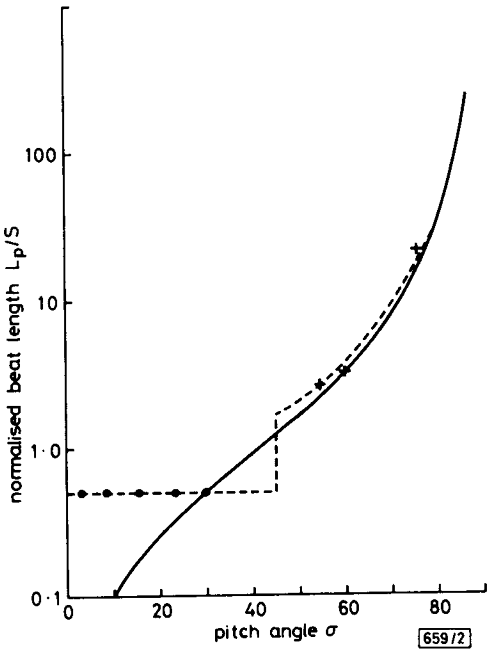



**Fig. 2** *Beat length of helical fibre against pitch angle $\sigma$*

Solid line corresponds to using the helix axis as reference and broken line corresponds to using the fibre axis as reference. Points are calculated by $L_p = \pi P/\theta$, where $\theta$ is the rotation angle of the plane of polarisation per unit and the value of the $\theta$ are from Ross.[2] Crossed points are from Varnham *et al.*[3,6]

*Conclusion:* The inconsistency of the formulas for the rotation angle in helical fibres, as presented in previous papers, arises from using a different co-ordinate reference frame, and has been resolved here using coupled-mode analysis. Different formulas are shown to apply for the different references axes. The formulas also provide a theoretical base for helical fibre design.

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J. QIAN* 
C. D. HUSSEY

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*Department of Electronics & Information Engineering*
*The University*
*Southampton S09 5NH, United Kingdom*

* On leave from the University of Science & Technology of China, Hefei, Anhui, China

## References

1 PAPP, A., and HARMS, H.: 'Polarisation optics of liquid core optical fibres', *Appl. Opt.*, 1977, **5**, pp. 1315–1319

2 ROSS, J. N.: 'The rotation of polarisation in low birefringent monomode optical fibres due to geometric effects', *Opt. & Quantum Electron.*, 1984, **16**, pp. 455–461

3 VARNHAM, M. P., BIRCH, R. D., and PAYNE, D. N.: 'Helical-core circularly-birefringent fibres'. Proc. IOOC-ECOC, Venice, 1985

4 FANG, X. S., and LIN, Z. Q.: 'Field in single-mode helically-wound optical fibres', *IEEE Trans.*, 1985, **MTT-33**, pp. 1150–1154

5 HUSSEY, C. D., BIRCH, R. D., and FUJII, Y.: 'Circularly birefringent single-mode optical fibres', *Electron. Lett.*, 1986, **22**, pp. 129–130

6 VARNHAM, M. P., BIRCH, R. D., PAYNE, D. N., and LOVE, J. D.: 'Design of helical-core circularly-birefringent fibres'. Proc. OFC/IGWO, Atlanta, 1986