

# NEW INTERPRETATION OF SPOT-SIZE MEASUREMENTS ON SINGLY CLAD SINGLE-MODE FIBRES

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By using the Petermann spot size rather than the Marcuse spot size we suggest a slight modification to the Millar method for ESI determination which will allow the three parameters necessary for the EESI to be determined. The method is particularly useful for triangular-profile fibres.

**Introduction:** One basic assumption in the Millar method for determining the two ESI parameters for single-mode fibres is that the spot size for arbitrary profiles behaves as simply a scaled version of the step-index spot size.<sup>1</sup> While this is true for most fibres used for telecommunications, it is not true in general, and in particular the method breaks down when applied to the triangular-type profile fibres which are currently receiving much attention.<sup>2</sup>

In its simplest form the Millar method measures the cutoff wavelength together with the spot size at cutoff. Unfortunately it is at the cutoff wavelength where the spot size is most sensitive (over the single-mode regime) to vagaries in the refractive index profile. Moreover, the Millar method has no built-in alarm to show that it breaks down for a specific fibre under measurement.

We will return to the above points in the course of our argument. Here we show how the Millar method can be adapted to cater for a more complex spot-size structure. Our approach uses the Petermann spot size<sup>3,4</sup> (or far-field RMS width) to find the three parameters necessary to specify the 'enhanced' ESI or EESI.<sup>5</sup> Having determined the three parameters we can then decide whether to use all three or only two in the simple ESI approximation, but now with a greater appreciation of the errors involved if we choose the latter.

The implementation of our approach is aided by our recently proposed analytic approximation to the Petermann spot size, which is a simple extension of the Marcuse spot-size formula.<sup>6</sup>

**Petermann spot size and EESI approximation:** The EESI is specified by three parameters.<sup>5</sup> For our purposes we will find it convenient to use the parameters  $\bar{V}$ ,  $a_c$  and  $|\Delta\bar{\Omega}_4|$ , where  $\bar{V} = (2\pi/\lambda)a_c NA_c = \sqrt{(2\Omega_0)V}$ ,  $a_c = a\sqrt{(2\Omega_2)}$  and  $NA_c = NA\sqrt{(\Omega_0/\Omega_2)}$ , where  $a$  is the core radius,  $NA$  is the numerical aperture,  $\Omega_0$  is the guidance factor and the  $\Omega$  are the moments of the profile. We call  $|\Delta\bar{\Omega}_4|$  the enhancement parameter; its magnitude gives an indication of the deviation of the refractive index from the step index (for which  $|\Delta\bar{\Omega}_4| = 0$ ). The triangular core fibre has an enhancement parameter of 0.19, for the parabolic profile it is 0.125, and for cusp-like profiles its value can approach 0.3.<sup>5</sup>

In terms of our three parameters, the near-field Petermann spot size  $\omega$  for fibres of arbitrary profile can be expressed as

$$\begin{aligned}\bar{\omega}^2 &= \left(\frac{\omega}{a_c}\right)^2 \\ &= (1/\bar{\omega}_{st}^2(\bar{V}) + |\Delta\bar{\Omega}_4| \{f(\bar{V})/\omega_{st}^2(\bar{V}) \\ &\quad + \bar{V}^3 f'(\bar{V})b_{st}(\bar{V})/4\})^{-1}\end{aligned}\quad (1)$$

where  $\bar{\omega}_{st}(\bar{V})$ ,  $f(\bar{V})$ ,  $b_{st}(\bar{V})$  are known.

The form of eqn. 1 illustrates dramatically the limitations of any procedure which arbitrarily attempts to fit a measured spot size with that of some step-index fibre. Additionally, the term in curly brackets in eqn. 1 is a monotonically increasing function of  $\bar{V}$ , so that by measuring the spot size at cutoff we are choosing the worst sampling point for fitting the simple step-index spot size for fibres which have a large enhancement parameter. These points are also illustrated in Fig. 1, where we plot  $\bar{\omega}$  as a function of  $\bar{V}$  for the triangular-profile fibre.

We will find it convenient to rewrite eqn. 1 as follows:

$$\omega^2 = \frac{a_c^2}{p + |\Delta\bar{\Omega}_4|q}\quad (2)$$

where  $p = 1/\bar{\omega}_{st}^2(\bar{V})$  and  $q = f(\bar{V})p + \bar{V}^3 f'(\bar{V})b_{st}(\bar{V})/4$ .  $p$  and  $q$  are specified once  $\bar{V}$  is known. Similarly the far-field spot size can be expressed as

$$\omega_{ff}^2 = \frac{2}{\omega^2} = \frac{2}{a_c^2} (p + |\Delta\bar{\Omega}_4|q)\quad (3)$$

**Adapted Millar procedure:** The starting point of the Millar method<sup>1</sup> is the fact that the effective normalised cutoff frequency  $\bar{V}_{co}$  is unlike any other fibre characteristic, because it is extremely insensitive to profile shape. The cutoff wavelength  $\lambda_{co}$  can therefore provide a fixed reference point for all single-mode fibre measurements.

Millar's method is perhaps the most standard technique for determining  $\lambda_{co}$  from observations of the near-field spot-size behaviour. Further refinements have produced more accurate determinations of  $\lambda_{co}$ .<sup>6</sup> The possibility of using the far-field spot-size behaviour in determining  $\lambda_{co}$  has also been suggested.<sup>7</sup> Such a technique could depend critically on the detector sensitivity.

By combining such a determination of the cutoff wavelength with the measurement of the Petermann spot size at two different wavelengths ( $\lambda_1$  and  $\lambda_2$ ) in the single-mode region, i.e.  $\lambda_1 \neq \lambda_2 \geq \lambda_{co}$ , the three necessary parameters for specifying the EESI are obtained as follows:

$$\bar{V}_{1,2} = 2.405\lambda_{co}/\lambda_{1,2}\quad (4a)$$

$$|\Delta\bar{\Omega}_4| = (p_1 - Rp_2)/(Rq_2 - q_1)\quad (4b)$$

$$\begin{aligned}a_c^2 &= \omega_1^2(p_1 + |\Delta\bar{\Omega}_4|q_1) \\ &= \omega_2^2(p_2 + |\Delta\bar{\Omega}_4|q_2)\end{aligned}\quad (4c)$$

where  $R$  is the ratio of the two spot-size measurements ( $\omega_2^2/\omega_1^2$ ),  $p_{1,2}$  and  $q_{1,2}$  are the values of  $p$  and  $q$  of eqn. 2 evaluated at  $\bar{V}_{1,2}$ , and  $\omega_{1,2}$  are the measured values of the near-field spot size at  $\lambda_{1,2}$ .

If we use the far-field RMS width measurements at the two different wavelengths, then the parameters  $\bar{V}_{1,2}$  and  $|\Delta\bar{\Omega}_4|$  are still given by eqns. 4a and b, while the equivalent core radius is now

$$\begin{aligned}a_c &= \frac{2}{\omega_{ff1}^2} (p_1 + |\Delta\bar{\Omega}_4|q_1) \\ &= \frac{2}{\omega_{ff2}^2} (p_2 + |\Delta\bar{\Omega}_4|q_2)\end{aligned}\quad (4d)$$

while  $R$  in eqn. 4b is now defined as  $(\omega_{ff1}^2/\omega_{ff2}^2)$ .

The implementation of this procedure is aided by the fact that functions  $\bar{\omega}_{st}(\bar{V})$ ,  $f(\bar{V})$ ,  $b_{st}(\bar{V})$  of eqn. 1 can be expressed in

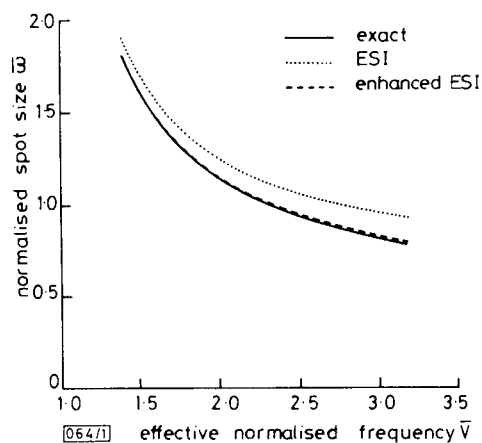


Fig. 1 Spot size  $\omega$  against  $\bar{V}$  for triangular-profile fibre

the approximate forms:<sup>8</sup>

$$\omega_{st}(\bar{V}) = 0.65 + 1.619\bar{V}^{-3/2} + 2.879\bar{V}^{-6} - (0.16 + 1.561\bar{V}^{-7}) \quad (5)$$

$$f(\bar{V}) = 0.313\bar{V} - 0.013\bar{V}^2 \quad (6)$$

$$b_{st}(\bar{V}) = (1.1428\bar{V} - 0.996)^2/\bar{V}^2 \quad (7)$$

*Conclusions:* The novelty of our procedure is that we introduce an extra parameter  $|\Delta\bar{\Omega}_4|$  which gives us additional accuracy in predicting propagation characteristics. Having determined the size of  $|\Delta\bar{\Omega}_4|$ , we can then decide on whether to use the three-parameter EESI or the two-parameter ESI, but now with *a priori* knowledge of the errors involved.

Our EESI approximation is perhaps unique in the sense that the three parameters can also be arrived at independently from measurements of the refractive index of the fibre or the preform.<sup>9</sup>

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