APPROXIMATE ANALYTIC FORMS FOR THE PROPAGATION CHARACTERISTICS OF SINGLE-MODE OPTICAL FIBRES

Indexing terms: Optical fibres, Single-mode fibres

An approximate expression for the Petermann spot size is derived in terms of an extended form of the Marcuse spot size expression. When used in conjunction with the Rudolph–Neumann approximation it yields accurate forms for the propagation characteristics on single-mode fibres.

Introduction: The Rudolph–Neumann approximation has been established as being the simplest and most accurate expression for the modal propagation constant on step index single-mode fibres.[1,2] Similarly, the Marcuse spot size formula has proved useful in determining microbending losses and those joint losses which are due to small tilts.[3]

There exists, however, no approximate formula for the Petermann spot size (i.e. the inverse of the far-field RMS width),[4] which is required for determining joint losses due to small lateral shifts,[5] for waveguide and total dispersion[6] and for determining the proportion of power propagating in the core.[6]

In this letter we present an approximate form for this Petermann spot size which was derived by adding only two extra terms to the Marcuse spot size expression. We have adopted this strategy from the observation that 'all spot sizes look the same'; we have shown that a simple extension of the tried and trusted Marcuse formula proves adequate.

Spot size and eigenvalue: The exact analytical formula for the normalised Petermann spot size, $\bar{\omega}_P$, $(2/W^2)J_1^2(U)/J_0^2(U)$ can be expressed approximately as follows:

$$\bar{\omega}_P = \bar{\omega}_M - (0{\cdot}016 + 1{\cdot}561V^{-7}) \tag{1}$$

where $\bar{\omega}_M$ is the Marcuse formula for the normalised spot size of an optimally exciting Gaussian beam, and is given by[3]

$$\bar{\omega}_M = 0{\cdot}650 + 1{\cdot}619V^{-3/2} + 2{\cdot}879V^{-6} \tag{2}$$

For our purposes we have optimised eqn. 1 to be accurate to within 1% in the range $1{\cdot}5 \leq V \leq 2{\cdot}5$, since this is the range of most practical interest in single-mode-fibre transmission. Like the Marcuse formula, eqn. 1 was determined empirically. Increased accuracy over a more extended range would require more additional terms and would be of questionable advantage.

Fig. 1 shows $\bar{\omega}_M$ and both the exact and approximate curves for $\bar{\omega}_P$; as mentioned above, both curves 'look the same'. The very good accuracy of eqn. 1 for larger V values ($V > 2{\cdot}5$) and the good qualitative behaviour for small V values ($V < 1{\cdot}5$) are additional bonuses.

For completeness and easy reference we also include here the Rudolph–Neumann approximation for the modal eigenvalue, which is given by

$$W = 1{\cdot}1428V - 0{\cdot}996 \qquad 1{\cdot}5 \leq V \leq 2{\cdot}5 \tag{3}$$

This is accurate to within 0·1% for our chosen range.[1]

Other propagation characteristics: The dispersion parameters b, b_1, b_2 and the fraction of power propagating in the core η, where $b = W^2/V^2$, $b_1 = d(Vb)/dV$, $b_2 = V\,db_1/dV$ and $\eta = \frac{1}{2}(b_1 + b)$ have been shown to be intimately related to the Petermann spot size $\bar{\omega}_P$,[6] namely

$$b_2 = 4\frac{d}{dV}\left(\frac{1}{V\bar{\omega}_P^2}\right) \tag{4a}$$

$$b_1 = \frac{4}{V^2\bar{\omega}_P^2} + b \tag{4b}$$

$$\eta = \frac{2}{V^2\bar{\omega}_P^2} + b \tag{4c}$$

We propose to use eqns. 1 and 3 in eqns. 4a–c, thereby eliminating the need for numerical methods in evaluating the eigenvalue or Bessel function terms.

The accuracy of this approach is illustrated in Fig. 2 for our four parameters b, b_1, b_2 and η. The curve for b is simply the well known Rudolph–Neumann approximation. Of remarkable accuracy are the curves for b_1 and η, which are within 1% for our chosen range and do not deteriorate appreciably beyond this range. For low V values we are clearly obtaining a compensating effect of a poor eigenvalue combined with a poor spot size to give good results. The parameter b_2 has always been the 'plague' of any approximation since it relies on derivatives for its determination. In this case, however, the result is clearly impressive; the accuracy at $V = 1{\cdot}5$ is 1·6% and is well within this over most of the range ($1{\cdot}5 \leq V \leq 2{\cdot}5$). The percentage accuracy at large V values becomes meaningless since the function is approaching zero; however, a good indication of its validity is that the approximation predicts the zero dispersion point ($V_{zd} \simeq 3$) to within 2%.

Conclusion: We have provided a very accurate approximation formula for the Petermann spot size by providing a simple extension to the Marcuse formula.

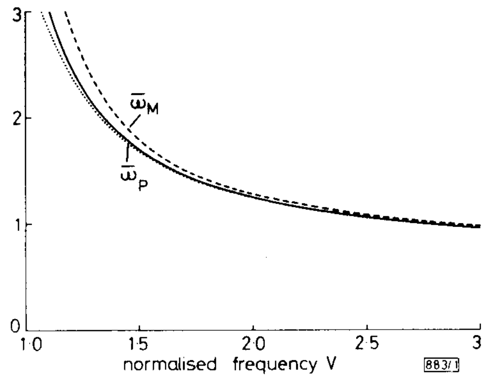

Fig. 1 *Petermann spot size $\bar{\omega}_P$ and Marcuse spot size $\bar{\omega}_M$ plotted as a function of normalised frequency V*

——— Petermann (exact)
······ Petermann (approximate)
– – – – Marcuse

This new formula allows us to determine the dispersion parameters b_2, b_1 and the relative core power with good accuracy when used in conjunction with the Rudolph–Neumann approximation. A complete accurate approximate description of the step index single-mode fibre is now available and will find application for fibres of arbitrary refractive index through the ESI and the EESI approximations.[7]

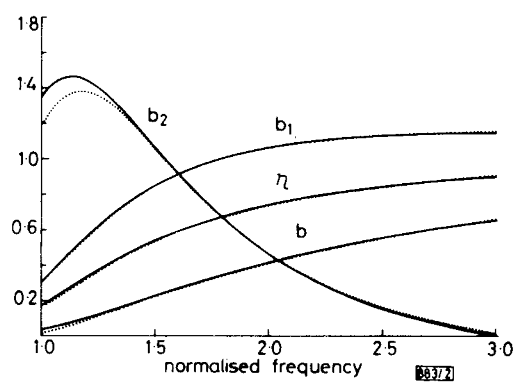

Fig. 2 *Approximate (broken) and exact (solid) dispersion and core power curves plotted as functions of normalised frequency V*

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