Numerical simulation of a modelocked laser with an intracavity nonlinear mirror based on second harmonic generation

J.R.M. Barr [1]

Department of Physics, Southampton University, Southampton SO9 5NH, UK

Received 12 July 1990

A simple computer model of modelocking using an intracavity nonlinear mirror based on second harmonic generation has been constructed. Saturation of the gain medium is included. The model exhibits experimentally observed features such as a second harmonic conversion efficiency threshold before modelocking occurs. Self Q-switching and modelocking occurs for sufficiently large values of fluorescence lifetime. The model predicts that there is a narrow region above the modelocking threshold which allows steady state modelocking to occur even in long fluorescence lifetime materials.

1. Introduction

Experimental results on modelocking Nd:YAG lasers using second harmonic generation to construct a nonlinear mirror have shown a number of features which need explanation [1–4]. In the first experimental demonstration of intracavity SHG modelocking a Nd:YAG pulsed laser was used to generate a Q-switched and modelocked pulse train with typical pulse durations of 100 ps measured at the second harmonic wavelength [2]. The single pass conversion efficiency was shown to be less than 1%. A subsequent experiment used SHG modelocking in a coupled cavity configuration to generate pulses as short as 30 ps at the fundamental wavelength [3]. The threshold in this case was estimated to be $\sim 0.2\%$. The Nd:YAG laser used in this experiment was a long pulse system with a 5 ms pulse duration. The laser system effectively performed as a continuous wave laser once the initial relaxation oscillations disappeared. This experiment showed that the modelocking process also initiated relaxation oscillations. These are due to the increase in reflectivity of the nonlinear mirror as the pulse forms which leads to a net round trip gain in materials with long fluorescence lifetimes. The computer model was designed to investigate in detail the origin of the threshold SHG level and the cause of the experimentally observed Q-switching.

Lasers with a nonlinear mirror based on second harmonic generation have consistently shown that they can evolve from noise without the intervention of an active form of modelocking [2,3]. The injection of a small signal fluctuating in amplitude was studied using the model to mimic spontaneous emission and demonstrated that a pulse can evolve from a very small perturbation to the laser. A laser containing a nonlinear mirror with a single pass conversion efficiency larger than the threshold level is very sensitive to small perturbations and is very unlikely to maintain a continuous wave steady state and will instead evolve to a modelocked condition.

The model involves a number of parameters which can be varied: fluorescence lifetime T_f; the magnitude of the conversion efficiency; the filter linewidth; as well as the level of the background spontaneous emission. It is a simple matter to use the computer model to predict the performance of a given laser material. However, it should be noted that there is no experimental evidence to confirm the validity of the model's predictions for short fluorescence lifetimes, unless the results of passive modelocking of dye lasers using saturable absorbers where a modelocked steady state occurs are an acceptable analogy.

[1] Also with the Opto-electronic Research Centre, Southampton University, Southampton SO9 5NH, UK.

2. The model

The model is based on the model used by New and Catherall [5] in their work on synchronously modelocked dye lasers and has also been applied to actively modelocked lasers. The ring laser is divided into a gain segment, a nonlinear segment, and a filter segment, which are sequentially applied to an array containing the magnitude of the electric field. Each element of the laser, shown schematically in fig. 1, will be described in turn.

2.1. The gain medium

For simplicity the gain segment has been separated from the spectral filtering associated with real materials. The gain element is assumed to be contained within a thin sheet and spatial saturation neglected. The gain element has been chosen to be homogeneously broadened, which simplifies the model, but still allows the conclusions to apply to important laser materials such as Nd:YAG. Each field element V_n^k is amplified and gain depletion, spontaneous emission and repumping calculated before the next element is processed:

$$V_n^{k+1} = V_n^k \exp(g_n/2) , \quad (1)$$

$$g_{n+1} = g_n - I_n^k[\exp(g_n) - 1]t_m/T_f - g_n t_m/T_f + \lambda t_m . \quad (2)$$

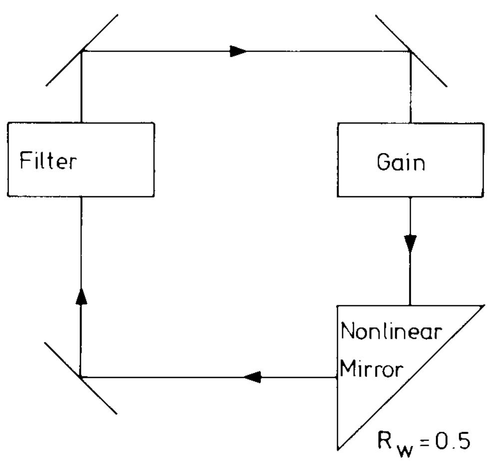

Fig. 1. The schematic diagram of the ring laser used in the computer model.

Here λ is the repumping rate, t_m is the time separation between each field element V_n^k, g_n is the intensity gain coefficient experience by field element V_n^k. The superscript k refers to the pass number while the subscript n refers to position in the array representing the cavity length. The normalised intensity is related to the field element via

$$I_n^k = \tfrac{1}{2}(\epsilon_0 c/I_s)(V_n^k)^2 , \quad (3)$$

where I_s is the saturation intensity in the gain medium. Since the gain is recalculated for each element this model could include pulse reshaping through gain saturation.

The pumping rate above threshold, r, is derived in the absence of the nonlinear mirror as

$$r = \frac{\lambda T_f}{\ln(1/R_\omega)} . \quad (4)$$

This quantity will be shown to play a role in the response of the laser to a transient perturbation.

2.2. The nonlinear element

The nonlinear element used in these calculations was based on the nonlinear mirror constructed by a second harmonic crystal and a retro reflecting mirror [1]. The reflectivity of this device can, under the correct conditions, increase with increasing intensity. Clearly this will result in the laser passively modelocking. The equations governing the nonlinear mirror are

$$V_n^{k+1} = V_n^k \sqrt{R_\omega} \exp[\alpha \tanh^2(\sqrt{\eta_0} V_n^k)/2] . \quad (5)$$

This is a simple approximation to the exact form of the nonlinear reflectivity, α is a fitting parameter which depends on the fundamental intensity reflectivity of the retro reflecting mirror R_ω [6]. The mirror reflectivity for the second harmonic is assumed to be unity. η_0 is related to the steady state continuous wave conversion efficiency η_{ss} of the laser. This quantity is accessible to direct experimental measurement and includes the nonlinear coefficient, the crystal length and other parameters relevant to second harmonic generation, η_0 is then given by

$$\eta_0 = \eta_{ss}/V_{ss}^2 \quad (6)$$

where V_{ss} is the field amplitude under steady state conditions. Experimentally η_{ss} is known to be in the region of 10^{-2} to 10^{-3} [2,3].

An estimate of the steady state pulse duration may be made using the approximations of ref. [6]. These assume that the pulse is gaussian in shape and the pulse bandwidth is sufficiently smaller than the filter bandwidth that a gaussian expansion may be made about line centre. It is clear that the response of the nonlinear mirror will in general produce a non-gaussian pulse. This feature is ignored and the curvature near the peak of the pulse which is quadratic in time is taken to be characteristic of the length of a gaussian pulse. The final result is

$$\tau=\frac{2}{\pi}\sqrt{2\ln(2)}\left(\frac{1+\alpha\eta_{p}}{\alpha\eta_{p}}\right)^{1/2}\frac{1}{f_{f}}, \quad (7)$$

where η_p is the peak conversion efficiency of the pulse and f_f is the fwhm of the filter discussed in section 2.3. This result is valid if the laser reaches steady state. In practice in lasers with long fluorescence lifetimes, e.g. Nd:YAG, the nonlinearity tends to drive relaxation oscillations resulting in self Q-switching and termination of the modelocking process.

At present the peak conversion efficiency cannot be calculated analytically. The exact value depends on the interaction between the nonlinear mirror and the saturation parameter of the gain medium. In the case where modelocking does not occur a continuous wave steady state may be calculated by iterating the following equations:

$$I_{ss}=\frac{\lambda T_{f}-g_{ss}}{\exp(g_{ss})-1}, \quad (8)$$

$$\exp(g_{ss})=\frac{1}{R_{\omega}\exp[\alpha\tanh^{2}(\sqrt{\eta_{ss}\exp(g_{ss})})]}. \quad (9)$$

This pair of equations is used to provide the starting conditions for the computer model. A small perturbation is added and the evolution of the system monitored. One test of the computer model was to initialise with the steady state conditions from eqs. (8) and (9) and check that no change in the intensity occurred over the number of iterations, typically up to 3000, used in the following calculations.

2.3. The filter

The filter is assumed to be a Fabry-Perot etalon with a plate separation equal to the mesh spacing t_m and an intensity reflection coefficient for each plate of R. In this approximation the equation characterising the filter transmission is

$$V_{n+1}^{k+1}=(1-R)V_{n}^{k}+RV_{n}^{k+1}. \quad (10)$$

For the purpose of this paper V_n^k has been chosen to be real to simplify the computation. This choice may be justified since the nonlinear mirror does not shift the frequency of incident light.

This filter has a delay time associated with it. For gaussian pulses, for example, this delay time may be calculated using a gaussian approximation to the filter pass band as discussed in ref. [7]. The delay time is

$$t_{f}=\left(\frac{R}{1-R}\right)t_{m}. \quad (11)$$

In the same approximation the filter linewidth is

$$f_{f}=\frac{1}{2\pi}\left(\frac{8}{R}\right)^{1/2}(1-R)\frac{1}{t_{m}}. \quad (12)$$

The circulating field is resonant with the filter so the model cannot give any information on off resonant operation of the laser as has been recently discussed with regard to amplitude modulation of modelocked lasers [8].

3. Discussion

Table 1 displays all the parameter values used in the following discussion. The computer model was

Table 1
The values of the parameters used in the various calculations.

Symbol	Value	Definition
t_m	10^{-3}	mesh size (units of T_{cav})
N	1000	number of elements V_n in cavity
T_{cav}	1	cavity round trip time
T_f	25–3200	fluorescence lifetime (units of T_{cav})
η_{ss}	10^{-3}–10^{-2}	steady state conversion efficiency
R	0.95	etalon mirror reflectivity
t_f	19 t_m	filter delay time for gaussian pulses
f_f	0.0231 $(t_m)^{-1}$	filter linewidth in gaussian approximation
α	0.6	fitting parameter for nonlinear mirror
R_ω	0.5	mirror reflectivity
r	2 or 8	above threshold parameter

in all cases started from initial conditions obtained using eqs. (8) and (9). First the steady state intensity in the absence of the nonlinear mirror is calculated and is given by

$$I_{ss}=\frac{r-1}{1-R_{\omega}}R_{\omega}\ln(1/R_{\omega})\,. \tag{13}$$

For the values used in the model $I_{ss}=0.693$ when $r=2$ and $I_{ss}=4.852$ for $r=8$. This intensity is assigned the steady state conversion efficiency η_{ss}. Next eqs. (8) and (9) are iterated using as starting conditions the values calculated above. This yields the steady state intensity in the presence of the nonlinear mirror. The intensity is slightly larger than the values calculated using eq. (13) since the effect of the nonlinear mirror is to increase the effective reflectivity of the output coupler. For example for $\eta_{ss}=0.01$ and $r=2$ the result is that $I_{ss}=0.723$, while for $r=8$ $I_{ss}=4.983$.

The steady state values of intensity were used to initiate the computer model. The first task was to investigate the evolution of the model as the system was perturbed from steady state. A small perturbation was included with one of the steady state values. As the model was iterated a quantity I_p/E was determined for each pass. I_p is the largest value of intensity in the array V_n^k while E is the total energy per unit area within the array. As the pulse develops I_p/E increases. It can be shown that for a gaussian pulse $I_p/E=939/\tau$ where τ is the fwhm pulse duration measured in units of t_m. This quantity is a useful measure of pulse duration particularly during the early stages of pulse development where the 'pulse' is longer than the cavity length and measures such as fwhm or rms variations are poorly defined. Once the pulse was well developed the usual measures of duration could be applied.

The results of this procedure for $r=2$ and $T_f=25$ are displayed in fig. 2. The steady state values of I_p/E which are related to pulse duration are plotted against η_{ss} which is varied from $\eta_{ss}=10^{-3}$ to 1.2×10^{-2}. It is clearly shown that there is a threshold in steady state conversion efficiency of 6×10^{-3} in order to allow modelocking to evolve. For the particular values of T_f chosen here, a long term steady state where the laser is modelocked is established. For $\eta_{ss}=0.01$ the peak intensity is $I_p=3.096$ corresponding to a peak conversion efficiency of $\eta_p=0.046$. The analytic approximation of eq. (7) would predict that the pulse duration would be 198 t_m whereas the duration obtained from the computer model was $219t_m$. The threshold conversion efficiency was also determined for $T_f=200$ and $T_f=3200$. In each case the threshold was found to be 6×10^{-3} and within the validity of the model there was no dependence of threshold on fluorescence lifetime. The existence of a threshold has been observed experimentally for both the intracavity case [2] and the coupled cavity case [3]. The experimental values were 0.01 and 2×10^{-3} respectively and agree remarkably well with the values obtained here.

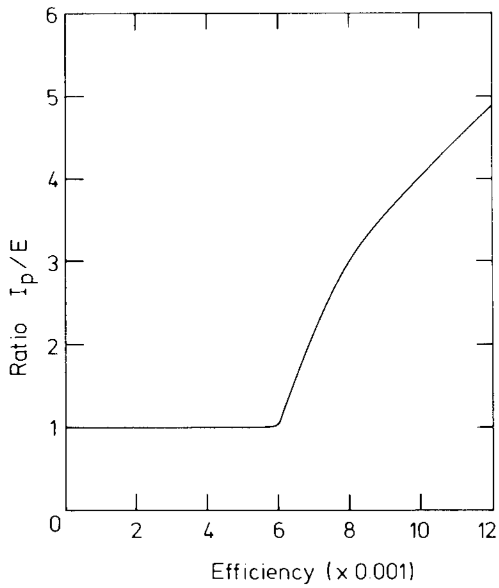

Fig. 2. The ratio of final intensity to total energy as a function of conversion efficiency. The fluorescence lifetime was $T_f=25$ and the pump rate $r=2$. In this case the laser reached steady state pulse generation.

The dependence of the evolution of the computer model on fluorescence lifetime was also studied. The results show that for a fluorescence lifetime shorter than a critical value the model reached a steady state in terms of pulse duration and pulse energy. For values of fluorescence lifetime longer than this critical value the evolution was unstable and resulted in Q-switching of the laser. The data are shown graphically in fig. 3, where I_p/E is plotted as a function of round trip number for a variety of values of T_f. In all cases $\eta_{ss}=0.01$.

The critical fluorescence lifetime T_{fc} is given by

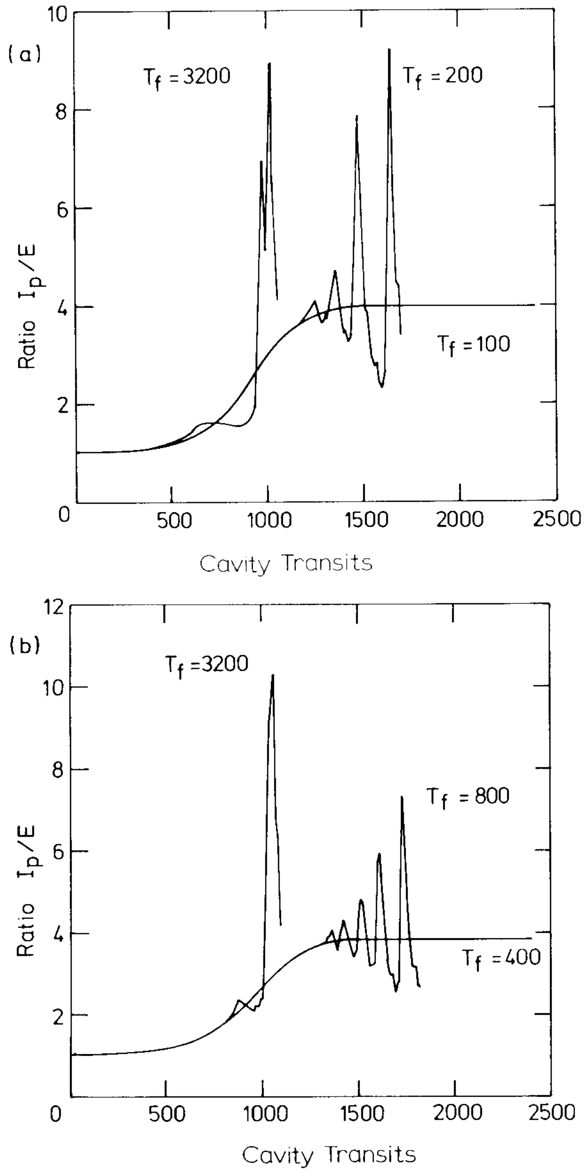

Fig. 3. The evolution of the laser pulse duration as measured using the ratio I_p/E for different pumping rates (a) $r=2$ and (b) $r=8$. Each figure shows the evolution for two values of fluorescence lifetime which bracket the critical fluorescence lifetime and also for a value of $T_f=3200$.

$100<T_{fc}\leqslant 200$ for $r=2$ and by $400<T_{fc}\leqslant 800$ for $r=8$. The result predicts that the critical fluorescence lifetime is directly proportional to the pumping rate r as would be expected if the origin of the Q-switching were to be explained by relaxation oscillations. For example, using the linearised theory of relaxation oscillations and using as a criterium for the critical fluorescence lifetime, that the damping rate should equal the relaxation oscillation period, it is possible to show that

$$T_{fc} \propto r^2/(r-1)\ . \tag{14}$$

A further interesting feature was noted. For $T_f=25$ and $T_f=3200$ the laser either evolved to a mode-locked steady state or Q-switched when the conversion efficiency was above threshold. However, for $T_f=200$ a more complex evolution was observed. For $\eta_{ss}=6\times10^{-3}$ (threshold) and $\eta_{ss}=7\times10^{-3}$ the laser evolved to a steady state without Q-switching. Larger values of η_{ss} lead to Q-switching similar to that shown in fig. 3. The explanation for this feature is presumably related to the interplay of gain saturation and pulse formation. Provided that the pulse formation and the resultant change in reflectivity of the nonlinear mirror is sufficiently slow, a long fluorescence lifetime material may remain in equilibrium and not undergo dramatic Q-switching. In support of this explanation it should be noted that for $\eta_{ss}=7\times10^{-3}$ the model was iterated for 10000 round trips before steady state was reached. This value is significantly longer than the fluorescence lifetime used in the calculation ($T_f=200$). The narrow extent of the stable region would limit the usefulness of this effect experimentally. No sign of this stable region was observed for $T_f=3200$ at the increment of η_{ss} of 10^{-3} used in the calculation.

The results of the computer model satisfactorily explain the self Q-switching observed experimentally in Nd:YAG [3] as being directly attributable to the long fluorescence lifetime of that material which causes relaxation oscillations. For a material which has a fluorescence lifetime close to the critical fluorescence lifetime T_{fc} stable operation is likely to occur at higher pump rates (larger values of r).

When the model exhibits Q-switching very short pulses result. This can be explained by the dramatic increase in pulse energy and intensity which causes increased pulse shortening in the nonlinear mirror. For example for $T_f=3200$ and $r=2$ the peak value of I_0/E is 8.9 and occurs at iteration 1010. The predicted pulse duration is 107 t_m, which compares favourably with the measured value of 104 t_m. However, the shortest pulses appear at the end of the Q-

switched envelope which peaked at iteration 970 and had a duration of ~20 iterations. This observation agrees with experimental results [9] where slightly shorter pulses were observed in the second half of the Q-switched pulse train. Note that the pulses are still more than a factor of 2 longer than the inverse of the filter linewidth ($f_f^{-1} = 43.3\ t_m$) indicating that shorter pulses might be obtained if the Q-switch duration could be extended.

Finally the question of the size of perturbation used to initiate the pulse formation has to be addressed. For $\eta_{ss} = 0.01$ and $r = 2$ a single element of the array V_n^0 was modified according to

$$V_n^0 = V_n^0 (1+\beta) . \tag{15}$$

During the course of this investigation β was varied from $\beta = 10^{-4}$ to $\beta = 10$. The lower figure corresponds to a perturbation similar in size to spontaneous emission within a typical laser cavity. In all cases the model evolved away from the initial steady conditions towards modelocked operation unless the conversion efficiency was below threshold. The long term evolution was controlled by the value of T_f as described earlier. The importance of the observation that fluctuations of the size of the spontaneous emission level cause the laser to self modelock is that steady state operation with a nonlinear mirror and a conversion efficiency larger than the threshold level is impossible. The main effect of varying the size of the perturbation was to alter the time taken for the system to reach equilibrium or to Q-switch.

4. Conclusion

A simple computer model which replicates the features of experiments based on modelocking with intracavity nonlinear mirrors is described. The computer model shows that there is a threshold second harmonic generation efficiency below which no modelocking occurs. Presumably below threshold a perturbation to steady state is strongly damped through spectral filtering and gain saturation. Above the threshold it is shown that a very small perturbation comparable in size to spontaneous emission readily causes the laser to self modelock. The threshold predicted by the computer model is in broad agreement with experimental values.

The self Q-switching experimentally observed in Nd:YAG lasers could be induced in the model. There is a critical value of fluorescence lifetime T_{fc}, which divides laser materials which allow a modelocked steady state to be reached, $T_f < T_{fc}$, from those materials which exhibit self Q-switching, $T_f > T_{fc}$. The value of T_{fc} may be altered experimentally by changing the pump rate to threshold pump rate ratio r. The self Q-switching is explained as a result of weakly damped relaxation oscillations. For fluorescence lifetimes slightly larger than the critical value the model predicts that for a restricted range of conversion efficiencies at and above the threshold a steady state modelocked laser output can be sustained. This region is however small and larger values of conversion efficiency lead to Q-switching as expected. The stable region is tentatively explained by the change in nonlinear mirror reflectivity taking place on a timescale longer than the fluorescence lifetime so that relaxation oscillations do not occur.

The objective of this work was to explain experimentally observed features such as the threshold effect and the existence of Q-switching. Future experimental work could examine the other predictions of the model; for example, the existence of a critical fluorescence lifetime below which steady state modelocking occurs. Suitable laser systems for this work include the Ti:sapphire laser where relaxation oscillations are strongly damped. The computer model could be adapted to include the coupled cavity situation. Simple analytical results for this case have already been obtained [4].

Acknowledgement

The author gratefully acknowledges partial support for this work from the Nuffield Foundation and from the Department of Physics, Southampton University.

References

[1] K.A. Stankov, Appl. Phys. B 45 (1988) 191.
[2] K.A. Stankov and J. Jethwa, Optics Comm. 66 (1988) 41.
[3] J.R.M. Barr and D.W. Hughes, Appl. Phys. B 49 (1989) 328.
[4] J.R.M. Barr and D.W. Hughes, J. Mod. Optics 4 (1990) 447.

[5] J.M. Catherall and G.H.C. New, IEEE J. Quant. Electron. QE-22 (1986) 1593.
[6] J.R.M. Barr, Optics Comm. 70 (1989) 229.
[7] D.A. Kuizenga and A.E. Siegman, J. Quant Electron. QE-6 (1970) 694.
[8] M. Morin, M. Piché and R. Tremblay, Optics Comm. 68 (1988) 213.
[9] K.A. Stankov, Optics Lett. 14 (1989) 359.