ENHANCED ESI FOR PREDICTION OF WAVEGUIDE DISPERSION IN SINGLE-MODE OPTICAL FIBRES

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We provide an enhanced equivalent step index (EESI) approximation for the accurate prediction of waveguide dispersion on monomode optical fibres. Using a quadratic 'enhancement function' the waveguide dispersion even for the triangular core profile fibre can be predicted to be between 92 and 100% of its exact value in the single-mode region.

Introduction: The accurate prediction of waveguide dispersion in single-mode fibres is the severest test to which any model or approximation can be subjected. The ESI which has proved to be the most useful model for predicting other fibre characteristics has been criticised for its failure to predict accurately waveguide dispersion for all monomode fibres.[1] As a consequence, other models, particularly those which are based on spot-size measurements, are currently receiving active consideration.[2-4]

In this letter we seek to reinstate the ESI by showing that it can be readily enhanced for the accurate prediction of waveguide dispersion. We propose therefore to keep the ESI for those parameters for which it is useful and to use an 'enhanced' ESI (EESI) for the dispersion parameters.

The model we use is based on the moments of the refractive index shape function and is much easier to use than a similar model proposed in Reference 5.

We use the power-law profiles as a test of our model because these profiles are reasonably familiar and also because the triangular core profile has recently been proposed as being very suitable for shifting the zero of total dispersion to higher wavelengths.[6]

ESI approximation: The ESI which is based on the moments requires that the first two even moments $(\Omega_0, \bar{\Omega}_2)$ of both the actual profile and the equivalent step profile be equal. This condition renders an equivalent step with a core radius of

$$\rho_e = \sqrt{(2\bar{\Omega}_2)} \qquad (1a)$$

and a profile height of

$$h_e = \Omega_0/\bar{\Omega}_2 \qquad (1b)$$

where $\bar{\Omega}_2 = \Omega_2/\Omega_0$.

The average waveguide parameter, $\bar{V}$, is related to the usual waveguide parameter V by

$$\bar{V} = \sqrt{(2\Omega_0)}V \qquad (2)$$

Using the average waveguide parameter and the ESI defined by eqn. 1 the dispersion expressions take the following form:

$$b(V) = W^2/V^2 \simeq (\Omega_0/\bar{\Omega}_2)b_{st}(\bar{V}) \qquad (3a)$$

$$b_1(V) = d(Vb)/dV \simeq (\Omega_0/\bar{\Omega}_2)b_{1st}(\bar{V}) \qquad (3b)$$

$$b_2(V) = Vd(b_1)/dV \simeq (\Omega_0/\bar{\Omega}_2)b_{2st}(\bar{V}) \qquad (3c)$$

where $b_{st}(\bar{V})$ is evaluated for the step index fibre at $V = \bar{V}$.

Remark: Waveguide dispersion is proportional to the parameter $b_2(V)$ which relies on the second derivative of the eigenvalue W. It is for this reason that waveguide dispersion provides such a strict test on any model or approximation used in determining W.

Philosophy of method: The ESI is a two-parameter model which in our case requires the first two moments Ω_0 and $\bar{\Omega}_2$ for its specification. Any error introduced in using this ESI is caused by neglecting the difference between the higher moments of the profile Ω_m and the higher moments of ESI, $\bar{\Omega}_{mst}$. We define the normalised difference as follows:

$$\Delta\bar{\Omega}_m = (\bar{\Omega}_m - \bar{\Omega}_{mst})/\bar{\Omega}_{mst} \qquad (4)$$

where $m \geq 4$ and m is even and

$$\bar{\Omega}_{mst} = (2\bar{\Omega}_2)^{m/2}\Big/\left(\frac{m}{2}+1\right) \qquad (5)$$

In extending the model we will seek:

(i) to keep the ESI intact so that its advantages for other fibre parameters are not lost in the enhanced model

(ii) to introduce only one extra parameter i.e. $\Delta\bar{\Omega}_4$

(iii) to provide a model which is both simple to use and is readily accessible.

We therefore adopt the following functional form for the dispersion parameter $b(V)$:[5]

$$b(V) \simeq \left(\frac{\Omega_0}{\bar{\Omega}_2}\right) b_{st}(\bar{V})(1 + |\Delta\bar{\Omega}_4|\, f(\bar{V})) \qquad (6)$$

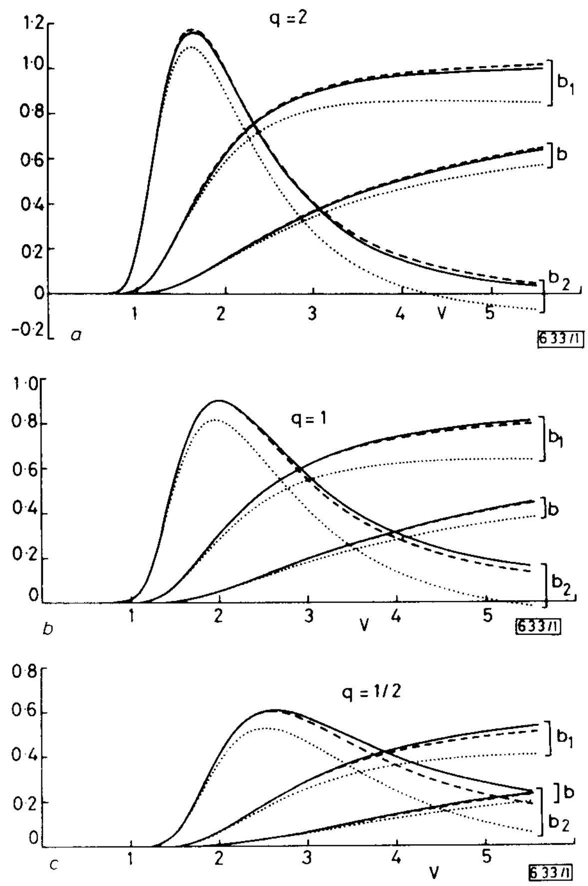

Fig. 1 *Comparison of ESI and EESI in predicting exact dispersion parameters for power-law profiles*

a $q = 2$
b $q = 1$
c $q = \frac{1}{2}$
······· ESI
- - - - EESI
——— exact

This is an enhancement of the simpler expression of eqn. 3*a*, where $f(\bar{V})$ is our 'enhancement function'. We will refer to eqn. 6 as the 'enhanced' ESI (EESI) approximation for $b(V)$. Unlike the model proposed in Reference 5, eqn. 6 holds for all fibres, i.e. $\Delta\bar{\Omega}_4$ can be positive or negative.

In line with our aims, we will seek the lowest-order polynomial as a 'best fit' approximation for $f(\bar{V})$ rather than follow rigorously the methods of Reference 5. It will then be straightforward to derive the enhanced expressions for $b_1(V)$ and $b_2(V)$.

The EESI and its prediction of dispersion: From the exact calculation of $b(V)$ for a range of profiles we found the lowest-order polynomial for $f(\bar{V})$ to be the quadratic

$$f(\bar{V}) = 0{\cdot}313\bar{V} - 0{\cdot}013\bar{V}^2 \qquad (7)$$

It is now necessary to see how this enhancement function improves the prediction of the dispersion parameters (particularly the waveguide dispersion parameter b_2) over the simple ESI. The power-law profiles with exponant q are chosen for the following examples.

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Profiles with $q \geq 2$: For profiles which range from the clad parabola ($q = 2$) to the step ($q = \infty$) index, the error in using our enhanced model of eqns. 6 and 7 in predicting the dispersion parameters is negligible in the single-mode region.

We illustrate the clad parabola case in Fig. 1*a*, where we show that, while the ESI breaks down completely for this profile, the the EESI is extremely accurate.

Profiles with $q < 2$: For these very extreme profiles we find that the EESI provides the bulk of the correction term for waveguide dispersion in the range of V-value for which the ESI breaks down.

In the case of the triangular core profile, as illustrated in Fig. 1*b*, the EESI provides an estimate of the waveguide dispersion parameter $b_2(V)$ of 92% of its exact value at the cutoff of the second mode, and improves rapidly for smaller V-values. In this case the estimates for b and b_1 are complete.

The EESI, when applied to the $q = \frac{1}{2}$ profile, provides an estimate of the waveguide dispersion of 80% at the second mode cutoff while the overall behaviour for the other parameters is similar to the triangular core case. The results for this profile are shown in Fig. 1*c*.

Table 1 VALUE OF PARAMETER $\Delta\bar{\Omega}_4$ FOR THE POWER-LAW PROFILES

q	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$
$\Delta\bar{\Omega}_4$	0.028	0.067	0.125	0.190	0.246	0·285

Discussion: We have provided a very simple enhancement to the ESI model which allows us to predict waveguide dispersion accurately even for extreme profiles like the $q = 1$ and the $q = \frac{1}{2}$ power-law profiles. The extent of the correction term is found to be directly proportional to a new parameter $\Delta\bar{\Omega}_4$. Table 1 shows that $\Delta\bar{\Omega}_4$ decreases as we approach the step-index fibre; the correction term decreases accordingly. The size of $\Delta\bar{\Omega}_4$ can therefore be used to decide whether or not the enhancement function is required. However, such a decision must also take into account the operating wavelength and, of course, the accuracy required.

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