# FIELD TO DISPERSION RELATIONSHIPS IN SINGLE-MODE FIBRES

*Indexing terms: Optical fibres, Single mode fibres*

Exact explicit expressions are obtained which relate (i) the modal spotsize and (ii) the core power to the dispersion parameters in single-mode optical fibres. Spotsize can therefore be expressed exactly in terms of the core power, with the degree of guidance playing a crucial role in the relationship.

*Spotsize:* Pask[1] has recently shown that the far-field RMS width $\omega_{ff}$ of the fundamental mode of an optical fibre can be used to provide a direct expression for the pure waveguide dispersion:

$$d\tau/d\lambda = \{\lambda/4\pi^2 cn_1\}\, d(\lambda\omega_{ff}^2)/d\lambda \tag{1}$$

Waveguide dispersion can also be expressed in terms of the dispersion parameter $b_2$:[2]

$$d\tau/d\lambda = \{-n_1\,\Delta/c\lambda\}b_2 \tag{2}$$

where $b_2 = V\,db_1/dV$, $b_1 = d(Vb)/dV$, $b = W^2/V^2$ and $W$ is the modal propagation parameter in the cladding, and the other parameters take their usual meaning.

The RMS width of the far field $\omega_{ff}$ has been shown to be inversely related to the near-field spotsize $\omega$, namely

$$\omega_{ff}^2 = (2/\omega^2) \tag{3}$$

$\omega^2$ is called the spotsize, and it provides a measure of the near-field size; however, it is not the RMS width of the near field. This last factor does not pose a problem since it is the far field that is the observable (and measurable) quantity.[1]

We will find it convenient to define a normalised near-field spotsize relative to the core radius $a$:

$$\omega_0^2 = \omega^2/a^2 \tag{4}$$

If we now use the definition for $V$,

$$V = \frac{2\pi a}{\lambda} n_1\sqrt{(2\Delta)} \tag{5}$$

to eliminate common terms between eqns. 1 and 2, we obtain the following equation:

$$d\left(\frac{2}{V\omega_0^2}\right)\bigg/ dV = \tfrac{1}{2}V\,db_1/dV \tag{6}$$

where we have used the relationship between $b_2$ and $b_1$ for the RHS. The LHS can be directly integrated, and the RHS can be integrated by parts. Using the relationship between $b_1$ and $b$ we derive the following expression for the spotsize:

$$\omega_0^2 = \frac{4}{V^2(b_1 - b)} \tag{7}$$

*Core power:* For the step index profile there is a well known expression relating the fraction of power propagating in the core to the dispersion parameters $b$ and $b_1$,[3] namely

$$\eta = \tfrac{1}{2}(b_1 + b) \tag{8a}$$

For the power-law profiles of exponent $\alpha$ this expression takes the form[4]

$$\eta = \frac{1}{2}\left[\left(\frac{\alpha+2}{\alpha}\right)b_1 + \left(\frac{\alpha-2}{\alpha}\right)b\right] \tag{8b}$$

By following the strategy of Reference 4, we can obtain a more general expression in the form

$$\eta = \frac{1}{2}\left[\left(\frac{1}{2\Omega_0}\right)b_1 + \left(\frac{4\Omega_0 - 1}{2\Omega_0}\right)b\right] \tag{8c}$$

where $\Omega_0$ is the degree of guidance[5] which is the first moment of the refractive index profile shape function.[6] Clearly eqn. 8$c$ reduces to eqns. 8$a$ and $b$ since $2\Omega_0 = 1$ and $\alpha/(\alpha+2)$ for the step and power-law profiles, respectively. [Note: eqn. 8$c$ is very important in its own right. It is the first equation, to the author's knowledge, where the guidance factor provides an exact characterisation for general refractive index profiles.]

*Spotsize and core power:* If we now eliminate $b_1$ from eqn. 6 and substitute $\eta$ from eqn. 8$c$, we find that the spotsize can now be expressed by

$$\omega_0^2 = \frac{1}{V^2\Omega_0(\eta - b)} \tag{9}$$

Eqn. 9 contains a considerable amount of information. The spotsize is inversely related to the following:

(*a*) *V-value:* This is the well known uncertainty relationship between field size and normalised frequency.

(*b*) *Degree of guidance* $\Omega_0$*:* The qualitative behaviour of field size to the degree of guidance, as discussed in References 5 and 6, is now explicitly expressed.

(*c*) *Core power* $\eta$*:* This is an intuitive relationship between the field size and the proportion of field power propagating in the core.

(*d*) *Normalised propagation constant* $b = W^2/V^2$*:* The field in the cladding is $K_0(WR)$. As $W$ decreases the cladding field decays more slowly so that the field becomes more extended.

The convenient forms of eqns. 7 and 9 are clearly a result of adopting the far-field RMS width $\omega_{ff}$ for the definition of the spotsize. Since eqn. 3 contains the clear 'inverse' relation of the far field to the near field, the far-field spotsize can now be simply written as

$$\omega_{ff}^2 = \frac{V^2(b_1 - b)}{2a^2} \tag{10a}$$

$$= \frac{2V^2\Omega_0(\eta - b)}{a^2} \tag{10b}$$

Eqn. 10 contains the direct relation of the far-field width to the degree of guidance and the core power.

*Conclusion:* The recent definition of spotsize in terms of the far-field RMS width has given rise to a radical simplification of the relationships between the field characteristics and the dispersion characteristics of single-mode optical fibres. Of the four parameters, $\omega_0$, $\eta$, $b$ and $b_1$, we have shown that any two are redundant. We have also shown that the degree of guidance is necessary to establish the exact relationship between these parameters for fibres with arbitrary refractive index profiles.

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