To specify the W-fibre completely we also require the parameters: 5

\[ S = \frac{b}{a} \quad 1 < S < \infty \] (3a)

\[ R = \frac{n_2^2 - n_1^2}{n_1^2 - n_2^2} \quad -1 < R < 0 \] (3b)

Fig. 1b defines the normalised refractive-index profile \( s(\rho) \), required for evaluating the parameter \( \Omega_0 \), which gives a quantative measure of the 'effective refractive index'. It is defined as:

\[ \Omega_0 = \int_0^\infty s(\rho)\rho \, d\rho \] (4)

For the step-index W-fibre \( \Omega_0 \) can be written

\[ \Omega_0 = \frac{1}{2S^2} (1 + RS^2) \] (5)

From the limits of possible \( R \) and \( S \) values given by eqns. 3a and b, \( \Omega_0 \) has the following range:

\[ 0.5 \geq \Omega_0 \geq -0.5 \] (6)

It is clear from eqn. 5 that fibres with the same 'effective refractive index' as defined by \( \Omega_0 \) can have radically different profiles (or different values of the \( R \) and \( S \) parameters).

Modal cutoff propagation constant and the parameter \( \Omega_0 \): If \( \Omega_0 \geq 0 \) then the fundamental mode has no cutoff, i.e. \( V_{11} = 0 \); if \( \Omega_0 < 0 \) then we have a finite cutoff so that \( V_{11} > 0 \).

Fibres with positive \( \Omega_0 \) can be dealt with in terms of some singly clad step-index fibre, 2,3 so their cutoffs and range of monomode operation are well defined. In this letter we concern ourselves with those fibres which have \( \Omega_0 < 0 \).

Lower bounds on \( V_{11} \) and \( V_{01} \) and the range of monomode operation: Any W-fibre will have particular values of \( R \) and \( S \) and consequently a particular value of \( \Omega_0 \). However, for a given \( \Omega_0 \) there is a range of possible values for \( R \) and \( S \), i.e.

\[ 2\Omega_0 > R > -1 \] (7a)

while

\[ \infty > S > (1 + 2\Omega_0)^{-1/2} \] (7b)

At one of these limits \( R = 2\Omega_0 \), \( S = \infty \), and as such only one may be used to provide bounds on the possible values of \( B(V) \) and \( V_{01} \) for a particular profile with a particular value of \( \Omega_0 \). The primed quantities in eqns. 8 and 9 denote these bounds.

The modal cutoffs \( (V_{01}^{\prime}) \) are given by \( B(V_{01}) = 0 \), or

\[ B_n \left( \frac{V_{01}^{\prime}}{\sqrt{(1 + 2\Omega_0)}} \right) = -2\Omega_0 \] (9)

We stress that eqns. 8 and 9 are only exact in the limit \( R = 2\Omega_0 \), \( S = \infty \), and as such only one may be used to provide bounds on the possible values of \( B(V) \) and \( V_{01}^{\prime} \) for a particular profile with a particular value of \( \Omega_0 \). The primed quantities in eqns. 8 and 9 denote these bounds.

The curves of \( V_{11}^{\prime} \) and \( V_{01}^{\prime} \), as given by eqn. 9 are plotted as a function of the effective refractive index \( \Omega_0 \) in Fig. 2. We notice two things in Fig. 2:

(a) In the extreme limit \( \Omega_0 = -0.5 \) \( (R = -1, S = \infty) \) we find the well known result that the frequency \( V_{11}^{\prime} \) for the fundamental-mode cutoff approaches 2.405 and that the fre-
frequency \( V_{01} \) for the second-mode cutoff approaches 3.832, so that the range of monomode operation can be written as

\[
V_{01} - V_{11} = 1.427
\]  

(10)

![Fig. 2 Plots of \( V_{01} \) and \( V_{11} \) as functions of \( \Omega_0 \)]

(h) Furthermore we notice that the curves for \( V_{01} \) and \( V_{11} \) remain remarkably parallel over the complete range of \( \Omega_0 \), and only diverge when \( \Omega_0 \) is small in magnitude. This is shown very clearly in Fig. 3, where the range of monomode operation \( (V_{01} - V_{11}) \) is plotted as a function of \( \Omega_0 \). Eqn. 10 therefore applies over most of the range of \( \Omega_0 \).

![Fig. 3 Range of monomode operation \( (V_{01} - V_{11}) \) obtained from curves of Fig. 2](image)

Remark: Indeed we find by exact calculation that the curve given in Fig. 3 is accurate for all W-fibres. This means that although the absolute values of \( V_{01} \) and \( V_{11} \) do change for particular fibres with the same value of \( \Omega_0 \), the difference between them stays constant and is given by the curve in Fig. 3.

We conclude that eqn. 10 provides a good ‘working’ expression for the range of monomode operation in all W-fibres, i.e.

\[
V_{01} - V_{11} \approx 1.427
\]  

(11)

We hope that this number (1.427) will prove as useful for W-fibres as its counterpart (2.405) in singly clad fibres.

Discussion: It is interesting that the range of monomode operation is virtually constant for all W-fibres. However, in practice the knowledge of this range is useful only if we know either \( V_{11} \) or \( V_{01} \) exactly.

Experimentally the second-mode cutoff frequency \( V_{01} \) is easier to determine than the fundamental-mode cutoff frequency \( V_{11} \). Having an expression for the range of monomode operation requires only that the simpler experiment be performed.

We are seeking to develop the theory presented here to provide more accurate expressions for \( B(V) \) and \( V_{01} \) by including the second moment \( \Omega_2 \) (keeping in mind the usefulness of the parameters \( \Omega_0 \) and \( \Omega_2 \) in describing singly clad fibres\(^5\)). We hope that, by using these more general parameters, these methods can be readily applied to W-fibres with arbitrary refractive-index profile.

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13th March 1984

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