189

EFFECTIVE REFRACTIVE INDEX AND RANGE OF MONOMODE OPERATION FOR W-FIBRES

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Through the concept of 'the effective refractive index' of a step-index W-fibre, we can define, in a very simple manner, lower bounds on the normalised cutoff frequencies of the fundamental and second modes of that particular fibre. This allows us to specify the 'normalised range of monomode operation' which for most W-fibres has a constant value of 1·427.

Introduction: The normalised modal cutoff frequency and the range of monomode operation in optical fibres is of great practical importance. For the singly clad fibre having an arbitrary refractive-index profile, monomode operation ranges from zero to the cutoff frequency of the second (TE_{01}) mode which can be written as $\bar{V}_{01} = 2{\cdot}405$, where $\bar{V}$ is the effective waveguide parameter.[1]

In the W-fibre, however, the range of monomode operation has so far eluded a simple determination for two reasons. The first is that the fundamental (HE_{11}) mode can have a nonzero cutoff frequency V_{11}, and secondly the cutoff frequency V_{01} of the second (TE_{01}) mode cannot be simply expressed.[2-4]

It is the purpose of this letter to propose a new parameter (denoted by Ω_0), which is a measure of the 'effective refractive index' and provides a simpler description of the behaviour of W-fibres.

The effective refractive index had previously only been used to determine whether or not the fundamental mode is cut off. However, we show that this new parameter allows us to define the lower bounds on the values of both V_{11} and V_{01}, for a given W-fibre, in a very simple manner. The range of monomode operation (i.e. the entity $V_{01} - V_{11}$) is then easily determined.

Throughout, we adopt Monerie's notation.[6,7]

Notation: Fig. 1*a* defines the refractive-index profile of a step-index W-fibre. The normalised frequency is taken as

$$V = k_0 a(n_1^2 - n^2)^{1/2} \tag{1}$$

and the normalised propagation constants are given by

$$B = (\beta^2 - k_0^2 n^2)/k_0^2(n_1^2 - n^2) \tag{2}$$

for the bound modes.

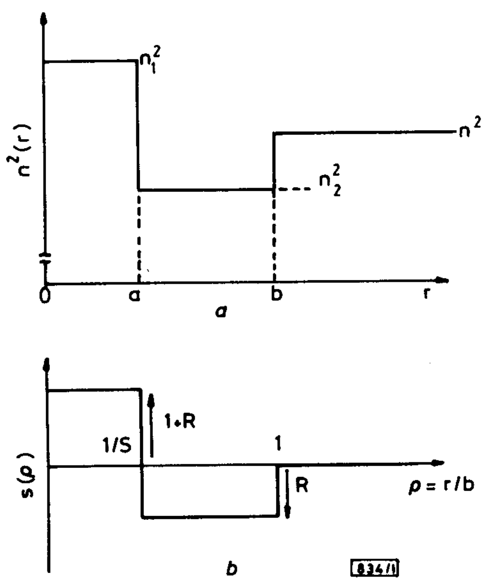

Fig. 1

a Refractive-index profile of W-fibres
b Normalised shape function $s(\rho)$ required for evaluating Ω_0

$s(\rho) = \dfrac{n^2(\rho) - n^2}{n_1^2 - n_2^2}; \quad \rho = r/b$

To specify the W-fibre completely we also require the parameters:[6]

$$S = b/a \qquad 1 < S < \infty \tag{3a}$$

$$R = \frac{n_2^2 - n^2}{n_1^2 - n_2^2} \qquad -1 < R < 0 \tag{3b}$$

Fig. 1*b* defines the normalised refractive-index profile $s(\rho)$, required for evaluating the parameter Ω_0 which gives a quantitative measure of the 'effective refractive index'. It is defined as[5]

$$\Omega_0 = \int_0^\infty s(\rho)\rho \, d\rho \tag{4}$$

For the step-index W-fibre Ω_0 can be written

$$\Omega_0 = \frac{1}{2S^2}(1 + RS^2) \tag{5}$$

From the limits of possible R and S values given by eqns. 3*a* and *b*, Ω_0 has the following range:

$$0{\cdot}5 \geq \Omega_0 \geq -0{\cdot}5 \tag{6}$$

It is clear from eqn. 5 that fibres with the same 'effective refractive index' as defined by Ω_0 can have radically different profiles (or different values of the R and S parameters).

Modal cutoff propagation constant and the parameter Ω_0: If $\Omega_0 \geq 0$ then the fundamental mode has no cutoff, i.e. $V_{11} = 0$; if $\Omega_0 < 0$ then we have a finite cutoff so that $V_{11} > 0$.[2-4]

Fibres with positive Ω_0 can be dealt with in terms of some singly clad step-index fibre,[2,3] so their cutoffs and range of monomode operation are well defined. In this letter we concern ourselves with those fibres which have $\Omega_0 < 0$.

Lower bounds on V_{11} and V_{01} and the range of monomode operation: Any W-fibre will have particular values of R and S and consequently a particular value of Ω_0. However, for a given Ω_0 there is a range of possible values for R and S, i.e.

$$2\Omega_0 > R > -1 \tag{7a}$$

while

$$\infty > S > (1 + 2\Omega_0)^{-1/2} \tag{7b}$$

At one of these limits ($R = 2\Omega_0$, $S = \infty$), the normalised propagation constants $B(V)$ for the bound modes may be written in terms of the step-index (singly clad) propagation constants:[7]

$$B'(V) = \frac{1}{1 + 2\Omega_0}\left[B_{st}\left(\frac{V}{\sqrt{(1 + 2\Omega_0)}}\right) + 2\Omega_0\right] \tag{8}$$

The modal cutoffs (V_{co}) are given by $B(V_{co}) = 0$, or

$$B_{st}\left(\frac{V'_{co}}{\sqrt{(1 + 2\Omega_0)}}\right) = -2\Omega_0 \tag{9}$$

We stress that eqns. 8 and 9 are only exact in the limit $R = 2\Omega_0$, $S = \infty$, and as such they can only be used to provide bounds on the possible values of $B(V)$ and V_{co} for a particular profile with a particular value of Ω_0. The primed quantities in eqns. 8 and 9 denote these bounds.

The curves of V'_{11} and V'_{01} as given by eqn. 9 are plotted as a function of the effective refractive index Ω_0 in Fig. 2. We notice two things in Fig. 2:

(*a*) In the extreme limit $\Omega_0 = -0{\cdot}5$ ($R = -1$, $S = \infty$) we find the well known result that the frequency V'_{11} for the fundamental-mode cutoff approaches 2·405 and that the frequency V'_{01} for the second-mode cutoff approaches 3·832,[4] so that the range of monomode operation can be written as

$$V'_{01} - V'_{11} = 1{\cdot}427 \tag{10}$$

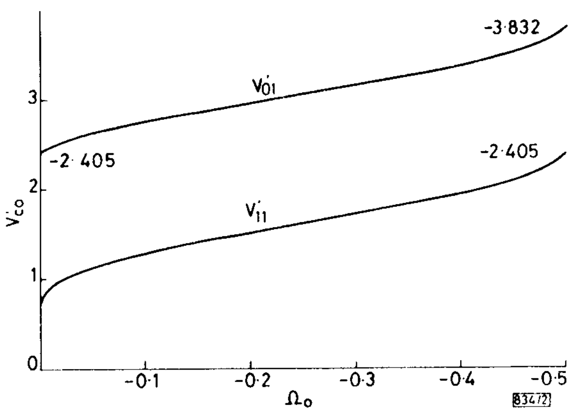

Fig. 2 *Plots of V'_{01} and V'_{11} as functions of Ω_0*

(*b*) Furthermore we notice that the curves for V'_{01} and V'_{11} remain remarkably parallel over the complete range of Ω_0, and only diverge when Ω_0 is small in magnitude. This is shown very clearly in Fig. 3, where the range of monomode operation ($V'_{01} - V'_{11}$) is plotted as a function of Ω_0. Eqn. 10 therefore applies over most of the range of Ω_0.

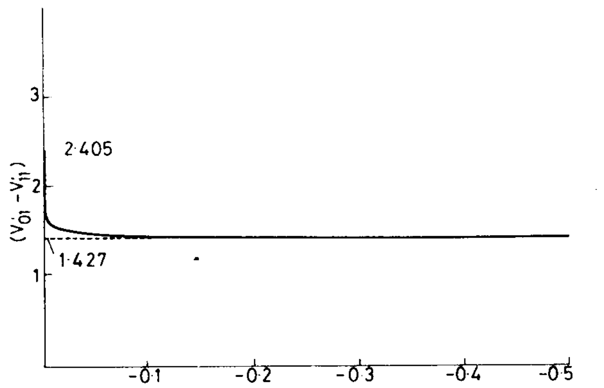

Fig. 3 *Range of monomode operation ($V'_{01} - V'_{11}$) obtained from curves of Fig. 2*

Remark: Indeed we find by exact calculation that the curve given in Fig. 3 is accurate for all W-fibres. This means that although the absolute values of V_{01} and V_{11} do change for particular fibres with the same value of Ω_0, the difference between them stays constant and is given by the curve in Fig. 3.

We conclude that eqn. 10 provides a good 'working' expression for the range of monomode operation in all W-fibres, i.e.

$$V_{01} - V_{11} \simeq 1{\cdot}427 \tag{11}$$

We hope that this number (1·427) will prove as useful for W-fibres as its counterpart (2·405) in singly clad fibres.

Discussion: It is interesting that the range of monomode operation is virtually constant for all W-fibres. However, in practice the knowledge of this range is useful only if we know either V_{11} or V_{01} exactly.

Experimentally the second-mode cutoff frequency V_{01} is easier to determine than the fundamental-mode cutoff frequency V_{11}. Having an expression for the range of monomode operation requires only that the simpler experiment be performed.

We are seeking to develop the theory presented here to provide more accurate expressions for $B(V)$ and V_{co} by including the second moment Ω_2 (keeping in mind the usefulness of the parameters Ω_0 and Ω_2 in describing singly clad fibres[5]). We hope that, by using these more general parameters, these methods can be readily applied to W-fibres with arbitrary refractive-index profile.

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