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Coupled cavity mode-locking using nonlinear amplitude modulators

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Abstract. A straightforward analysis of the process by which nonlinear amplitude modulators in coupled cavities can lead to enhanced mode-locking is presented. The application of the theory is demonstrated by using second-harmonic generation in an external cavity to mode-lock a Nd : YAG laser yielding 30 ps pulses in a Q-switched and mode-locked pulse train.

1. Introduction

Nonlinear effects in external cavities have received a lot of attention since it was demonstrated that enhanced mode-locking can occur for this configuration. The nonlinearity may be self-phase-modulation in one arm of a Michelson interferometer used as an output coupler [1, 2] or self-phase modulation in an optical fibre within a Fabry–Perot cavity [3–6]. Nonlinear amplitude modulators have also been demonstrated to cause enhanced mode-locking. A saturable diode amplifier resulted in pulse shortening of a colour-centre laser [4]. Second-harmonic generation (SHG) in an external cavity has been used to mode-lock a Nd : YAG laser [7, 8]. This latter result is of interest because it was the first demonstration of coupled cavity mode-locking which was 'self-starting' and did not need a primary active mode-locker within the main laser cavity.

A theoretical study of the process of mode-locking using nonlinear elements (saturable amplifiers and saturable absorbers) in coupled cavities predicted that enhanced mode-locking could occur and explained the process as a nonlinear coupling of the laser modes leading to shorter pulses [9]. Another analysis pictures the pulse-shortening process as interference between the pulse which has passed through the nonlinear element (in this case a nonlinear phase modulator) and the intracavity laser pulse when they are recombined on the laser output coupler. This process, called additive pulse mode-locking, will be studied in this paper and extended to the case of nonlinear amplitude modulators. The results of this analysis will be compared with an experimental investigation into second-harmonic mode-locking of a Nd : YAG laser [7].

2. Theory

The optical configuration that will be analysed is shown in figure 1. The nonlinear material is contained within a resonant reflector matched in length to the laser cavity to length detunings $\sim \lambda$. This ensures that the round trip time of a pulse within the laser cavity matches the round trip time within the resonant reflector.

The starting point for the investigation of nonlinear modulators in external cavities is the coupling together of the two cavities at the laser output coupler. The

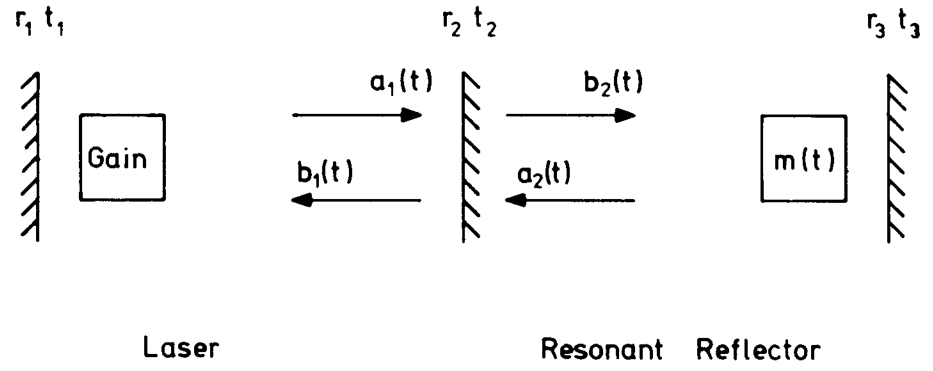

Figure 1. A schematic of the experimental arrangement showing the quantities used in the analysis. The nonlinear amplitude modulator is labelled $m(t)$.

equations are known to be [5, 9, 10]

$$\left.\begin{aligned} b_1(t) &= r_2 a_1(t) + \mathrm{i} t_2 a_2(t), \\ b_2(t) &= \mathrm{i} t_2 a_1(t) + r_2 a_2(t), \end{aligned}\right\} \tag{1}$$

where r_2 and t_2 are the electric field amplitude reflection and transmission coefficients respectively for mirror 2. $a_1(t)$ and $a_2(t)$ are the fields incident on the mirror while $b_1(t)$ and $b_2(t)$ are the reflected fields. The subscript 1 refers to fields within the laser cavity while subscript 2 refers to fields within the coupled cavity. An analytic solution will be found by making the assumption that the fields within the cavities are in steady state. The reflection coefficient $r(t)$ from the cavity containing the nonlinear medium is obtained by noting that

$$a_2(t) = b_2(t) m(t) \exp(-\mathrm{i}\phi), \tag{2}$$

where ϕ is the resonant reflector round-trip phase change and $m(t)$ is the double-pass transmission coefficient of the nonlinear modulator. This can be used to solve equation (1) for the reflection coefficient of the resonant reflector $r(t)$ in the form

$$r(t) = \frac{b_1(t)}{a_1(t)} = \frac{r_2 - r_3 m(t) \exp(-\mathrm{i}\phi)}{1 - r_2 r_3 m(t) \exp(-\mathrm{i}\phi)}. \tag{3}$$

Note that the reflection coefficient is minimum for $\phi = 2q\pi$ and maximum for $\phi = (2q+1)\pi$ where q is an integer. The reflection coefficient may be simplified since most nonlinear modulators are relatively lossy and the effective reflection coefficient r_3, which includes coupling losses in fibres, is much smaller than r_2. For the case of SHG mode-locking r_3 is small in order to maximize the pulse-shortening mechanisms [11, 12]. Assuming that $r_3 \ll r_2$ the effective reflectivity is

$$r(t) = r_2 - r_3 t_2^2 m(t) \exp(-\mathrm{i}\phi). \tag{4}$$

To proceed further requires a specific form for the modulator transmission, $m(t)$.

The nonlinear amplitude modulator caused by second-harmonic generation will be considered. The reflectivity of a second-harmonic crystal separated from a retroreflecting mirror can be shown to have a reflectivity that depends exponentially on intensity as shown in figure 2 for the case where the reflectivity decreases with intensity. The other case, where the nonlinear reflectivity increases with intensity, is

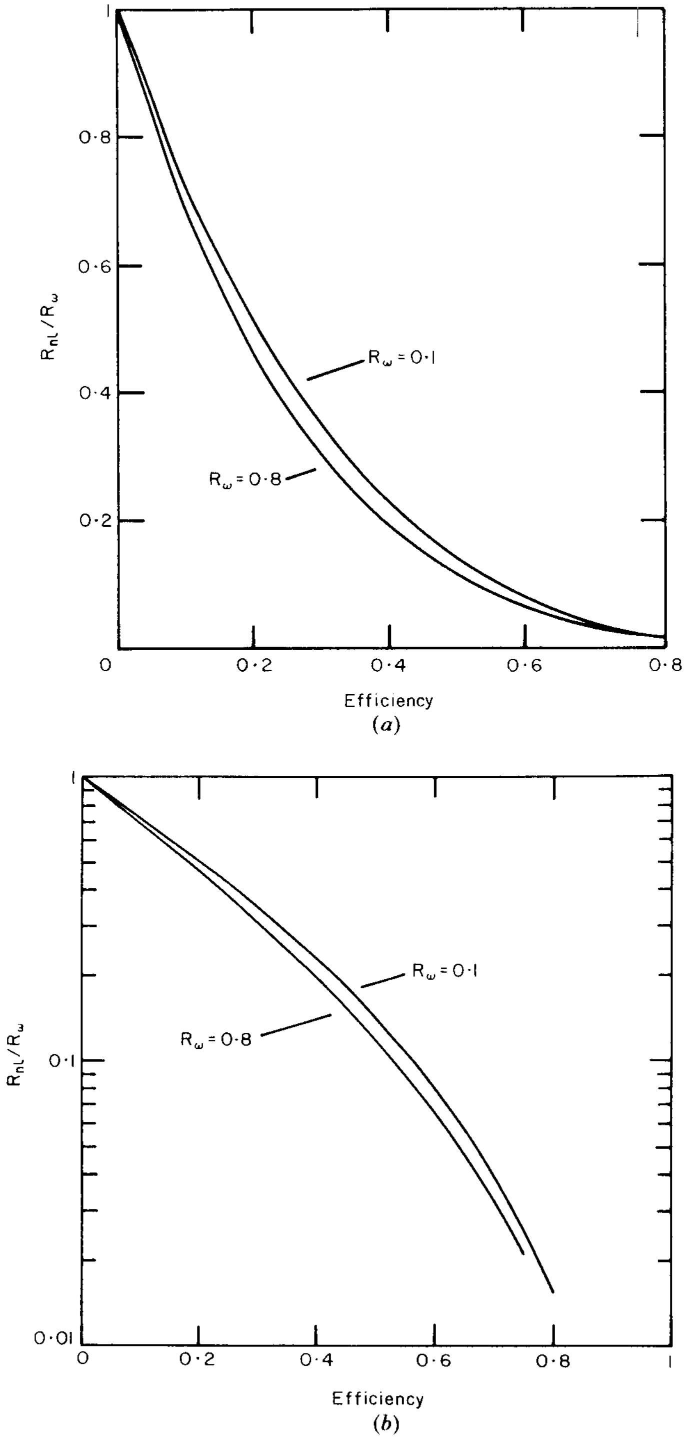

Figure 2. The nonlinear reflectivity R_{nl} depends on the fundamental reflectivity R_ω, reflectivity at the second harmonic $R_{2\omega}$ and the conversion efficiency η; (*a*) shows the variation on a linear scale of the nonlinear reflectivity when the nonlinear mirror is configured as a pulse stretcher, while (*b*) shows the region of validity of the exponential approximation by replotting (*a*) on a logarithmic scale. In each case $R_{2\omega} = 1$.

shown in [12]. The nonlinear reflectivity, r_{nl} is approximated by [12]

$$r_{nl} = r_3 \exp(\sigma\eta/2), \tag{5}$$

when η is the conversion efficiency to the second harmonic on the first pass through the crystal, σ is a fitting parameter which depends on r_3 and is shown in the table. The reflectivity of the mirror for the second harmonic is assumed to be 100% to give optimum operation of the nonlinear mirror. The nonlinear mirror can act either as a pulse stretcher or pulse compressor depending on the phase change experienced by the harmonic with respect to the fundamental due to dispersion in air and by reflection from the mirror. The two modes of operation are described by equation (5) where $\sigma > 0$ for pulse compression and $\sigma < 0$ for pulse stretching.

The pulse reshaping caused by the nonlinear mirror is found by using the appropriate reflectivity given by equation (5) for each point on the pulse temporal profile. Group velocity dispersion between the fundamental and second harmonic is neglected. In general group velocity dispersion is only important for long crystals or very short pulses. The conversion efficiency for a pulse described by its electric field amplitude $f(t)$ in the absence of saturation is [12]

$$\eta(t) = \eta_0 \frac{f^2(t)}{f^2(0)}, \tag{6}$$

where η_0 is the conversion efficiency at the peak of the pulse $f(0)$. The transmission coefficient of the nonlinear modulator is then

$$m(t) = \exp\left(\frac{\sigma\eta_0}{2}\frac{f^2(t)}{f^2(0)}\right). \tag{7}$$

Clearly the nonlinear mirror changes the pulse shape near the peak of the pulse with negligible change in the wings. However to obtain an analytic expression for the pulse duration, a Gaussian pulse shape approximation will be made [12]. The pulse shape will be given by

$$f(t) = f_0 \exp(-yt^2) \exp(i\omega_a t), \tag{8}$$

Fundamental mirror reflectivity $R_3 = \lvert r_3 \rvert^2$	'Saturable absorber' $\sigma > 0$	'Saturable amplifier' $\sigma < 0$	
		$r_{2\omega} = 0$	$r_{2\omega} = 1$
0·1	1·0	−1·4	−3·5
0·2	0·9	−1·45	−3·6
0·4	0·7	−1·6	−3·9
0·8	0·2	−1·9	−4·1

The error in the fit, defined by

$$\varepsilon = \max\left\{1 - \left[\frac{r_3}{r'_{nl}(\eta)}\exp\left(\frac{\sigma\eta}{2}\right)\right]^2\right\},$$

where $r'_{nl}(\eta)$ is the exact expression for the nonlinear reflectivity [11], is always less than 5% except for the 'saturable amplifier' with $r_{2\omega} = 1$, where the limit is 10% for $0 < \eta < 0{\cdot}5$.

where ω_a is the centre frequency of the gain medium. Substituting $f(t)$ into equation (7) and expanding the exponential yields

$$m(t)=\exp(\tfrac{1}{2}\sigma\eta_0)\exp(-\sigma\eta_0 y t^2). \tag{9}$$

Notice that this has the same form as the transmission of an amplitude modulator used in the standard active mode-locking treatment of homogeneously broadened lasers [13]. Substituting equation (9) into equation (4) after some rearranging gives

$$r(t)=r\exp(\Gamma\sigma\eta_0 y t^2), \tag{10}$$

where

$$r=r_2-r_3 t_2^2\exp(\tfrac{1}{2}\sigma\eta_0)\exp(-\mathrm{i}\phi), \tag{11}$$

$$\Gamma=\frac{r_3}{r_2}t_2^2\exp(\tfrac{1}{2}\sigma\eta_0)\exp(-\mathrm{i}\phi). \tag{12}$$

The approximation $r_3 \ll r_2$ has been made and the exponent in equation (9) was expanded to the leading term in t^2. Once again equation (10) has a form suitable for use with the standard active mode-locking analysis. However, for mode-locking to occur through amplitude modulation requires that Γ is real and $\sigma\Gamma$ is negative. This can be achieved in two distinct ways: either $\exp(-\mathrm{i}\phi)=1$, $\phi=2q\pi$ and $\sigma<0$ so the nonlinear amplitude modulator increases the pulse duration or $\exp(-\mathrm{i}\phi)=-1$, $\phi=(2q+1)\pi$ and $\sigma>0$ so the nonlinear amplitude modulator shortens the pulse duration. Other values of ϕ lead to a complex value of Γ and combined amplitude and phase modulation. In particular $\phi=(2q+1)\pi/2$ leads to pure phase modulation since Γ is imaginary.

The pulse duration may be calculated using a steady state analysis in the following way. The pulse duration τ for a standard active modelocker is [13]

$$\tau=\left(\frac{2\ln 2}{\pi^2}\right)^{1/2}\left(\frac{2g_2(4\pi^2)}{\Delta\omega_m^2}\right)^{1/4}\frac{1}{\sqrt{f_a}}, \tag{13}$$

where g_2 is the double pass amplitude gain coefficient, f_a is the gain bandwidth, Δ is the modulation depth and ω_m is the modulation frequency. The modulator transmission used in the standard analysis is

$$m(t)=\exp(-\tfrac{1}{2}\Delta\omega_m^2 t^2). \tag{14}$$

Comparison of equation (14) with equation (10) gives the following relationship:

$$\frac{\Delta\omega_m^2}{2}=-\Gamma\sigma\eta_0 y. \tag{15}$$

Equivalently in terms of the pulse intensity f.w.h.m. τ,

$$\frac{\Delta\omega_m^2}{2}=\frac{-\Gamma(2\ln 2)\sigma\eta_0}{\tau^2}. \tag{16}$$

Essentially the response of the nonlinear amplitude modulator is being compared with a fictitious amplitude modulator whose modulation frequency ω_m depends on the incident pulse duration τ. This enables the nonlinear amplitude modulator to generate pulses shorter than could be obtained using conventional amplitude modulation techniques. Substituting equation (16) into (13) yields

$$\tau = \frac{2}{\pi}\left(\frac{2g_2 \ln 2}{-\Gamma\sigma n_0}\right)^{1/2}\frac{1}{f_a}. \tag{17}$$

Notice that the pulse duration is now proportional to f_a^{-1}, rather than to $f_a^{-1/2}$ as in the normal analysis. This is important because it means that nonlinear modulators make better use of the available bandwidth than active modulators with a fixed modulation frequency ω_m. This equation represents the final stage of the analysis of nonlinear amplitude modulators. Before comparing the theoretical results with experiment it is worth recapping the approximations inherent in the use of equation (17).

(*a*) The system is homogeneously broadened and the final pulse bandwidth is less than f_a.
(*b*) The pulse shape is Gaussian.
(*c*) The peak conversion efficiency η_0 is linear with intensity (no saturation of second-harmonic generation).
(*d*) Group velocity dispersion in the second-harmonic crystal may be neglected.
(*e*) The system can evolve into steady state.
(*f*) Mode-locking through amplitude modulation occurs when Γ is real and $\Gamma\sigma < 0$.

A full derivation of equation (17) using the methods of reference [12] gives essentially the same result.

3. Experiment

The experimental arrangement shown in figure 3 is discussed elsewhere and will be summarized here [7, 8]. A long pulse (5 ms) Nd : YAG laser operating at 5 Hz had a mirror placed a distance precisely equal to the laser cavity length beyond the output coupler. This formed the external cavity. The output coupler transmission was 50% ($r_2 = 0{\cdot}71$) while the retroreflecting mirror had a reflectivity of 10% at 1·064 μm ($r_3 = 0{\cdot}32$) and was a high reflector at 532 nm. A lens, $f = 20$ cm, formed a waist on the retroreflecting mirror and the second harmonic crystal, KTP cut for type II doubling, $3 \times 3 \times 5\ \text{mm}^3$, was placed 2 to 6 cm from the mirror.

Typically the laser was operated with peak powers of 16 W corresponding to a (non-mode-locked) single-pass conversion efficiency of 2×10^{-3}. The $1/e^2$ radius within the doubling crystal was measured to be 110 μm. The laser was observed to be

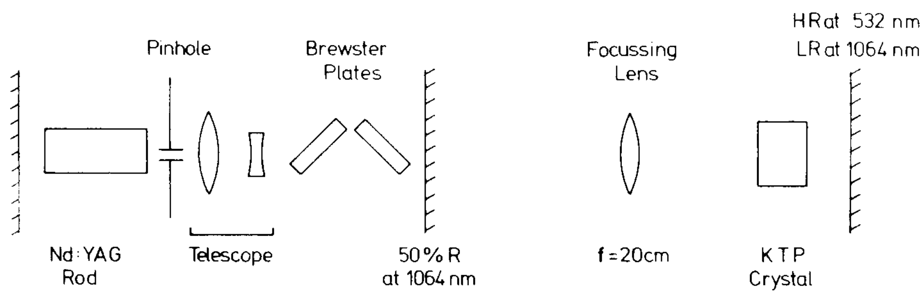

Figure 3. A schematic diagram of the experimental laser system.

Q-switched and mode-locked when the resonant reflector had the correct phase change ϕ. This occurred over the range $0<\phi<\pi$ but especially for $\phi\sim 0$. The explanation for the value of phase $\phi\sim 0$ is that in this case the nonlinear mirror is acting as a pulse stretcher so that $\sigma<0$ and Γ, given by equation (12), is real and positive as required for the net reflectivity of the resonant reflector to act as a pulse compressor. The fact that the laser Q-switched, giving a 700 ns f.w.h.m. envelope, is not surprising and happens since the nonlinear modulator either increases or decreases the amplitude at the peak of the pulse while leaving the wings of the pulse unaffected. This leads to a net increase in reflectivity of the resonant reflector at the peak of the pulse provided that either $\phi=0$, $\sigma<0$ or $\phi=\pi$, $\sigma>0$ as may be seen from equations (10)–(12). This reflectivity increase leads to a relaxation oscillation spike, the duration of which is limited by saturation of the gain medium. The Nd : YAG laser clearly does not reach steady state so in principle the steady state analysis given in the preceding section does not apply. It does however predict pulse durations which are close to the experimentally determined values. These were measured using an autocorrelator and are averaged over the Q-switched pulse. The shortest pulse duration was 30 ps (assuming a Gaussian pulse shape) and ranged from 30 to 50 ps depending on the length of the resonant reflector. Taking appropriate values of the parameters $r_2=0{\cdot}71$, $r_3=0{\cdot}32$, $t_2^2=0{\cdot}5$, $\sigma=-3{\cdot}6$, $\phi=0$, $\eta_0=0{\cdot}1$ yields $\Gamma\sigma\eta=-0{\cdot}068$. The gain g is calculated using

$$g=\ln(1/r)=0{\cdot}6, \tag{18}$$

where r is given by equation (11) evaluated for $\eta_0=0$. The pulse duration is then $\tau=19$ ps if $f_a=120$ GHz. Taking into account the experimental uncertainty in the peak conversion efficiency, η_0, which ranged from 2–20% the calculated pulse duration could be in the ranges 14 ps to 38 ps. Thus the measured pulse durations may be adequately explained by the theory presented in the last section.

The calculations use an analytic formula derived using a number of approximations. The validity of the process for intracavity SHG mode-locking has been examined by numerically integrating the equations governing the pulse evolution on a pass by pass basis. The results, briefly described in reference [12], show that the true pulse duration may be longer than the analytic results by about 30%. The temporal profile is non-Gaussian in the wings of the pulse. Part of the discrepancy between the experimental result and the simple theory may be explained by the approximations used in the analysis.

4. Conclusion

A temporal analysis of enhanced mode-locking using a nonlinear amplitude modulator has been presented. Using an extension to the standard analysis of the active mode-locking of homogeneously broadened lasers an analytic approximation to the pulse duration in steady state was obtained. A number of approximations were made to obtain this result, in particular that the laser could reach a steady state. This latter feature was not supported by experiments where Q switching and mode-locking events were observed. However the pulse duration calculated from the theory agreed reasonably with experiment. Other laser materials which have different values of the saturation energy density and fluorescence lifetimes may reach steady state. Ti : sapphire with its large linewidth and relatively short fluorescence lifetime could be mode-locked using SHG in $KNbO_3$. The final point worth noting is that the analysis demonstrates that the pulse duration is inversely proprotional to the laser gain bandwidth. It is expected that a similar form would result for the analysis of the nonlinear phase modulator and be sufficient to explain some of the results obtained for coupled cavity mode-locking where fibres were used as the nonlinear medium [2, 6].

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References

[1] Ouellette, F., and Piche, M., 1986, *Optics Commun.*, **60,** 99–100.
[2] Ouellette, F., and Piche, M., 1988, *Can. J. Phys.*, **66,** 903–913.
[3] Mollenaur, L. F., and Stolen, R. H., 1984, *Optics Lett.*, **9,** 13.
[4] Blow, K. J., and Nelson, B. P., 1988, *Optics Lett.*, **13,** 1026–1028.
[5] Keen, P. N., Zhu, X., Crust, D. W., Grant, R. S., Langford, N., and Sibbett, W. S., 1988, *Optics Lett.*, **14,** 39–41.
[6] Mark, J., Lie, L. Y., Hall, K. L., Haus, H. A., and Ippen, E. P., *Optics Lett.*, **14,** 48–50.
[7] Barr, J. R. M., Hanna, D. C., and Hughes, D. W., 1989, *Digest of the conference on Lasers and Electro-Optics.* Baltimore U.S.A. Paper FQ5.
[8] Barr, J. R. M., and Hughes, D. W., 1989, *Appl. Phys.* B, **49,** 323.
[9] Blow, K. J., and Wood, D., 1988, *J. opt. Soc. Am.*, **5,** 629.
[10] Barr, J. R. M., 1989, *Optics Commun.*, **73,** 484.
[11] Stankov, K., 1988, *Appl. Phys.* B, **45,** 191.
[12] Barr, J. R. M., 1989, *Optics Commun.*, **70,** 229.
[13] Kuizenga, D. A., and Siegman, A. E., 1970, *J. quant Electron*, **6,** 694.