ENHANCEMENT OF THE MODULATION DEPTH OF AN ACTIVE MODELOCKER USING AN EXTERNAL CAVITY

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An external cavity containing a standard active modelocker can result in enhanced modelocking of lasers. The pulse length reduction depends on the configuration chosen and can be as large as 50%.

1. Introduction

Active modelocking is a well established technique for generating ultrashort pulses from many types of lasers [1]. The main drawback of this method is that it is relatively insensitive to variations in the parameters that control the pulse duration – modulation depth, modulation frequency, laser gain and laser bandwidth. For a given laser material the two parameters that are under experimental control are the modulation frequency and modulation depth. Unfortunately significantly increasing either of these quantities to generate shorter pulses is technically difficult.

In this paper a procedure which enables the modulation depth to be artificially increased is explored. The active modelocker is placed in a coupled cavity matched to the length of the laser. The increase in modulation depth can be a factor of 5 or more resulting in a pulse length reduction of at least 33%. Larger increases in modulation depth are possible in favourable circumstances. It will be shown that the increase in modulation depth under optimum conditions depends only on the reflectivity of the mirrors comprising the coupled cavity in which the modulator is placed and not on its modulation depth. The technique works for both acousto-optic amplitude modulators and for phase modulators. Like other experimental configurations where nonlinear elements are placed inside external cavities the enhanced modelocking only occurs for particular values of the round trip phase of the coupled cavity [2–7].

Previously an acousto-optic frequency shifter has been used in a coupled cavity arrangement to modelock a HeNe laser [8]. Mode-coupling was caused by giving the injected radiation a frequency shift equal to the mode spacing of the laser. This paper concentrates on developing a formalism applicable to homogeneously broadened lasers.

2. Theory

The optical configuration that will be analysed will be shown in fig. 1. The modulator is placed close to the end mirror of a resonant reflector matched in length to the laser cavity to within length detunings $\sim\lambda$. This ensures that the longitudinal modes of the resonator have the same spacing as the laser cavity or, equivalently, the round trip time for a pulse in the laser cavity matches the round trip time of a pulse

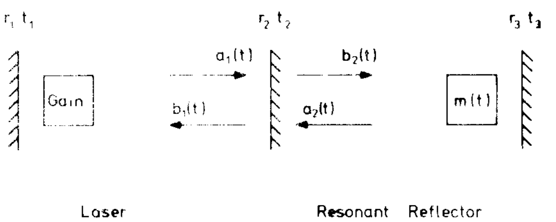

Fig. 1. A schematic diagram of the laser system and resonant reflector.

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in the coupled cavity. The modulation frequency of the modelocker ω_m is equal to the mode spacing of the laser. As in the case of standard active modelocking this ensures that the pulse passes through the modelocker at the same point on the modulation cycle so it experiences the same loss or phase shift.

The reflectivity of the resonant reflector containing the modulator will now be calculated. The coupling equations on the input mirror are well known to be [9]

$$b_1(t)=r_2 a_1(t)+\mathrm{i}t_2 a_2(t)\,,$$

$$b_2(t)=\mathrm{i}t_2 a_1(t)+r_2 a_2(t)\,, \quad (1)$$

where r_2 and t_2 are the electric field reflection and transmission coefficients respectively for mirror 2. $a_1(t)$ and $a_2(t)$ are the electric field incident on the mirror while $b_1(t)$ and $b_2(t)$ are the reflected fields. The subscript 1 refers to fields in the laser cavity while the subscript 2 refers to fields in the resonant reflector.

The reflection coefficient $r(t)$ is obtained by noting that

$$a_2(t)=b_2(t)m(t)\exp(-\mathrm{i}\phi)\,, \quad (2)$$

where ϕ is the resonant reflector round trip phase change and $m(t)$ is the double pass transmission of the modulator,

$$r(t)=\frac{b_1(t)}{a_1(t)}=\frac{r_2-r_3 m(t)\exp(-\mathrm{i}\phi)}{1-r_2 r_3 m(t)\exp(-\mathrm{i}\phi)}\,. \quad (3)$$

The results of this analysis apply to a laser in steady state when the field in the external resonator has reached an equilibrium value.

2.1. Amplitude modulation

The transmission $m(t)$ for an amplitude modulator will be taken to be

$$m(t)=\exp[-\Delta(1-\cos\omega_m t)]\,. \quad (4)$$

Generally the laser pulse will be short compared to the modulation period $T=2\pi/\omega_m$ and the pulse will pass through the modelocker when $|r(t)|$ is maximised. Calculating $\mathrm{d}r/\mathrm{d}t=0$ shows that the extremum values of $r(t)$ occur when $\mathrm{d}m/\mathrm{d}t=0$ at $\omega_m t=q\pi$. Thus the reflectivity may be written as a Taylor expansion about the extremum

$$r(t)=r_{ex}+\tfrac{1}{2}\left.\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right|_{ex} t^2=r_{ex}\exp\left(\frac{1}{2r_{ex}}\left.\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right|_{ex} t^2\right)\,, \quad (5)$$

where the following quantities are evaluated at the extremum of $r(t)$,

$$\left.\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}\right|_{ex}=-\frac{r_3(1-r_2^2)\exp(-\mathrm{i}\phi)}{(1-r_2 r_3 m_{ex})\exp(-\mathrm{i}\phi)}\left.\frac{\mathrm{d}^2 m}{\mathrm{d}t^2}\right|_{ex}\,, \quad (6)$$

$$r_{ex}=\frac{r_2-r_3 m_{ex}\exp(-\mathrm{i}\phi)}{1-r_2 r_3 m_{ex}\exp(-\mathrm{i}\phi)}\,, \quad (7)$$

$$m_{ex}=1\,,\quad \omega_m t=2q\pi,\ \text{positive mode}\,,$$
$$=\exp(-2\Delta)\,,\quad \omega_m t=(2q+1)\pi,\ \text{negative mode}\,, \quad (8)$$

$$\mathrm{d}^2 m/\mathrm{d}t^2|_{ex}=-\Delta\omega_m^2 m_{ex}\,,\quad \text{positive mode}\,,$$
$$=+\Delta\omega_m^2 m_{ex}\,,\quad \text{negative mode}\,. \quad (9)$$

The labels positive and negative mode have been introduced by analogy with phase modulators [1]. In this context the positive mode refers to the pulse which passes through the modulator at its maximum transmission while negative mode corresponds to the pulse that passes through with minimum transmission. In use in coupled cavities both modes can correspond to modelocking and have to be distinguished. Eq. (5) may be written in the suggestive form

$$r(t)=r_{ex}\exp(-\Gamma\Delta\omega_m^2 t^2/2)\,, \quad (10)$$

where Γ is the increase in modulation depth due to the resonant reflector

$$\Gamma=\mp\frac{r_3(1-r_2^2)\mathrm{e}^{-\mathrm{i}\phi}m_{ex}}{(r_2-r_3 m_{ex}\,\mathrm{e}^{-\mathrm{i}\phi})(1-r_2 r_3 m_{ex}\,\mathrm{e}^{-\mathrm{i}\phi})}\,. \quad (11)$$

The upper sign is taken for positive mode while the lower sign is taken for negative mode. For pulse compression to result from reflection from the resonant reflector by amplitude modulation Γ has to be real and positive. The sign of Γ depends on the value of $\exp(-\mathrm{i}\phi)$ and $r_2-r_3 m_{ex}\exp(-\mathrm{i}\phi)$. The magnitude of Γ is clearly enhanced if $r_2-r_3\approx 0$. A possible solution is to take $r_3>r_2$, $\exp(-\mathrm{i}\phi)=1$, and to consider the positive mode so Γ is real and given by

$$\Gamma=\frac{r_3(1-r_2^2)}{(r_3-r_2)(1-r_2 r_3)}\,. \quad (12)$$

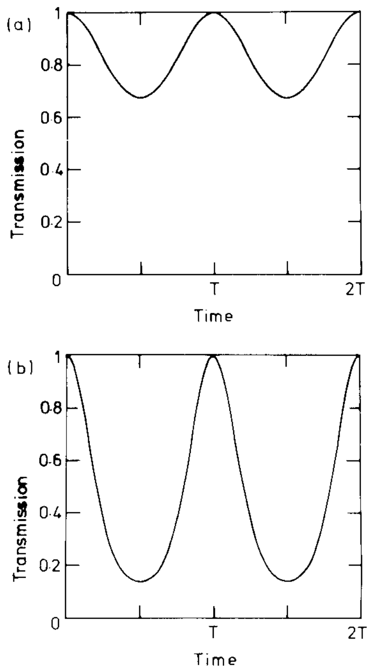

Fig. 2. The transmission function of an amplitude modulator, (a) $\Delta=0.1$, (b) $\Delta=0.5$.

Fig. 2 shows the transmission of two amplitude modulators with (a) $\Delta=0.1$ and (b) $\Delta=0.5$. Figs. 3 and 4 show the intensity reflectivity $R(t)=|r(t)|^2$ for a number of configurations. In fig. 3 the modulation depth is $\Delta=0.1$ and the mirror reflectivities are 3a $r_2=\sqrt{0.5}$, $r_3=\sqrt{0.9}$ and 3b $r_2=\sqrt{0.05}$, $r_3=\sqrt{0.99}$. A number of different phase values are shown. In fig. 3a $\Gamma=5.96$ while in fig. 3b $\Gamma=5.8$. Notice that fig. 2b is very similar to the curves in fig. 3 for $\phi=0$ indicating that the modulation depth has indeed been increased by a factor ~ 5. Fig. 4 shows the reflectivity for $\Delta=0.5$ for each combination of mirror reflectivity. Notice that the maximum reflectivity $|r_{cx}|^2$ has increased from 0.54 in fig. 3a to 0.94 in fig. 4b. This is an illustration of one of the practical trade offs inherent in this mode of operation, that is maximising Γ may minimise the resonant reflector reflectivity. Whether or not large values of Γ are feasible depends on the gain available in the laser. The alternative solution $\exp(-i\phi)=-1$ with no constraints on r_2 and r_3 can be shown to have a maximum value of $\Gamma=1$ which occurs for $r_2=0$, with r_3 indeterminate and is generally worse than using the modelocker intracavity.

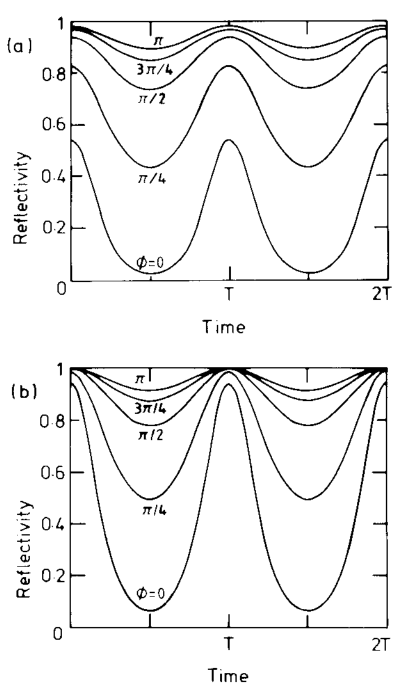

Fig. 3. The intensity reflectivity of a resonant reflector with mirror reflectivities, (a) $r_2=\sqrt{0.5}$, $r_3=\sqrt{0.9}$, and (b) $r_2=\sqrt{0.5}$, $r_3=\sqrt{0.99}$ containing an amplitude modulator with $\Delta=0.1$.

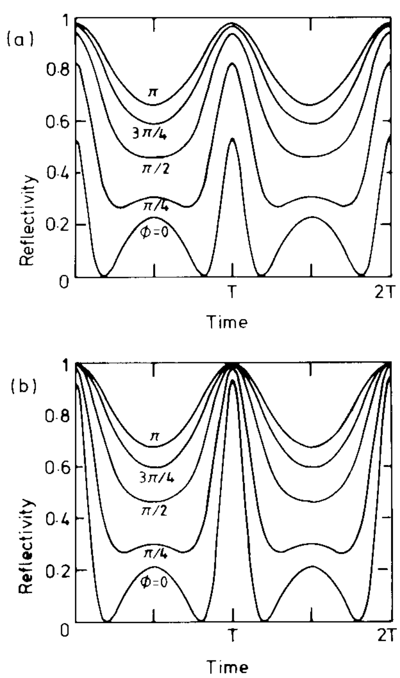

Fig. 4. The intensity reflectivity of a resonant reflector with mirror reflectivities, (a) $r_2=\sqrt{0.5}$, $r_3=\sqrt{0.90}$, and (b) $r_2=\sqrt{0.5}$, $r_3=\sqrt{0.99}$. The modulation depth is $\Delta=0.5$.

The other set of possible solutions, the negative mode, where pulse passes through the modelocker at minimum transmission, $m_{ex}=\exp(-2\Delta)$, will now be briefly discussed. This would normally correspond to pulse stretching if used intracavity but pulse broadening mechanisms can lead to net pulse shortening if reinjected into the laser cavity with the correct phase. The value of Γ for the negative mode is found by taking the lower sign in eq. (11). If $\exp(-i\phi)=1$ then Γ is real and given by

$$\Gamma=\frac{r_3(1-r_2^2)\exp(-2\Delta)}{[r_2-r_3\exp(-2\Delta)][1-r_2r_3\exp(-2\Delta)]} \quad (13)$$

and the mirror reflectivities have to obey

$$r_2>r_3\exp(-2\Delta)$$

for Γ to be positive.

Notice that Γ is now reduced by the modulation depth $\exp(-2\Delta)$ so the negative mode is a less favourable mode of operation than the positive mode.

2.2. *Phase modulation*

For the case of phase modulation the modulation function is

$$m(t)=\exp[-i\Delta\cos(\omega_m t)]\ .$$

The expansion of the resonant reflector reflectivity $r(t)$ around extremal values yields

$$r(t)=r_{ex}\exp\left(\frac{1}{2r_{ex}}\left.\frac{d^2r}{dt^2}\right|_{ex}t^2\right), \quad (15)$$

where

$$\left.\frac{d^2r}{dt^2}\right|_{ex}=-\frac{r_3(1-r_2^2)\exp(-i\phi)}{[1-r_2r_3m_{ex}\exp(-i\phi)]^2}\left.\frac{d^2m}{dt^2}\right|_{ex}, \quad (16)$$

$$r_{ex}=\frac{r_2-r_3m_{ex}\exp(-i\phi)}{1-r_2r_3m_{ex}\exp(-i\phi)}, \quad (17)$$

$$\begin{aligned} m_{ex}&=\exp(-i\Delta),\quad \omega_m t=2q\pi,\ \text{positive mode},\\ &=\exp(+i\Delta),\quad \omega_m t=(2q+1)\pi,\ \text{negative mode}, \end{aligned} \quad (18)$$

$$\begin{aligned} d^2m/dt^2|_{ex}&=+i\Delta\omega_m^2 m_{ex},\quad \text{positive mode},\\ &=-i\Delta\omega_m^2 m_{ex},\quad \text{negative mode}. \end{aligned} \quad (19)$$

If eq. (15) is written as

$$r(t)=r_{ex}\exp\left(i\Gamma\frac{\Delta\omega_m^2t^2}{2}\right), \quad (20)$$

then

$$\Gamma=\mp\frac{r_3(1-r_2^2)\,e^{-i\phi}m_{ex}}{(r_2-r_3m_{ex}e^{-i\phi})(1-r_2r_3m_{ex}e^{-i\phi})}. \quad (21)$$

This is identical with the enhancement factor for the amplitude modulator given by eq. (11).

For pure phase modulation to occur requires that Γ is real which gives the condition

$$m_{ex}\exp(-i\phi)=\pm 1 \quad (22)$$

which is satisfied when $|-\phi\mp\Delta|=2q\pi$ or $(2q+1)\pi$, where the upper sign is taken for the positive mode.

3. Discussion

The results outlined in the last section show that it is possible to enhance the modulation depth of an active modelocker by placing the modulator in an external cavity. The enhancement depends, in the optimum case, on the reflectivity of the mirrors used in the resonant reflector. An important result of the analysis is that the round trip phase change in the cavity must have a particular value for the enhancement to occur.

For the case of amplitude modulation if no net phase modulation is to occur then Γ must be real and $\phi=q\pi$ where q is an integer. The value of the enhancement is larger for $\phi=2q\pi$ than for $\phi=(2q+1)\pi$ as is clearly shown in figs. 3 and 4. The phase of the resonant reflector must be stabilised in order to achieve reliable enhancement of the modelocking. This feature is shared with the other coupled cavity modelocking schemes. One effect of the enhancement factor Γ is to increase the sensitivity of the laser and resonant reflector to phase detunings $\Delta\phi$ from the optimum phase value ($\phi=2q\pi$ for the amplitude modulator). It is straightforward to show from eq. (3) that if the round trip phase is detuned by $\Delta\phi$ the phase change on reflection from the resonant reflector is $\Gamma\Delta\phi$. This will set quite stringent limits on the stability of the laser and resonant reflector.

A further important result of this analysis of amplitude modulators is that the negative mode, which is normally suppressed in intracavity use, can result in modelocking when used in a coupled cavity arrangement. In particular when the pulse passes through the amplitude modulator at minimum transmission its duration is increased. However, as is shown in fig. 4 the net change in reflectivity of the resonant reflector as a function of time has a maximum for $\omega_m t=(2q+1)\pi$ and can act as a pulse compressor. With reference to fig. 4b by choosing values of r_1, r_2 and Δ it is possible to ensure that the reflectivity of the positive mode is less than the negative mode so that the laser operates on the negative mode and modelocking results from a pulse broadening mechanism. In practice this operating condition is unlikely to be useful.

The use of a phase modulator was briefly examined and similar features noted as for the amplitude modulator. For pure phase modulation to result with no amplitude modulation component requires that the net round trip phase change be 0 or π. Maximum enhancement occurs for a net phase change, $-\phi\pm\Delta=0$.

The expected pulse length reduction can be easily calculated. The analysis of active modelocking [1] yields

$$\tau \propto (g/\Delta)^{1/4}(f_a f_m)^{-1/2}\,, \tag{23}$$

where Δ is the modulation depth, g is the amplitude gain coefficient, f_a is the laser bandwidth and f_m is the modulation frequency, $\omega_m=2\pi f_m$. With a modulation depth enhanced by a factor Γ the resultant pulse duration τ' is related to τ via

$$\tau'=\tau/(\Gamma)^{1/4} \tag{24}$$

for both amplitude and phase modulation. For the specific examples illustrated in figs. 3 and 4, $\Gamma=5.96$ and $(\Gamma)^{1/4}=1.56$ giving a pulse length reduction from τ to $r'=0.64\tau$. A larger pulse length reduction may be found from the following set of conditions $r_2=\sqrt{0.8}$, $r_3=\sqrt{0.99}$, $\phi=0$ where $\Gamma=17.98$ and $(\Gamma)^{1/4}=2.06$. The reflectivity of r_3 (which includes any passive losses from the modulator as well as losses from poor modematching into the coupled cavity) is large to ensure that the extremal reflectivity r_{ex} calculated from eq. (7) is $r_{ex}=-\sqrt{0.83}$. If r_3 were reduced to 0.95 then $\Gamma=18.94$ and $r_{ex}=-\sqrt{0.4}$ which is a large reduction in r_{ex} and could only be sustained for a high gain laser.

Values of the enhancement Γ which are complex lead to a combination of amplitude modulation and phase modulation. Whether there is any advantage in this mode of operation has not been investigated.

4. Conclusion

An investigation into the use of active modulators within coupled cavities or resonant reflectors has been presented. Under certain circumstances large enhancements of the modulation depth can be obtained, both for amplitude and phase modulators. By choosing the round trip phase of the resonant reflector correctly the reflected signal can have the character of the modulator within the resonant reflector. In general, however, the reflected signal experiences both amplitude and phase modulation.

A second feature of the analysis that is worth noting is that for the amplitude modulator it is possible to find operating conditions where the pulse passing through at minimum transmission can lead to modelocking.

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