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Modes in graded noncircular multimode optical fibers

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Individual leaky modes have been launched on a noncircular multimode fiber by a prism-coupling technique. The resulting field patterns have been interpreted on the basis of a new analytical ray theory of noncircular fibers.

Introduction

The near-field-scanning (NFS) technique[1] and its reciprocal[2] are convenient methods for determining the refractive-index profiles of multimode fibers when used with a set of correction factors[3] that allow for the attenuation of tunneling leaky modes. However, there is some indication[4] that the correction factors may not be necessary in all cases, and it has been suggested[5,6] that fiber noncircularity is responsible since leaky modes in noncircular graded-index fibers may suffer high attenuation. The object of the present contribution is to study leaky modes on noncircular fibers both experimentally and theoretically and to compare their properties with the corresponding modes of circular fibers. Our conclusions indicate that the leaky-mode behavior is closely related to fiber-index profile for noncircular fibers and that leaky modes exist without exceptionally high attenuation in many fibers.

Experiment

Individual leaky modes were excited on multimode fibers by a prism-coupled side-launch technique developed first by Stewart.[7,8] Field distributions of the modes thus launched were observed after a few centimeters at the fiber end with a microscope; an example is shown in Fig. 1. The fiber used in this case was fabricated by the conventional chemical-vapor-deposition process and had a normalized frequency V of 75 at He–Ne wavelength, a core ellipticity estimated to be of the order of 0.1%, and a power-law index profile with exponent $\alpha = 1.89$ (including correction) as measured by the NFS technique. For such fibers the modes form two classes[9,10]—elliptic (E) and hyperbolic (H)—and Fig. 1 shows an E-type mode whose linearly polarized designation would be (32,6). A range of different E modes and some of the H modes were excited by varying the angle of the incident laser beam relative to the fiber. Clearly, the leaky modes that are most easily launched by this technique are those with sufficient loss to ensure that the radiation field can be accessed externally and yet that are not so heavily attenuated that they would not be observed at the fiber end. Identification of the mode designation from the field pattern yields a simple means for comparison with the predictions of the theory presented below.

Analytic Ray Theory for Graded Noncircular Fibers

The mathematical representation of refractive-index profiles in noncircular, graded-index fibers has been discussed in recent publications.[9,10,15] For the present work we use elliptical cylinder coordinates specified by (ξ,η,z) and a focus parameter f. Many realistic noncircular index profiles can be written in the form[10]

$$n^2 = \begin{cases} n_0^2 - \dfrac{g(\xi) + h(\eta)}{\sinh^2 \xi + \sin^2 \eta}, & \text{in the core} \\ n_{cl}^2, & \text{in the cladding} \end{cases} \qquad (1)$$

by suitable choice of the functions g and h and the focus position f; the latter is important in that its position relative to the core–cladding interface determines the mode patterns. In Ref. 10 mode patterns and caustics for such profiles are found by analyzing the scalar-wave equation. Here we give a ray-optics treatment, commencing with the eikonal equation[11]

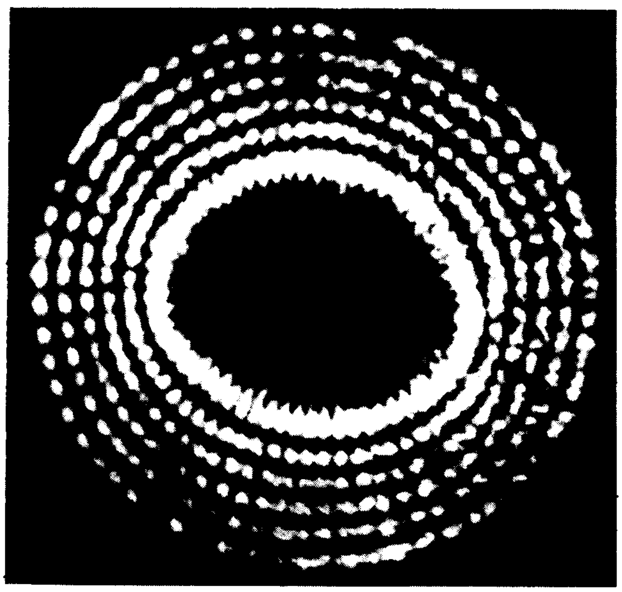

Fig. 1. Elliptic mode pattern observed in a noncircular fiber.

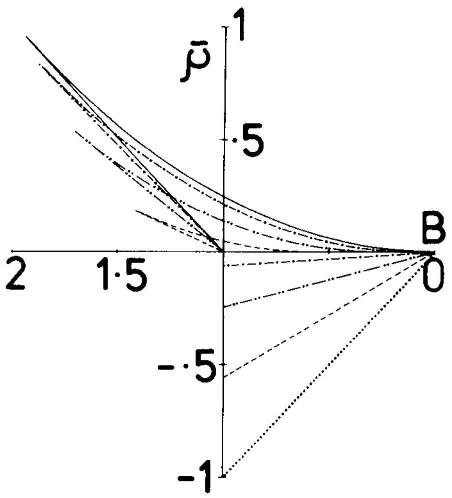

Fig. 2. Allowed domains for leaky and guided modes in a noncircular fiber for $\alpha = 1.9$: —, $f/\rho = 0$; - - - -, $f/\rho = 0.25$; — - - — - -, $f/\rho = 0.5$; - - - -, $f/\rho = 0.75$; ·····, $f/\rho = 1$.

$$(\nabla S)^2 = n^2. \tag{2}$$

We use the Hamilton–Jacobi method[12] by taking Eq. (1) and attempting to find $S = S_1(\xi) + S_2(\eta) + S_3(z)$ such that the eikonal equation in elliptical cylinder coordinates is satisfied. After separation of variables, we obtain:

$$\begin{aligned} S_3'(z) &= \bar{\beta}, \\ [S_1'(\xi)]^2 &= f^2[(n_0^2 - \bar{\beta}^2)\sinh^2\xi - g(\xi)] \\ &\quad - (n_0^2 - n_{cl}^2)\rho^2\bar{\mu} \equiv G(\xi), \\ [S_2'(\eta)]^2 &= f^2[(n_0^2 - \bar{\beta}^2)\sin^2\eta - h(\eta)] \\ &\quad + (n_0^2 - n_{cl}^2)\rho^2\bar{\mu} \equiv H(\eta), \end{aligned} \tag{3}$$

where ρ is the distance from the center to the core–cladding interface measured along the major axis, n_{cl} is the index at the interface, and $\bar{\beta}$ and $\bar{\mu}$ are constants. The ray direction at any point is found by using $\nabla S = n(\mathrm{d}\mathbf{R}/\mathrm{d}s)$, where $\mathbf{R}$ is the position vector and s is the distance along the ray path. Hence

$$\left(\frac{\mathrm{d}\xi}{\mathrm{d}z}\right)^2 = \frac{G(\xi)}{\bar{\beta}^2 f^4 (\sinh^2\xi + \sin^2\eta)^2}; \quad \left(\frac{\mathrm{d}\eta}{\mathrm{d}z}\right)^2 = \frac{H(\eta)}{\bar{\beta}^2 f^4 (\sinh^2\xi + \sin^2\eta)^2}. \tag{4}$$

Clearly a turning point occurs for each zero of G or H, and the ray path is confined to the region where G and H are both positive. For physically reasonable profiles there are two basic patterns:

(1) If $\bar{\mu} > 0$, then G has two zeros, $\xi_{\min}$ and ξ_{tp}, and H has no zeros; the ray projection is contained between two ellipses, $\xi = \xi_{\min}$ and $\xi = \xi_{tp}$ (E type).

(2) If $\bar{\mu} < 0$, then G has one zero, ξ_{tp}, and H has one zero, $\eta_{\min}$; the ray projection is bounded by the ellipse $\xi = \xi_{tp}$ and the hyperbola $\eta = \eta_{\min}$ (H type).

This theory explains the ray projections found numerically in Ref. 9; the ray invariants $\bar{\beta}$ and $\bar{\mu}$ can be written in terms of ξ, η, and two angles. It is also possible to write $\bar{\beta} = \lambda\beta/2\pi$ and $\bar{\mu} = \mu/V^2$, where β and μ are the modal invariants, λ is the wavelength, and V is the normalized frequency. With this identification, G and H are the same as the functions found by using the modal analysis.[10]

In Ref. 10 it is shown that suitable functions g and h can be chosen to represent power-law profiles (exponent α) having elliptical index contours; the focus position f is given in terms of α and the ellipticity. This allows us to delineate on the $(\bar{\beta}, \bar{\mu})$ plane the region in which the modes or rays can exist. We define the normalized propagation wave number $B = (n_0^2 - \bar{\beta}^2)/(n_0^2 - n_{cl}^2)$; the resulting domains shown in Fig. 2 for various focus positions are a generalization of the regions for circular power-law index profiles.[7,13] In the limit $f/\rho \to 0$, the circular results are recovered; for the case in which $f > \rho$, the E region ($\bar{\mu} > 0$) no longer exists and there is a larger H region ($\bar{\mu} < 0$). Note that for the parabolic profile ($\alpha = 2$), the focus position $f \to \infty$, and therefore the E region no longer exists.[5]

Standing-Wave Conditions

The phase part of the field is given by $\exp(ikS)$, where S is found from Eq. (3). The (extended) line through the foci divides the fiber core into two halves. The number of modes in one half in the ξ direction (m) and in the η direction (ν) can be found from standing-wave conditions, noting that the phase change at a caustic is $-\pi/2$:

$$\left.\begin{aligned} \int_{\xi_{\min}}^{\xi_{tp}} G^{1/2}\mathrm{d}\xi &= \frac{\lambda}{2}(m + 1/2), & m &= 0, 1, 2, \ldots \\ \int_{\eta_{\min}}^{\pi/2} H^{1/2}\mathrm{d}\eta &= \frac{\lambda\nu}{4}, & \nu &= 1, 2, 3, \ldots \end{aligned}\right\}. \tag{5}$$

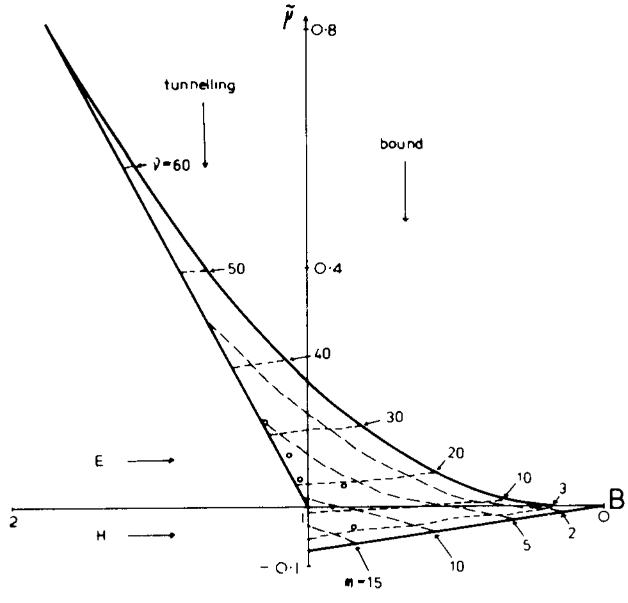

Fig. 3. Lines of constant m and constant ν for the noncircular fiber with $\alpha = 1.9$, $f/\rho = 0.27$, and $V = 75$. Small circles indicate the positions of the experimentally observed modes. Note that the parameters used for the calculation imply that the ratio of major to minor axes of the core is 1.005.

When $\bar{\mu} < 0$, $\xi_{min} = 0$, and, strictly speaking, ν should be replaced by $\nu + 1/2$; when $\bar{\mu} > 0$, we have $\eta_{min} = 0$. To illustrate these results, we use them to find integers m and ν by numerical integration (Fig. 3). The equations reduce to the known results for fibers with circular symmetry when $f \rightarrow 0$; e.g., when $n = n(r)$, the second condition in Eq. (5) is trivial and the lines of constant ν become horizontal on Fig. 2.

Discussion and Conclusions

The nature of the experiment indicates that the leaky modes we have observed are of medium loss. Whereas for circular fibers it is possible to find tunneling modes with low ν values, the above theory shows that for noncircular fibers all leaky modes must have ν relatively large. This prediction agrees with our experimental observations; some of the observed leaky modes are indicated in Fig. 3 and serve to illustrate this point. In addition, it was possible also to launch some E and H modes in the guided-mode region of Fig. 3, as illustrated. This was due to the existence of a low-index buffer layer in the fiber cladding, which had the effect of converting these modes (which would be guided for a uniform cladding) into local leaky modes. The observed mode numbers ($m = 13$, $\nu = 4$) of the H mode indicated on Fig. 3 are in the region predicted by the theory given here. A further point arising from the theory is that the relation $2m + \nu + 1 = VB/2$, which is exact for circular fibers with $\alpha = 2$ (except for a few modes near cutoff),[14] is found to hold approximately for many noncircular fibers of interest.

The good agreement obtained between experimentally observed mode patterns of individual leaky modes with the predictions of the analytic ray theory presented here is encouraging. We conclude that leaky modes do indeed exist without exceptionally high attenuation on noncircular graded-index fibers except for the special case in which $\alpha = 2$; this case is rarely achieved in practice with sufficient precision to determine whether the singular nature of its behavior is relevant to real fibers.

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