ATTENUATION OF LEAKY MODES IN GRADED NONCIRCULAR MULTIMODE FIBRES

Indexing term: Optical fibres

A simple formula for the attenuation constant of leaky modes in a generalised class of noncircular graded-index fibres is derived both from geometrical optics and a WKB method. The leaky mode attenuation in near-parabolic profile fibres is shown to increase rapidly with increasing eccentricity of the fibre.

Introduction: Leaky-mode attenuation in circular and noncircular multimode fibres is of relevance to the measurement of refractive-index profiles using the near-field scanning technique.[1] The generalised form of Fresnel's law[2] can be used to calculate the attenuation in slightly elliptical step-index fibres;[3] an approximate solution of the eigenvalue equation gives the same result.[3] However, these approaches are not directly applicable to graded-core fibres, and the only calculation reported to date[4] for this case uses coupled-mode theory for a specific form of profile. In this contribution, we report a simpler technique based on earlier studies of propagation in graded noncircular fibres;[5–7] our principal result is an expression for leaky-mode attenuation which is deduced both from a ray approach and the WKB method.

In elliptical co-ordinates (ξ, η), the refractive-index profile is expressed[6,7] as

$$n^2(\xi, \eta) = n_0^2 - \frac{g(\xi) + h(\eta)}{\sinh^2 \xi + \sin^2 \eta} \tag{1}$$

where n_0 is the index at core centre. In order to approximate a power-law profile (exponent α) having concentric ellipses in the core and a uniform cladding of index n_{cl}, the functions g, h are chosen as

$$g(\xi) = \begin{cases} (n_0^2 - n_{cl}^2)(f/\rho)^\alpha \sinh^2 \xi \cosh^\alpha \xi, & \xi \le \xi_a \\ (n_0^2 - n_{cl}^2) \sinh^2 \xi, & \xi \ge \xi_a \end{cases} \tag{2}$$

$$h(\eta) = (n_0^2 - n_{cl}^2)(f/\rho)^\alpha \sin^2 \eta \cos^\alpha \eta, \quad \text{all } \eta \tag{3}$$

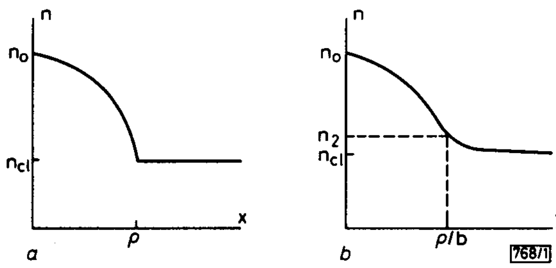

Fig. 1 *Refractive-index profile along (a) x-axis, (b) y-axis. b is the ratio of major to minor axis of (concentric ellipse) contours in the core*
$n_2^2 = n_0^2 - (n_0^2 - n_{cl}^2)/(1 + b^2 f^2/\rho^2)$, where n_2 is the index at the point where the y-axis intersects the core/cladding interface. In this case, $f/\rho = 0{\cdot}424$ and $b = 1{\cdot}005$ ($\alpha = 2{\cdot}05$)

where $\cosh \xi_a = \rho/f$, ρ is the distance from the core centre to the core-cladding interface along the x-axis, and the focus position f is given as a function of α and the core eccentricity in Reference 6. Fig. 1 shows plots of the index profile from eqns. 1–3 along the major and minor axes; the cladding is well represented provided that f/ρ is not too close to unity. With the aid of this profile we may discuss leaky-ray attenuation either in terms of a physically intuitive ray model or from the more algebraic WKB method.

Ray optics approach: From the geometric optics viewpoint, a tunnelling ray spirals down the fibre, always remaining between its two caustics, ξ_{min} and ξ_{tp}.[5–7] The axial distance between a point where the ray touches its inner caustic and the point where it next touches its outer caustic is defined as z_a. If T is the transmission coefficient[8] then the power attenuation coefficient γ is given by $T/\langle 2z_a \rangle$, where $\langle\ \rangle$ indicates that an average is taken over many periods, since the periodic distance is not constant for noncircular fibres. The expression for T is well known:[8]

$$T = \exp\left[-2k \int_{\xi_{tp}}^{\xi_{rad}} [-G(\xi)]^{1/2}\, d\xi\right] \tag{4}$$

where $\xi = \xi_{rad}$ is the surface from which energy loss occurs by radiation, k is the free-space wave number and G is given by[7]

$$G(\xi) = f^2[(n_0^2 - \bar{\beta}^2) \sinh^2 \xi - g(\xi)] - (n_0^2 - n_{cl}^2)\rho^2 \tilde{\mu} \tag{5}$$

where $\bar{\beta}$, $\tilde{\mu}$ are the ray invariants. Using the ray optics equations[7] we find for the average $\langle z_a \rangle$

$$\langle z_a \rangle = \bar{\beta} f^2 \int_{\xi_{min}}^{\xi_{tp}} [G(\xi)]^{-1/2} \times [D(\bar{\beta}, \tilde{\mu}) + \sinh^2 \xi]\, d\xi \tag{6}$$

where

$$D(\bar{\beta}, \tilde{\mu}) = \frac{\int_0^{\pi/2} [H(\eta)]^{-1/2} \sin^2 \eta\, d\eta}{\int_0^{\pi/2} [H(\eta)]^{-1/2}\, d\eta} \tag{7}$$

$$H(\eta) = f^2[(n_0^2 - \bar{\beta}^2) \sin^2 \eta - h(\eta)] + (n_0^2 - n_{cl}^2)\rho^2 \tilde{\mu} \tag{8}$$

and we have used the fact that $G^{-1/2}\, d\xi = H^{-1/2}\, d\eta$. Note that $D(\bar{\beta}, \tilde{\mu}) \le \frac{1}{2}$ and $D \to \frac{1}{2}$ as $f/\rho \to 0$, i.e. of vanishingly small eccentricity.

WKB method: The WKB approach uses the fact that γ is twice the imaginary part of the propagation constant $\beta = \bar{\beta} k$. Using a similar approach to that applied for circular fibres,[9] we find:

$$\gamma = \frac{1}{2} \left| \frac{\partial I}{\partial \beta} \right|^{-1} \exp\left[-2k \int_{\xi_{tp}}^{\xi_{rad}} |G(\xi)|^{1/2}\, d\xi\right] \tag{9}$$

where

$$\left| \frac{\partial I}{\partial \beta} \right| = \frac{1}{2k} \int_{\xi_{min}}^{\xi_{tp}} [G(\xi)]^{-1/2} \times \left[2\beta f^2 \sinh^2 \xi + \frac{\partial \mu}{\partial \beta}\right] d\xi \tag{10}$$

and $\mu = \tilde{\mu} k^2 \rho^2 (n_0^2 - n_{cl}^2)$. Using the standing-wave condition[7]

$$k \int_0^{\pi/2} [H(\eta)]^{1/2}\, d\eta = \frac{\pi \nu}{2} \tag{11}$$

it follows that

$$\frac{\partial \mu}{\partial \beta} = 2\beta f^2 D(\bar{\beta}, \tilde{\mu}) \tag{12}$$

where $D(\bar{\beta}, \tilde{\mu})$ is given by eqn. 7, and ν is the number of nodes in the η-direction. Thus the WKB method gives an identical result for γ to that found from the ray approach above.

Numerical results: Fig. 2 shows the leaky-mode attenuation calculated from eqn. 9 for a fibre with $\alpha = 2{\cdot}05$ and three different values of core eccentricity. The mode groups are labelled by the principal mode number[9] $M = 2m + \nu + 1$, where m and ν are the numbers of nodes in the ξ and η directions, respectively. The relation[9] $2M(n_0^2 - n_{cl}^2)^{1/2} = k\rho(n_0^2 - \beta^2)$ is found to hold approximately. For very small eccentricities the minimum attenuation within a mode group (i.e. that of the $m = 0$ mode) is roughly the same as for the circular case. However, for moderate values of eccentricity and $\alpha \approx 2$, quite large increases of minimum attenuation are found. As the ellipticity increases, the value of f increases, and the number of leaky modes having an attenuation less than some fixed value decreases; this is in qualitative agreement with the results of Reference 4. Eventually, for a certain value of ellipticity, no leaky modes can propagate in the fibre.[6] The principal merit of the present approach lies in its applicability to a range of profiles of the form of eqn. 1, as opposed to the limitations of the specific form of profile used in Reference 4.

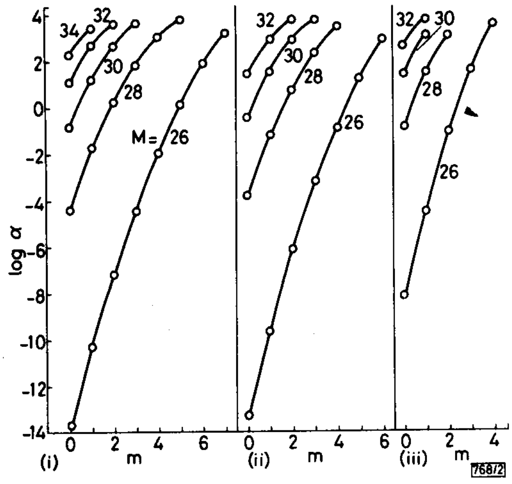

Fig. 2 *Leaky-mode attenuation γ(dB/m) for $\alpha = 2{\cdot}05$ for various mode groups: (i) $b = 1{\cdot}00001$ ($f/\rho = 0{\cdot}017$), (ii) $b = 1{\cdot}001$ ($f/\rho = 0{\cdot}175$), (iii) $b = 1{\cdot}005$ ($f/\rho = 0{\cdot}424$). Parameters are $\rho = 30$ μm, relative index difference $\Delta = 0{\cdot}01$, and normalised frequency $V = 50$*

Conclusion: We have shown how to calculate the attenuation constant of leaky modes in a general class of graded-index noncircular fibres. Geometrical optics and WKB analyses give the same result. The implications for near-field scanning profile measurement are that a reduced correction factor may be needed for elliptical fibres with $\alpha \approx 2$; experimental evidence, however, indicates that the full circular correction factor is required. It is thought this discrepancy may be associated with deviations from α-law profiles, in particular those associated with 'ripples' resulting from the c.v.d. layers.

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