

# An optimized technique for backscatter attenuation measurements in optical fibres

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A technique which gives maximum range and sensitivity for estimating fibre attenuation from the backscattered light has been evolved. With this technique stability problems in the source and receiver are overcome by taking two different time samples for each pulse launched into the fibre, the attenuation being derived from the ratio of the two samples. An analysis indicates that an optimum operating strategy exists in terms of the choice of system parameters such that the range over which attenuation can be measured is maximized. Results indicate that local loss measurements can be performed on fibre sections which are remote from the source by a distance corresponding to a fibre attenuation of 27 dB. Furthermore, preliminary experimental results are presented to demonstrate the viability of the technique.

## 1. Introduction

Light backscattered from an optical pulse launched periodically into an optical fibre provides information from which the attenuation characteristics of the fibre can be derived [1]. Instruments for backscatter measurement typically use an optical time-domain reflectometer and extract the return signal from noise introduced in the receiver by point-by-point averaging with a boxcar integrator [2, 3]. Such an approach, however, necessitates the use of a very stable optical source and detector since any variation in signal level during averaging limits the accuracy of the measurement. Furthermore boxcar integrators are expensive, bulky and not generally suited to field use.

We propose in this paper an alternative technique for backscatter analysis which does not require a rigidly controlled and stable source and detector. With this technique stability problems are overcome by taking two different time samples for each pulse launched into the fibre, the attenuation being derived directly from the ratio of the two samples. In addition, the availability of inexpensive microprocessors suggests numerical averaging as a better alternative to boxcar integration. Accordingly, parameters pertaining to digital data acquisition are taken into account.

It is shown that with the double sampling or two-point technique the accuracy with which the attenuation can be determined depends markedly on the time separation of the samples, and that an optimum spacing exists. The effects of other relevant parameters such as the receiver gain, timing jitter, and averaging period are also considered.

## 2. Two-point attenuation measurements

An optical pulse propagating in a fibre scatters light and some of this is trapped by the fibre and returned to the source. The returned signal is observed as an exponentially decaying pulse, the rate of decay being determined by the fibre attenuation. Subject to the condition that the exciting pulse is sufficiently narrow for insignificant local attenuation to occur over its spatial width in the fibre, i.e.

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$$W \ll \frac{1}{\alpha v_g} \quad (1)$$

the backscattered signal,  $p(t)$ , is given by [4, 5]

$$p(t) = \frac{P_0 W S \alpha_s v_g}{2} \exp(-\alpha v_g t) \quad (2)$$

where  $P_0$  is the peak power launched,  $W$  the optical pulse width,  $S$  the backscatter factor, i.e. the proportion of the scattered light collected by the fibre in the reverse direction,  $\alpha_s$  the Rayleigh scattering loss,  $v_g$  the average group velocity,  $\alpha = (\alpha' + \alpha'')/2$  with  $\alpha'$  and  $\alpha''$  being the respective average attenuations of the forward travelling and backscattered pulses.

It has been shown [4, 6] that the backscatter factor  $S$  for a step-index fibre is

$$S = \frac{3(NA)^2}{8n_1^2} \quad (3a)$$

and that for a graded-index fibre

$$S = \frac{(NA)^2}{4n_1^2} \quad (3b)$$

where  $(NA)$  is the numerical aperture and  $n_1$  the refractive index at core centre.

Equation 1 can also be interpreted in spatial terms by the variable transformation

$$t = \frac{2l}{v_g} \quad (4)$$

where  $l$  is the distance in metres to the scattering point.

With the standard point-by-point approach to attenuation measurement [2, 3] an estimate of  $p(t)$ , denoted  $\hat{p}(t)$ , is extracted from the receiver noise by a boxcar integrator which progressively samples and averages each point on the waveform. An average  $\hat{\alpha}(l)$  is then derived from the slope of  $\ln[\hat{p}(t)]$ . When near maximum range, the receiver signal-to-noise ratio is low and the scan time must be long to obtain adequate noise averaging. This imposes rigid limits on the stability of the source, detector and integrator if the measured accuracy in  $\hat{\alpha}$  is to be maintained.

The configuration of the proposed two-point technique is shown in Fig. 1. An optical pulse obtained from a semi-conductor laser-diode is launched into the fibre and the resulting backscattered power is detected on an avalanche photodiode and amplified for analysis. A small fraction of the launched pulse is reflected from the beam splitter and detected to provide a timing reference for the time-delay generator. The latter provides timing pulses to enable the sample and hold circuits, S-H 1 and 2, to sample the amplified backscatter waveform at times  $t_1$  and  $t_2$  respectively. Note that since the time-delay generator is synchronized to the optical pulse, jitter in the laser pulse does not result in uncertainty in the time position of the sampling windows. The analogue samples denoted  $\hat{p}(t_1)$  and  $\hat{p}(t_2)$  are converted into digital form (A - D converted) for processing. The results of many such samples may be numerically averaged to provide an estimate of the fibre attenuation

$$\hat{\alpha}(t_1, t_2) = \frac{\ln[\hat{p}(t_1)/\hat{p}(t_2)]}{v_g(t_2 - t_1)} \quad (5)$$

where  $\hat{\alpha}(t_1, t_2)$  represents the measured attenuation over the section of fibre in the region  $t_1 v_g/2 \leq l \leq t_2 v_g/2$ . An advantage of the above scheme is that variations in the level of backscatter signal from pulse to pulse does not affect the accuracy of the calculation since the two sample sets are averaged over the same group of laser pulses. Accordingly, the stability of the laser-diode source, the avalanche photo-detector and amplifier is not critical and simple economic circuits may be employed. Furthermore, in

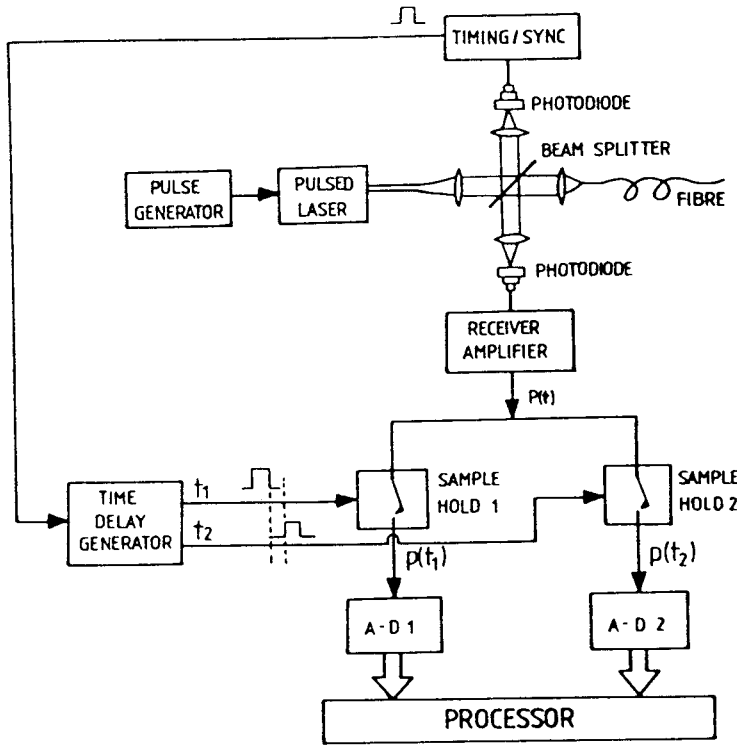


Figure 1 Schematic of the two-point sampling technique for backscatter attenuation measurement.

order to eliminate any gain errors in the sampling and the conversion circuits, the sample gates could on alternate pulses operate at times of  $t_1$  and  $t_2$  and then at times  $t_2$  and  $t_1$ .

To obtain the attenuation characteristics of the whole fibre the sample pair is scanned across  $p(t)$  and further application of Equation 5 yields  $\hat{\alpha}(t, t + \Delta T)$ , where  $\Delta T$  is the time separation of the samples, i.e.  $\Delta T = t_2 - t_1$ .

### 3. Optimization of the two-point sampling method

If  $\epsilon$  represents the worst-case error in the estimate of  $p(t_1)$  and  $p(t_2)$ , the maximum error in the measured attenuation  $\hat{\alpha}(t_1, t_2)$  occurs when

$$\frac{\hat{p}(t_1)}{\hat{p}(t_2)} = \frac{p(t_1) + \epsilon}{p(t_2) - \epsilon} \quad (6)$$

Defining  $\delta$  as the maximum error in  $\hat{\alpha}(t_1, t_2)$ , i.e.

$$\delta = |\hat{\alpha}(t_1, t_2) - \alpha(t_1, t_2)|_{\max} \quad (7)$$

it follows from Equations 2, 5 and 6 that

$$\delta = \frac{1}{v_g \Delta T} \ln \left[ \frac{1 + \epsilon/p(t_1)}{1 - \epsilon/p(t_2)} \right]. \quad (8)$$

If the error  $\epsilon$  is small in comparison to the samples  $p(t_1)$  and  $p(t_2)$  then Equation 8 reduces to

$$\delta \simeq \frac{1}{v_g \Delta T} \left[ \frac{\epsilon}{p(t_1)} + \frac{\epsilon}{p(t_2)} \right]. \quad (9)$$

From Equation 5

$$\Delta T = \frac{1}{\alpha v_g} \ln \frac{p(t_1)}{p(t_2)} \quad (10)$$

If the receiver gain is always adjusted such that the first sample  $p(t_1)$  spans the full A - D converter range, we may normalize  $p(t_1)$  to unity. Substituting for  $\Delta T$  in Equation 9 results in

$$\delta \approx \frac{-\alpha\epsilon}{\ln p(t_2)} \left[ 1 + \frac{1}{p(t_2)} \right] \quad (11)$$

From an instrumentation viewpoint the larger the sample error  $\epsilon$  can be, the less stringent are the circuit requirements on such parameters as linearity, offsets and quantization error. The value of  $\epsilon$  which is tolerable in order to obtain a measurement result to within  $\delta$  of its true value  $\alpha$  is shown in Fig. 2 as a function of sample separation  $\Delta T$  for fibres having attenuations of  $3 \text{ dB km}^{-1}$  and  $15 \text{ dB km}^{-1}$ . Consider, for example, the curve for the  $3 \text{ dB km}^{-1}$  fibre ( $\alpha = 6.9 \times 10^{-4} \text{ np m}^{-1}$ ); if the attenuation is to be measured to within  $0.1 \text{ dB km}^{-1}$  ( $\delta = 2.3 \times 10^{-5} \text{ np m}^{-1}$ ), then for sample separations,  $\Delta T$ , of  $0.1 \mu\text{s}$ ,  $10 \mu\text{s}$  and  $30 \mu\text{s}$ , the tolerable error  $\epsilon$  is  $0.02\%$ ,  $1\%$  and  $0.22\%$  respectively.

It is clear from the curve that there is an optimum sample separation,  $\Delta T_{\text{opt}}$ , which maximizes the allowable  $\epsilon$  and thereby minimizes the accuracy requirements of the instrumentation system. Thus for a fibre having  $3 \text{ dB km}^{-1}$  attenuation the sample separation which maximizes attenuation accuracy is approximately  $10 \mu\text{s}$ , corresponding to a  $1 \text{ km}$  length of fibre between the two sample points; the use of other gate separations will necessarily invoke a penalty in measurement accuracy.

The power ratio  $p(t_1)/p(t_2)$  which maximizes attenuation accuracy may be found by differentiating Equation 11. It is found that the optimum is independent of fibre attenuation  $\alpha$  and has a value  $\sim 5.6 \text{ dB}$ . Substituting this value into Equation 10 results in the optimum sample separation

$$\Delta T_{\text{opt}} = \frac{1.28}{\alpha v_g} \quad (12)$$

Therefore, test equipment which is able to adjust the value of  $\Delta T$  until the ratio of the samples is  $5.6 \text{ dB}$  will automatically maximize attenuation measurement accuracy.

The tolerable sampling error  $\epsilon$  at optimum  $\Delta T$  can be determined from Equation 8 and is plotted in Fig. 3 with fibre attenuation  $\alpha$  and required measurement accuracy  $\delta$  as parameters.

It should be noted that the foregoing analysis assumes a measurement in which the signal level  $p(t_1)$  is set to unity, i.e. the gain of the receiver is adjusted such that the first sample is at the maximum level acceptable by the measurement system. If this is not the case then the tolerable sample error  $\epsilon$  is decreased proportionally.

In some cases the optimum sample separation will be difficult to achieve; for example, it is inadvisable to site the two samples within different sections of a fibre link owing to the possible variation of the

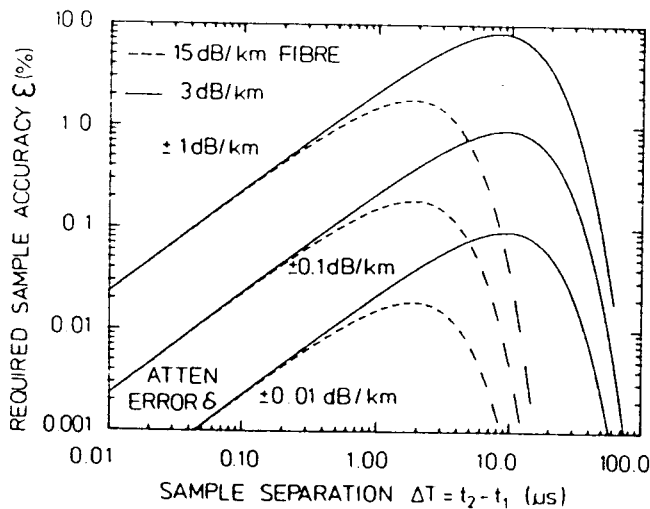


Figure 2 Required sample accuracy,  $\epsilon$ , (expressed as a percentage of measurement system full-scale deflection) versus sample separation for fibres having attenuations of  $3 \text{ dB km}^{-1}$  and  $15 \text{ dB km}^{-1}$ . The turning points in the curves represent an optimum operating condition with respect to sample separation. The curves are drawn for attenuation measurement errors of  $0.01$ ,  $0.1$  and  $\pm 1.0 \text{ dB km}^{-1}$ .

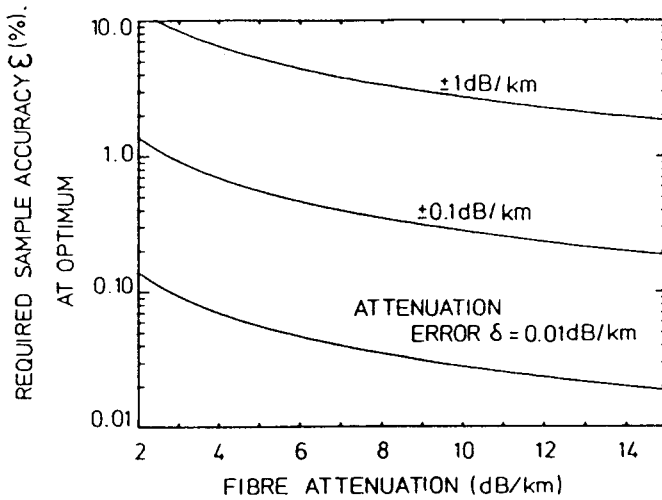


Figure 3 Sampling accuracy,  $\epsilon$  (%), required at optimum sample separation  $\Delta T_{\text{opt}}$  for attenuation measurement errors of 0.01, 0.1 and 1.0 dB km<sup>-1</sup>.

backscatter factor  $S$  between the sections [6]. In addition, the resolution of short-length attenuation variations requires a small gate separation. For non-optimal gate spacings we define the penalty  $P$  to be paid in the tolerable sample error  $\epsilon$  in order to achieve a given attenuation accuracy as

$$P = 10 \log_{10} (\epsilon / \epsilon_{\text{OPT}}) \quad (13)$$

where  $\epsilon_{\text{OPT}}$  is the sample accuracy required at the optimal gate separation. Curves giving the penalty  $P$  as a function of attenuation and  $\epsilon$  are given in Fig. 4. Note that the curve  $P = 0$  corresponds to that for optimal gate separation as given by Equation 12. As an example, let us take a fibre having an attenuation of 3 dB km<sup>-1</sup> for which the optimum sample separation  $\Delta T_{\text{opt}}$  is 10  $\mu$ s. The sample accuracy required for an attenuation error of  $\delta = \pm 0.1$  dB km<sup>-1</sup> is  $\epsilon_{\text{OPT}} = 1\%$  (Fig. 3). Suppose, however, that we require to inspect the fibre over a 250 m section ( $\Delta T = 2.5 \mu$ s). Fig. 4 shows that for the same error in attenuation we now require a 3 dB greater sample accuracy, i.e.  $\epsilon = 0.5\%$ .

The implication of the foregoing is that a compromise is necessary between the achievement of overall attenuation accuracy and the resolution of short-length fluctuations, since the latter requires small gate separations. This results from the difficulty in determining the slow decay in the backscatter signal when the difference between the power levels  $p(t_1)$  and  $p(t_2)$  is small. On the other hand, relatively large localized power variations are found at fault points and imperfections [2], leading to significant differences in signal levels if gates are positioned immediately before and after the point of interest. Thus rela-

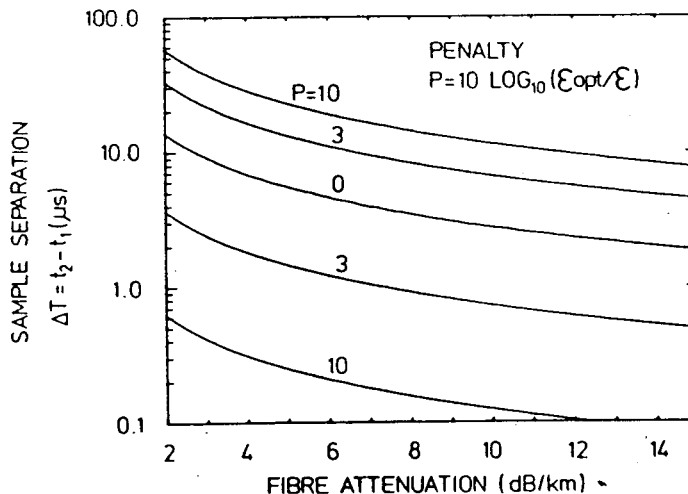


Figure 4 Increased sample accuracy requirements,  $P$  (dB), relative to the accuracy required at optimum spacing  $\Delta T_{\text{opt}}$  as a function of sample separation and fibre attenuation.

tively small gate separations are permitted and the accurate loss measurement of such discontinuities is not precluded.

A further consequence of the existence of an optimum gate separation is that greatest accuracy in the determination of overall link attenuation is obtained by summing the attenuation of individual fibre sections having lengths corresponding to  $\Delta T_{\text{opt}}$ . This approach may well have a further advantage in that it should allow both sample points to be sited within the same fibre section in a jointed link, thus avoiding the problems which can result from variations in the backscatter factor  $S$  between fibre sections. The latter may be caused by changes in numerical aperture or dopant level [6].

#### 4. Noise averaging

When the predominant source of noise on the signal at the processor input is due to quantization,  $\epsilon$  is governed by the accuracy of the A - D converter. Accordingly for an  $n$ -bit converter with  $p(t_1)$  set at maximum level [i.e.  $p(t_1) = 1$ ]

$$\epsilon = \frac{1}{2^{n+1}} \quad (14)$$

When the amplifier thermal noise predominates, numerical sample averaging can be used to reduce  $\epsilon$  to the level required. The noise at the amplifier output may be assumed Gaussian with variance  $\sigma^2$ . Averaging  $N$  samples reduces the variance to  $\sigma^2/N$ . Then taking  $\epsilon$  as the error in  $p(t_1)$  or  $p(t_2)$  which is exceeded with a probability of 5% it follows, from the Gaussian distribution that

$$\frac{\epsilon}{\sigma/\sqrt{N}} = 1.96. \quad (15)$$

Defining the signal-to-noise ratio (SNR) before averaging as

$$(SNR) = \left[ \frac{p(t_1)}{\sigma} \right]^2 \quad (16)$$

we have from Equations 15 and 16

$$\epsilon = \frac{1.96}{\sqrt{N(SNR)}} \quad (17)$$

Fig. 5 is a plot of  $\epsilon$  against  $N$  and (SNR). The graph of Fig. 3 indicates the value of  $\epsilon$  required to obtain a given measurement accuracy in  $\alpha$ . In combination with Fig. 5 the number of samples that must be aver-

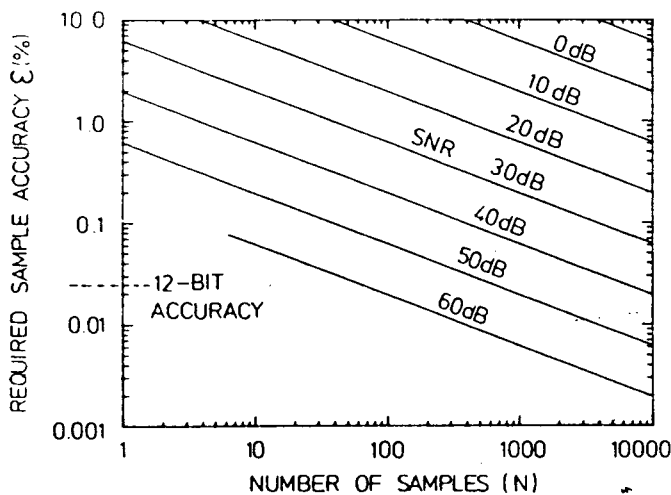


Figure 5 Number of samples,  $N$ , required to reduce the sample error to  $\epsilon$  (%) for various signal-to-noise ratios (SNR).

aged to obtain that  $\epsilon$  can be found. A design example in a later section of the paper will serve to clarify the procedure.

#### 4.1. Receiver amplifier gain

When the signal-to-noise ratio is low the amplifier gain must not be set such that the magnitude of  $p(t_1)$  is so close to the range limit of the A – D converter as to truncate the noise distribution. This will cause a bias in the mean of the samples since the noise cannot average to zero, even after an infinite number of samples.

If  $\pm K$  represents the range limits of the A – D converter and  $\sigma^2$  the variance of the noise  $n(t_1)$  at the amplifier output, then for low (SNR) operation the bias produced in the mean of  $p(t_1) + n(t_1)$  due to truncation is

$$\begin{aligned} \text{bias} = & [K - p(t_1)] \int_{[K-p(t_1)]/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\beta^2/2} d\beta \\ & - [K + p(t_1)] \int_{[K+p(t_1)]/\sigma}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\beta^2/2} d\beta \\ & - \sigma \int_{[K-p(t_1)]/\sigma}^{[K+p(t_1)]/\sigma} \beta \frac{1}{\sqrt{2\pi}} e^{-\beta^2/2} d\beta. \end{aligned} \quad (18)$$

To keep this bias to an acceptable level of less than  $10^{-5} p(t_1)$  it is necessary that

$$\frac{K - p(t_1)}{\sigma} \geq 4 \quad (19)$$

From Equations 16 and 19 it follows that the A – D converter range utilization is

$$\frac{p(t_1)}{K} = \frac{\sqrt{(SNR)}}{4 + \sqrt{(SNR)}} \quad (20)$$

At low (SNR) as will arise when measuring backscatter signal levels from sections of fibre remote from the launch point, the permitted utilization of the full A – D converter range is necessarily reduced. In this case the amplifier gain must be lowered to decrease the level of  $p(t_1)$ . Equation 9 shows that a reduction in  $p(t_1)$  requires that the sample error  $\epsilon$  be reduced proportionately in order to maintain the same attenuation accuracy. It is this effect that will ultimately determine the maximum attenuation range of any backscatter apparatus.

#### 5. Timing sensitivity

Sample and Hold aperture uncertainty,  $U$ , (i.e. timing jitter) produces an amplitude error when sampling fast slewing signals. For the sampling of  $p(t_1)$  the error,  $\epsilon_a$ , is equal to

$$\frac{\epsilon_a}{p(t_1)} = U \alpha v_g \quad (21)$$

where, from Equation 2,  $1/\alpha v_g$  is the time constant of  $p(t)$ .

Fig. 6 plots  $\epsilon_a$  against  $U$  for a range of fibre attenuations for the case  $p(t_1) = 1$ . From these curves, and those of Fig. 3, it can be seen that if the aperture uncertainty is not to be limiting, an uncertainty time of some 100 ps is required. This fortunately is possible with current technology.

#### 6. Design example

To demonstrate the utility of the analysis reported in this paper we derive the maximum acceptable fibre attenuation after which local loss measurements can be performed, subject to the assumption that the sample separation is set optimally at  $\Delta T_{\text{opt}}$  and that the number of samples averaged,  $N$ , is limited to

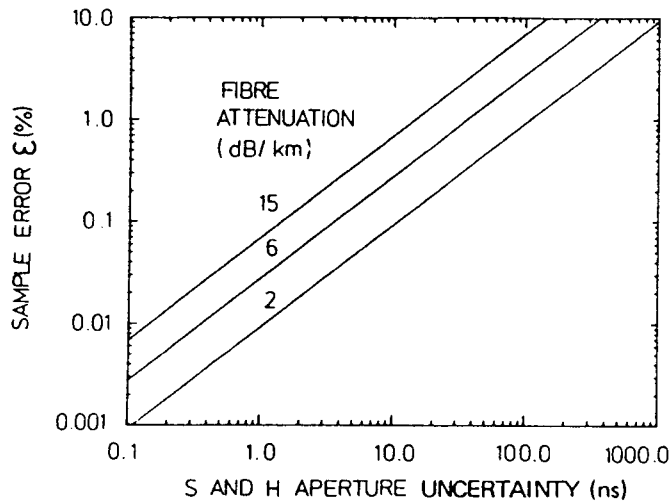


Figure 6 Sample error  $\epsilon$ (%) as a function of aperture timing uncertainty for fibre attenuations 2, 6 and 15 dB km<sup>-1</sup> under optimum operating conditions.

1000. With a pulse repetition rate of 1000 pps the averaging time for each fibre section would as a consequence be 1 s.

For an optical pulse width of 100 ns and a peak power of 1 W launched into a graded-index fibre of 0.2 numerical aperture the backscatter response from Equations 2 and 3b is

$$p(t) = (4.4 \times 10^{-2}) \alpha_s \exp [(-2 \times 10^8) \alpha t] \text{ watts} \quad (22)$$

where  $n_1 = 1.5$ ,  $v_g = 2 \times 10^8 \text{ m s}^{-1}$ .

Taking into account the 3 dB loss in the beam splitter and assuming a photodetector responsivity of 70 A W<sup>-1</sup> the detected photocurrent is

$$i_{p(t)} = 1.56 \alpha_s \exp [(-2 \times 10^8) \alpha t] \text{ amps} \quad (23)$$

or

$$i_{p(t)} = 1.56 \alpha_s \exp (-2 \alpha l) \text{ amps} \quad (24)$$

The backscatter signal,  $i_{p(t)}$ , has a 3 dB (electrical) bandwidth which ranges from 22 to 110 kHz for fibres having attenuations of 3–15 dB km<sup>-1</sup>. For the determination of attenuation the receiver bandwidth must be such as to pass  $p(t)$  without additional dispersion. A greater demand for receiver bandwidth arises when there is a need for accurate fault location for which the receiver must accommodate pulses of width  $W$  without dispersion. In either case it is advantageous to keep the receiver bandwidth as low as possible to maximize the signal-to-noise ratio. For the purposes of this example we choose a receiver with 10 MHz bandwidth and an input equivalent noise current of 1 pA Hz<sup>-1/2</sup>. It follows that the variance of the noise is

TABLE I Maximum fibre loss after which localized attenuation measurements can be made to the accuracy stated

Required attenuation accuracy (dB km <sup>-1</sup> )	Measurable insertion loss (dB)
0.01	22
0.1	27
1	31



$$\sigma_{n(t)}^2 = 10^{-17} (\text{amp}^2) \quad (25)$$

For a sample at  $t_1 = 2l/v_g$  the signal-to-noise ratio is then

$$(SNR) = (1.56)^2 \alpha_s^2 \times 10^{17} \exp(-4\alpha l) \quad (26)$$

The decay of  $(SNR)$  with length determines the maximum fibre attenuation after which measurements can be made to a desired accuracy (the 'range'). From Fig. 3 we can find the tolerable sample accuracy  $\epsilon$  for a measurement resolution  $\delta$  in a fibre having nominal attenuation  $\alpha$ . With this value of  $\epsilon$  and with  $N = 1000$ , Fig. 5 yields the minimum allowable signal-to-noise ratio  $(SNR)_{\min}$ . Then solving Equation 26 with  $(SNR)_{\min}$  gives  $l_{\max}$  from which the range,  $R$ , is

$$R = l_{\max} + \frac{1}{2} v_g \Delta T_{\text{opt}}. \quad (27)$$

The range can be expressed in terms of the maximum allowable one-way fibre loss by multiplying by the fibre attenuation  $\alpha$ . Table I shows the range for measurement accuracies  $\delta = \pm 0.01, \pm 0.1, \pm 1 \text{ dB km}^{-1}$ . It has been assumed in the calculation that  $\alpha = \alpha_s$ , i.e. that the fibre attenuation results predominantly from Rayleigh scattering. We see that for a measurement accuracy of  $\delta = \pm 0.1 \text{ dB km}^{-1}$  the maximum fibre attenuation after which a two-point measurement with optimum gate spacing (5.6 dB) could be made is 27 dB. The realization of this figure would permit the determination of the attenuation of long fibre links with high accuracy.

## 7. Experimental results

As an example of the implementation of the two-point sampling technique a preliminary measurement of the length dependence of attenuation in a 2.8 km multimode graded-index test fibre is given in Fig. 7. In this case a non-optimal gate spacing of 480 ns (50 m) was used in order to demonstrate the ability of the method to resolve localized attenuation fluctuations. It may be seen that the attenuation is revealed directly with a high degree of accuracy and that the resolution is sufficient to detect the series of small ( $\pm 0.5 \text{ dB km}^{-1}$ ) fluctuations occurring at the remote end of the fibre. The complete measurement could be reproduced to within  $\pm 0.05 \text{ dB km}^{-1}$  on subsequent occasions.

Work is continuing to further experimentally characterize the accuracy, reproducibility and ultimate range performance of the two-point technique.

## 8. Conclusions

We have proposed and demonstrated in this paper a new technique for backscatter attenuation measure-

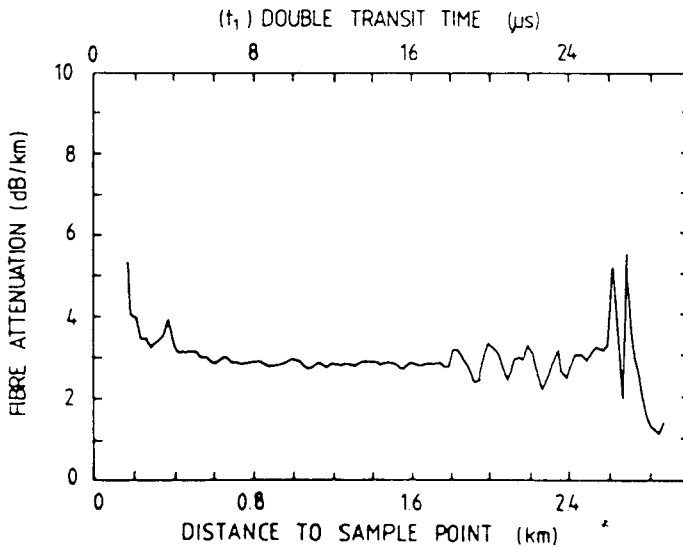


Figure 7 Experimentally-determined length-dependence of fibre attenuation obtained at a wavelength of 870 nm for a multimode, graded-index fibre.

ment which does not require a complex optical source or receiver. With the technique, stability problems are overcome by taking two different time samples for each pulse launched into the fibre, the attenuation being derived from the ratio of the two samples. An optimum operating condition was found to exist in terms of the time interval between the samples. Optimum operation provides for maximum accuracy in the estimated attenuation with the least sensitivity to measurement error.

System design for processor control has been considered and the effects of parameters such as receiver gain and timing sensitivity analysed, together with the operation of numerical averaging for improved accuracy. The theoretical and experimental results indicate that the two-point sampling technique is attractive for the development of an automated field or laboratory instrument for fibre attenuation measurement and fault location.

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### References

1. M. K. BARNOSKI and S. M. JENSEN, *Appl. Opt.* 15 (1976) 2112.
2. M. K. BARNOSKI, M. D. ROURKE, S. M. JENSEN and R. T. MELVILLE, *Appl. Opt.* 16 (1977) 2375.
3. B. COSTA and B. SORDO, *3rd European Conference on Optical Communication*, Munich (1977).
4. E.-G. NEUMANN, *Appl. Opt.* 17 (1978) 1675.
5. S. D. PERSONICK, *Bell System Tech. J.* 56, (1977) 355.
6. E.-G. NEUMANN, to be published.