Phase and group indices for double heterostructure lasers

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Abstract: Various models for calculation of refractive indices in semiconductor laser materials are discussed and the results for GaInAsP compounds are compared. It is shown that the modified single-oscillator model appears to be the most reliable. The group index is also considered since it can be found easily from measurements of laser spectra. The value of the group index is more model sensitive than the refractive index, but it is also a function of the active-layer thickness in double heterostructures as a consequence of waveguide dispersion. A simple formula connecting group and phase indices for a symmetrical dielectric slab waveguide is derived and used to calculate the effective group index for double-heterostructure lasers.

1 Introduction

Semiconductor lasers emitting in the wavelength range 1·3–1·6 μm are of considerable interest for use in optical communications systems since for silica-based fibres at these wavelengths both attenuation[1] and material dispersion[2] are much smaller than for the conventional GaAs emission wavelength. The most promising material used for such lasers at present is the quaternary GaInAsP lattice-matched to InP substrates. For analysis and optimisation of double-heterostructure (d.h.) lasers the values of the refractive indices of the active and passive layers are important parameters. However, reliable index data is lacking as yet for GaInAsP. The object of the present contribution is to review several models available for calculating refractive indices, based on interpolation between such experimental data as are available for relevant binary and ternary materials. The models, to be described more fully in the following Section, are (1) single oscillator,[3] (2) modified single oscillator,[4] (3) interpolated Sellmeier equation and (4) interpolation of index data at the bandgaps of the relevant binaries.[5] In Section 3, models 2–4 are applied to GaInAsP (at compositions which give the same lattice constant as InP) and the results are discussed with reference to experimental methods of measuring the index difference in d.h. lasers.

Another relevant quantity is the group index, which can be found from the longitudinal mode spacing when the laser length is known; calculations of the group index for GaInAsP/InP lasers are presented in Section 4. Since the values predicted for the group index are strongly dependent on the specific model used to calculate the refractive index, it might be possible to distinguish the most accurate model simply by measurements of the longitudinal mode spectra. However, for the detailed interpretation of such measurements it is necessary to take into account waveguide dispersion effects resulting from the d.h. laser structure. This means that the effective group index for a d.h. laser is a function of the active-layer thickness. In Section 5, therefore the relationship between effective d.h. group index and material refractive indices is discussed, and a simple formula is derived which connects these quantities; appropriate results are then given for GaInAsP/InP and GaAs/AlGaAs lasers. Provided that a reliable refractive index model is determined, the above argument may also be applied in the reverse direction; i.e. accurate determination of active region thickness may be found from measurements of the effective group index.

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2 Refractive-index models

2.1 Single oscillator

This model yields a very simple expression[3] for the refractive index n at photon energy E:

$$n^2 - 1 = \frac{E_0 E_d}{E_0^2 - E^2} \tag{1}$$

where $\pi E_d/2$ is the strength of an oscillator at energy E_0. Since this model assumes the imaginary part of the dielectric permittivity ϵ_2 to be a delta function at energy E_0, it will give reasonable results for photon energies well below the absorption edge in semiconductors. Since these energies are not of immediate interest for d.h. lasers, this model will not be considered further.

2.2 Modified single oscillator

Afromowitz[4] has suggested an improved model for the variation of ϵ_2 with photon energy in semiconductors:

$$\epsilon_2 = \begin{cases} \eta E^4 & E_G \leqslant E \leqslant E_f \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

where the new parameters E_f and η can be determined by requiring that the corresponding expression for n^2 agrees with eqn. 1 for $E \ll E_G$. The resultant expression for n^2 is

$$n^2 - 1 = \frac{E_d}{E_0} + \frac{E_d E^2}{E_0^3} + \frac{\eta E^4}{\pi} \ln\left(\frac{2E_0^2 - E_G^2 - E^2}{E_G^2 - E^2}\right) \tag{3}$$

where

$$\eta = \frac{\pi E_d}{2E_0^3(E_0^2 - E_G^2)} \tag{4}$$

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The parameters E_0, E_d and E_G appearing in eqns. 4 and 5 are known for many binary semiconductors and mav be calculated for ternary and quaternary compounds by interpolation. Afromowitz[4] gives values for AlGaAs, GaAsP and GaInP, e.g., for $Al_xGa_{1-x}As$, the results are

$$\left.\begin{aligned} E_0 &= 3{\cdot}65 + 0{\cdot}871x + 0{\cdot}179x^2 \\ E_d &= 36{\cdot}1 - 2{\cdot}45x \\ E_G &= 1{\cdot}424 + 1{\cdot}266x + 0{\cdot}26x^2 \end{aligned}\right\} \quad (5)$$

Recently, values of these parameters have been determined from measurements of refractive index on GaInAs.[6]

For the quaternary $Ga_xIn_{1-x}As_yP_{1-y}$, the values of E_0, E_d and E_G may be estimated from a combination of the values for the binaries GaAs, GaP, InAs and InP (Table 1). Note that the values of E_0 and E_d for InAs differ substantially from those for the other binaries in Table 1; however, for GaInAsP we will consider photon energies above the bandgap of InAs but below the bandgap for the other binaries. A simple interpolation scheme for the quaternary values is given, for example, by

$$\begin{aligned} E_0(Ga_xIn_{1-x}As_yP_{1-y}) &= xyE_0(\mathrm{GaAs}) \\ &+ x(1-y)E_0(\mathrm{GaP}) \\ &+ (1-x)yE_0(\mathrm{InAs}) \\ &+ (1-x)(1-y)E_0(\mathrm{InP}) \quad (6) \end{aligned}$$

and the results may then be used in eqns. 3 and 4. Other methods of interpolation are conveniently reviewed and applied to energy gap and lattice constant in Reference 7; experimental results for these quantities will be discussed below in Section 3.

The principal achievement of the modified single-oscillator model has been its success in yielding excellent agreement (i) with refractive-index measurements on bulk AlGaAs[8] and (ii) with index-difference measurements in d.h. lasers with GaAs active layers.[9] More recently, refractive-index measurements on bulk GaInAs[6] have been interpreted in terms of this model with a generally good level of agreement. An example is shown in Fig. 1, where refractive-index values for photon energies just below the bandgap are plotted as a function of Ga content x in $Ga_xIn_{1-x}As$. The encouraging level of agreement with experimental data for ternary compounds implies that this model should yield good results for quaternaries.

2.3 Interpolated Sellmeier equation

For the four binaries of Table 1, values of refractive indices as functions of wavelength λ are given in Reference 10. It is possible to fit these values by Sellmeier expressions of the form

$$n^2(\lambda) = A_i + \frac{B_i\lambda^2}{\lambda^2 - C_i} \quad (7)$$

Table 1: Values of E_0, E_d and E_G for binary semiconductors at room temperature

Material	E_0 eV	E_d eV	E_G eV
GaAs	3·65	36·1	1·42
GaP	4·51	36·45	2·77
InAs	1·50	16·2	0·36
InP	3·39	28·91	1·35

Table 2: Values of Sellmeier coefficients for binary semiconductors at room temperature

i	Material	A_i	B_i	C_i
1	GaAs	8·95	2·054	0·390
2	GaP	4·54	4·31	0·220
3	InAs	7·79	4·00	0·250
4	InP	7·255	2·316	0·3922

The units are such that the wavelength λ is measured in μm in eqn. 7

where the parameters A_i, B_i, C_i for the material denoted by subscript i are given in Table 2. The refractive index for the quaternary $Ga_xIn_{1-x}As_yP_{1-y}$, n_q, is then given by the interpolated Sellmeier equation

$$n_q^2(\lambda) = \sum_{i=1}^{4}\left(A_i + \frac{B_i\lambda^2}{\lambda^2 - C_i}\right) f_i \quad (8)$$

with

$$f_1 = xy, \quad f_2 = x(1-y), \quad f_3 = (1-x)y,$$
$$f_4 = (1-x)(1-y).$$

Although eqns. 7 and 8 neglect effects due to the absorption band edge, they become better approximations for wavelengths away from the bandgap.

2.4 Interpolated bandgap indices

In Reference 5 the refractive index of GaInAsP at photon energies near the absorption edge is found by interpolation of the index values for the four constituent binaries at their respective bandgaps. This method is known to be unreliable for the ternaries AlGaAs[8] and GaInAs[6] and hence is unlikely to yield very accurate results for quaternaries. In addition, this approach neglects the variation of the refractive index with wavelength for each of the binaries, and hence it is not possible to deduce an expression for the group index of the quaternary.

3 Refractive index for GaInAsP lattice-matched to InP

The requirement for lattice-matching of active and passive layers in a d.h. laser yields a relation between the composition parameters x and y in the quaternary $Ga_xIn_{1-x}As_yP_{1-y}$. Experimental evidence at present

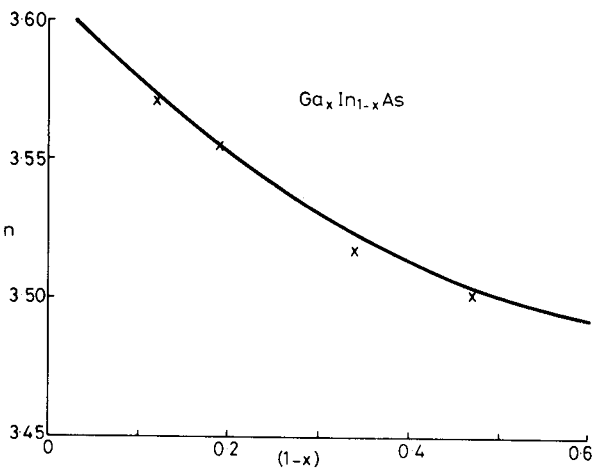

Fig. 1 *Refractive index of $Ga_{1-x}In_xAs$, at a photon energy of 0·03 eV below the bandgap, against composition parameter $(1-x)$*

——— modified single-oscillator model
x experimental data, Reference 6

available[11, 12] indicates that Vegard's law applies for the lattice constants, with the result that, for lattice-matching,[12]

$$x = \frac{0{\cdot}1894y}{0{\cdot}4184 - 0{\cdot}013y} \tag{9}$$

The published experimental data[11, 12] for variation of energy gap E_G with composition y in the lattice-matched quaternary form a better basis for interpolation than the scheme discussed above in Section 2.2, eqn. 6. Hence for the variation of E_G with y we use[12]

$$E_G(y) = 1{\cdot}35 - 0{\cdot}72y + 0{\cdot}12y^2 \tag{10}$$

Assuming that the photon energy E corresponding to the lasing wavelength λ is displaced by a constant energy ΔE from the bandgap E_G, then we have

$$E = E_G(y) - \Delta E \tag{11}$$

with

$$\lambda = \frac{1{\cdot}237}{E} \qquad (\lambda \text{ in } \mu\text{m}, E \text{ in eV}) \tag{12}$$

3.1 Modified single oscillator

For the variation of E_0 and E_d with y, in the absence of better data, we use the form of eqn. 6 which yields (assuming eqn. 9 to be satisfied)

$$E_0(y) = 3{\cdot}39 - 1{\cdot}395y + 0{\cdot}506y^2 \tag{13}$$

$$E_d(y) = 28{\cdot}91 - 9{\cdot}415y + 5{\cdot}978y^2 \tag{14}$$

The refractive index of GaInAsP lattice-matched to InP may now be calculated by the modified single-oscillator model by insertion of eqns. 10, 13 and 14 in eqns. 3 and 4, and use of eqns. 11 and 12. The refractive index for InP is found by using the values of E_G, E_0 and E_d for InP in eqns. 3 and 4, with the wavelength given by eqns. 10–12. The results are shown in Fig. 2, curve *a*.

3.2 Interpolated Sellmeier equation

Using the wavelength of laser emission from eqns. 10–12, the refractive index $n_q(\lambda)$ may be calculated directly from eqn. 8; the result is curve *b* on Fig. 2.

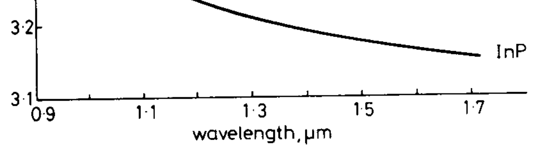

Fig. 2 *Calculated refractive index for InP and $Ga_xIn_{1-x}As_yP_{1-y}$ lattice-matched to InP, with $\Delta E = 0{\cdot}03$ eV*

a Modified single oscillator, eqn. 3, 4, 13 and 14
b Sellmeier interpolation, eqn. 8
c Interpolated bandgap indices, eqn. 15
The wavelength corresponding to a given composition is given by eqns. 10–12

3.3 Interpolated bandgap indices

In Reference 5 the bandgap refractive index found by interpolation is given as

$$n_q(y) = 3{\cdot}4 + 0{\cdot}256y - 0{\cdot}095y^2 \tag{15}$$

Curve *c* on Fig. 2 shows this variation as a function of emission wavelength via eqns. 10–12 and allowing a small offset in n_q as a consequence of the energy displacement ΔE of the lasing emission from the bandgap.

3.4 Comparison of results

The curves *a* and *b* of Fig. 2 differ by a constant value (approximately 0·06) over a large part of the wavelength range. This is explained by the fact that the Sellmeier interpolation does not include the effect of the photon energy lying close to the bandgap of the active layer (except near the InP limit). This omission is of course rectified by the modified single-oscillator model (curve *a*). However, the index variation found from this latter model depends to some extent on the value assumed for ΔE in eqn. 11 and the model cannot be used for ΔE very close to zero. Heavy doping and the presence of high densities of free carriers in d.h. lasers also change the values of refractive index for photon energies close to the bandgap[13] and these effects are not included in any of the present models. In spite of these deficiencies it is felt, on the basis of the arguments presented, that curve *a* is likely to give the best predictions for d.h. laser analysis.

It is interesting to note that for bandgap differences $\Delta E_G = E_G(\text{InP}) - E_G(y)$ between 0·2 and 0·5 eV the ratio $\Delta\epsilon/\Delta E_G$ ($\Delta\epsilon = n_q^2(y) - n_{InP}^2$) using curve *a* of Fig. 2 only changes from 4·3/eV to 3·8/eV. These values are close to the corresponding values in d.h. lasers with GaAs active layers and AlGaAs passive regions (4·6/eV to 3·8/eV), found from the modified single oscillator model for AlGaAs with data from Reference 4. The possibility of this similarity has already been pointed out on the basis of experimental measurements by the authors of Reference 14.

The value of the refractive index in the active layer, n_q, cannot be measured directly, but since the refractive index for InP is well established[15] it is sufficient to measure the dielectric constant step $\Delta\epsilon$. This quantity is found indirectly by using the fact that the far-field beam angle perpendicular to the junction plane depends on wavelength λ, active layer thickness d, and $\Delta\epsilon$[9, 14, 16–19] The method can therefore only be used in this case if reliable data for active layer thickness are available. The experimental results have to be corrected for the obliquity factor,[20, 21] and if the measurements are made by scanning linearly instead of rotating the laser then further corrections are needed. Thus the small amount of available data[14, 18, 19] on $\Delta\epsilon$ for GaInAsP d.h. lasers shows considerable scatter. At 1·31 μm a value[18] for $\Delta\epsilon$ of 2·0 has been reported, as compared with $\Delta\epsilon = 1{\cdot}43$, 1·04 and 1·96 from curves *a*, *b*, *c*, of Fig. 2, respectively. For lasers emitting at 1·25 μm, the published value[19] of 2·0 is again larger than all three results from Fig. 2 (1·31, 0·91, 1·81). At 1·15 μm an experimentally obtained value[14] of $\Delta\epsilon = 0{\cdot}75$ is to be compared with 1·04, 0·66 and 1·46 from Fig. 2. We conclude that verification of one or other of the present models for refractive index must await further measurement over a wider wavelength range.*Alternatively, a different technique for inferring index values from measurements other than far fields might perhaps be useful in this respect; one such technique is suggested in Sections 4 and 5.

4 Group index calculations

The group index $\bar{n}$ is defined by

$$\bar{n} = n - \lambda\frac{dn}{d\lambda} = n + E\frac{dn}{dE} \qquad (16)$$

and is related to the longitudinal modespacing, $\Delta\lambda$, by

$$\bar{n} = \frac{\lambda^2}{2L\Delta\lambda} \qquad (17)$$

where L is the laser length.

4.1 Modified single oscillator

Using eqn. 3 in eqn. 16 we find

$$\bar{n} = 3n - \frac{2}{n}\left[1 + \frac{E_d}{E_0} \times \left\{1 + \frac{E^2}{2E_0^2}\left(1 - \frac{E^4}{(2E_0^2 - E_G^2 - E^2)(E_G^2 - E^2)}\right)\right\}\right] \qquad (18)$$

the group indices for InP and GaInAsP lattice-matched to InP are found by using eqns. 10–14 in eqn. 18; the results are shown in Fig. 3 as functions of wavelength (solid lines).

4.2 Interpolated Sellmeier equation

Using eqn. 8 in eqn. 16 yields:

$$\bar{n}_q = n_q\left\{1 + \left(\frac{\lambda}{n_q}\right)^2 \sum_{i=1}^{4} \frac{B_i C_i f_i}{(\lambda^2 - C_i)^2}\right\} \qquad (19)$$

Results for InP and GaInAsP lattice-matched to InP are shown as broken lines on Fig. 3.

It is clear from Fig. 3 that there is a large difference between the values for group index predicted by the two models. However, the group index given by eqn. 19 is less likely to be correct than that from eqn. 18 due to the omission of the absorption edge effect on n in the model leading to eqn. 19. Since the group index is relatively simple to measure, it would appear that the easiest way of determining which model is most reliable will be from measurements of longitudinal mode-spacing rather than those of far fields.

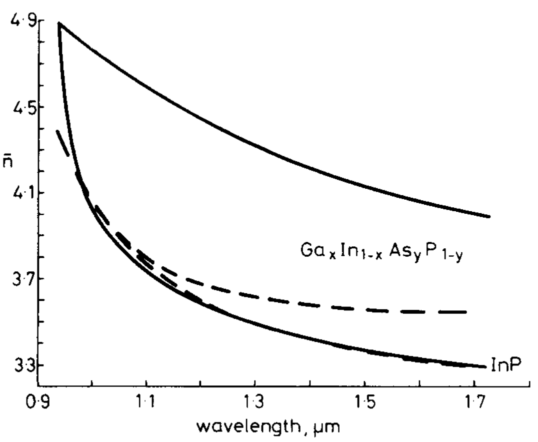

Fig. 3 *Calculated group index for InP and $Ga_xIn_{1-x}As_yP_{1-y}$ lattice-matched to InP, with $\Delta E = 0{\cdot}03$ eV*

Solid lines – Modified single-oscillator, eqn. 18
Broken lines – Sellmeier interpolation, eqn. 19

*More recent measurements at 1·15 μm, 1·2 μm and 1·3 μm have confirmed curve *a* of Fig. 2, (References 22 and 23)

5 Group index for symmetric slab waveguides

In a d.h. laser some fraction of the optical field propagates in the passive layers and it is necessary therefore to consider an effective index which is a weighted mean of the indices in the active and passive layers (each assumed to be of semi-infinite extent). With d as the active layer thickness and λ as the wavelength, the normalised frequency v is defined as

$$v = \frac{\pi d}{\lambda}(n_1^2 - n_2^2)^{1/2} \equiv \frac{\pi d}{\lambda}\Delta\epsilon^{1/2} \qquad (20)$$

We restrict the discussion here to TE modes since TE polarisation is usually observed in d.h. laser emission; however, in 'weakly-guiding' situations ($n_1 - n_2 \ll n_1$) the TM and TE modes have almost identical propagation constants. The effective phase index, n_{eff}, is now defined by

$$n_{eff}^2 = bn_1^2 + (1 - b)\,n_2^2 \qquad (21)$$

where b is a normalised propagation constant[24] related to v by the characteristic equation:

$$b^{1/2} = (1 - b)^{1/2} \tan\{v(1 - b)^{1/2}\} \qquad (22)$$

Since we are only concerned with the fundamental transverse mode, we have

$$0 \leqslant v(1 - b)^{1/2} < \frac{\pi}{2} \qquad (23)$$

It remains to find an effective group index for the waveguide, defined by analogy with eqn. 16 as

$$\bar{n}_{eff} = n_{eff} - \lambda\frac{dn_{eff}}{d\lambda} \qquad (24)$$

The calculation of $\bar{n}_{eff}$ is simplified by introducing the intensity filling factor[25] Γ:

$$\Gamma = \frac{\int_{-d/2}^{d/2} |E(x)|^2 dx}{\int_{-\infty}^{\infty} |E(x)|^2 dx} \qquad (25)$$

where $E(x)$ is the field distribution in the x-direction, i.e. perpendicular to the junction plane. Using the relations[26]

$$\Gamma = b + vb^{1/2}\left(\frac{1 - b}{1 + vb^{1/2}}\right) \qquad (26)$$

and[24]

$$\frac{db}{dv} = 2b^{1/2}\left(\frac{1 - b}{1 + vb^{1/2}}\right) \qquad (27)$$

it follows that Γ may also be expressed as

$$\Gamma = \frac{1}{2}\left(b + \frac{d(vb)}{dv}\right) \qquad (28)$$

Fig. 4 shows plots of b, $d(vb)/dv$ and Γ against v calculated for symmetric slab waveguides from the solutions of eqn. 22 for the fundamental TE mode. Inserting eqn. 21 in the definition of $\bar{n}_{eff}$ in eqn. 24 and using eqn. 28 yields the simple relationship

$$\bar{n}_{eff} n_{eff} = \Gamma \bar{n}_1 n_1 + (1-\Gamma) \bar{n}_2 n_2 \qquad (29)$$

where $\bar{n}_1$, $\bar{n}_2$ are the group indices inside and outside the active layer, respectively. We note in passing that eqn. 29 is merely the specific form for a symmetric-slab waveguide of a more general relationship which holds for guides of arbitrary index profile.[27]

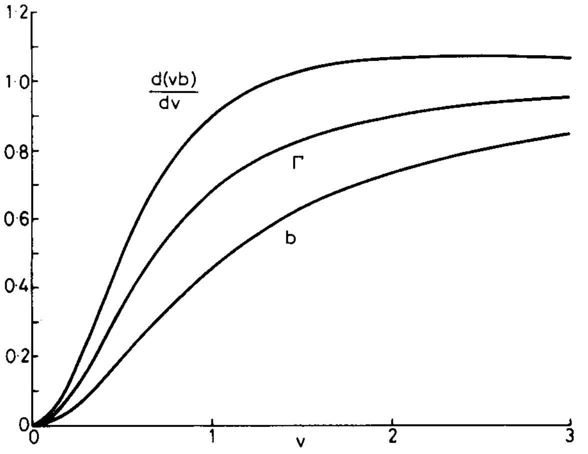

Fig. 4 *Waveguide parameters b, d (vb)/dv and Γ against v for the fundamental TE mode of a symmetric slab waveguide calculated from the solutions of eqn. 22*

Since Γ depends on the thickness d of the active layer, it follows that $\bar{n}_{eff}$, as measured from laser spectra, is also dependent on the thickness. Results calculated for $\bar{n}_{eff}$ of GaInAsP d.h. lasers are given in Fig. 5 as a function of composition parameter y (related to wavelength λ) and thickness d.

Eqn. 29 is simpler than the approximate expression given in eqn. 5.5.17 of the book by Kressel and Butler[28] where the assumption $(d/d\lambda)(n_1^2 - n_2^2) \simeq 2(n_1 - n_2)(dn_1/d\lambda)$ was made. This assumption is invalid since the material dispersion in the active layer will always be substantially different from that in the passive layers because the photon energy is always close to the bandgap in the former case and not in the latter. This point is immediately obvious from Fig. 3 for GaInAsP/InP lasers but applies equally well to the AlGaAs/GaAs system. Eqn. 29 is also more general than the expression given in eqn. 12 of Reference 29 where the modes were assumed to be those of a rectangular box with metallic walls. The latter assumption ignores the effects of power travelling within the passive layers of the laser and cannot therefore predict the correct thickness dependence of $\bar{n}_{eff}$.

The calculated results of Fig. 5 are presented in a different form in Fig. 6 where $\bar{n}_{eff}$ is plotted against active layer thickness d for wavelengths of 1·1 μm, 1·3 μm and 1·55 μm. As regards comparison with experimental measurements, Hsieh *et al.*[30] have reported a value of $\bar{n}_{eff} = 4{\cdot}1$ at 1·1 μm for a d.h. laser with $d = 0{\cdot}5\,\mu$m whereas Fig. 6 gives $\bar{n}_{eff} = 4{\cdot}4$. However, it should be noted that the experimental value was obtained on a 25 μm stripe-contact laser whilst the calculations presented above assume broad-contact devices. In principle, eqn. 29 could be adapted to deal with stripe contacts, but in practice the details of the lateral guiding mechanism would introduce a high degree of uncertainty into the calculation. For broad contact lasers at

Table 3: Values for $\bar{n}_{eff}$ published or derived from published spectra compared with values found using the modified single-oscillator model

λ	d	Expt $\bar{n}_{eff}$	Theory	Stripe width	Ref.
μm	μm			μm	
1·1	0·5	5·1	4·45	broad area	33
1·1	0·5	4·1		25	30
1·145	0·6	4·2	4·44	5	34
1·234	0·15	4·2	3·81	12	19
1·295	0·5	3·8	4·23	20	35
1·31	0·5	4·3	4·21	20	31, 32
1·675	1·5	4·2	4·02	20	36

1·25–1·35 μm with $d = 0{\cdot}4$-$5\,\mu$m Yamamoto *et al.*'s[31] measurements indicate $\bar{n}_{eff} \simeq 4{\cdot}3$ which lies within the range 4·1–4·4 given by Fig. 6 for these parameters. Further comparisons of the theoretical predictions with experimental data are given in Table 3. Good agreement can only be expected if the published values for length and thickness are precise. It is seen from Table 3 that, in spite of some disagreement, no systematic discrepancies are seen to be present. Note the result for 1·145 μm is for a rib-guide structure which could give some extra index difference.

The thickness dependence of $\bar{n}_{eff}$ is strongest for $d < 0{\cdot}4\,\mu$m and may give a simple method for estimating d

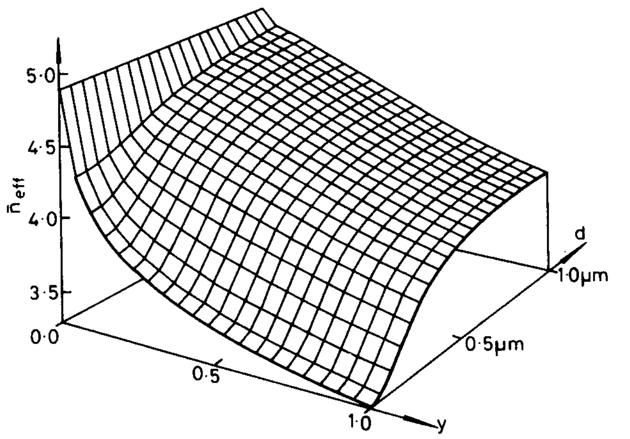

Fig. 5 *Effective group index $\bar{n}_{eff}$ for lattice-matched GaInAsP/InP d.h. lasers as a function of composition parameter y and active layer thickness d, calculated by the modified single-oscillator model with ΔE = 0·03 eV*

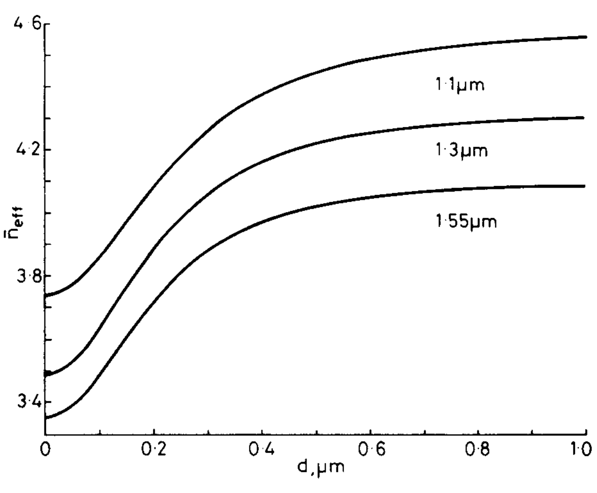

Fig. 6 *Effective group index $\bar{n}_{eff}$ for lattice match GaInAsP/InP d.h. lasers as a function of active-layer thickness d for λ = 1·1, 1·3 and 1·55 μm, calculated by the modified single-oscillator model with ΔE = 0·03 eV*

from measurements of longitudinal mode spacing if a reliable refractive index model is established. In AlGaAs lasers the value of $\bar{n}_{eff}$ will also be thickness dependent and an example calculated for this system is shown in Fig. 7. The sharp rise of $\bar{n}_{eff}$ with d in the plots of Figs. 6 and 7 imply that spectral measurements may provide a sensitive means of determining accurate active layer thicknesses in d.h. lasers. The most sensitive variation of $\bar{n}_{eff}$ is given by the maximum value of $d\Gamma/dv$ which occurs for $v \simeq 0{\cdot}4$. Note that this is also the typical v-value where minimum values of threshold current occur, i.e. $\bar{n}_{eff}$ is most sensitive to d for the most interesting d-values.

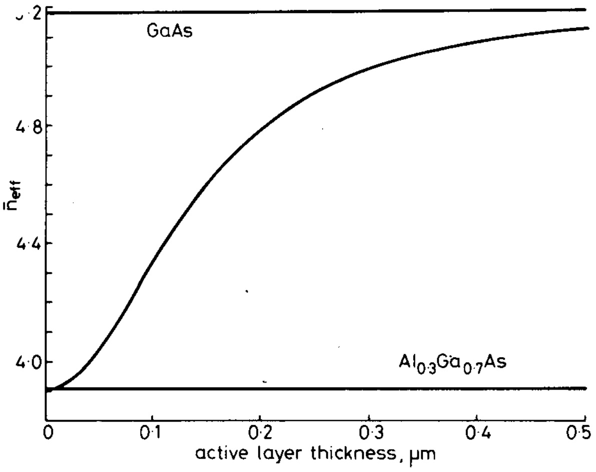

Fig. 7 *Thickness variation of the effective group index for a symmetric d.h. laser with GaAs active layer and $Al_{0·3}Ga_{0·7}As$ passive layers calculated by the modified single-oscillator model with $\Delta E = 0{\cdot}03\,eV$*

6 Guiding properties of GaInAsP lasers

The guiding properties of d.h. lasers depend on the v-value (eqn. 20), i.e. for a given thickness d the best confinement is achieved for a high value of $\pi\sqrt{\Delta\epsilon}/\lambda$. Using curve a in Fig. 2 this quantity has a rather flat maximum for $\lambda = 1{\cdot}35\,\mu m$ ($\pi\sqrt{\Delta\epsilon}/\lambda = 2{\cdot}89\,\mu m^{-1}$); values within 1% of the maximum are found in the range $1{\cdot}25\,\mu m < \lambda < 1{\cdot}50\,\mu m$. A typical value for $Al_xGa_{1-x}As$ d.h. lasers is $\pi\sqrt{\Delta\epsilon}/\lambda \simeq 4{\cdot}5\,\mu m^{-1}$ (30% difference in Al content). This means that the optimum thickness, giving minimum threshold current density (assuming the same gain parameters, the optimum thickness depends on v only), will be a factor ~ 1·5 higher in GaInAsP/InP lasers; Consequently the minimum threshold current density will also be a factor of ~ 1·5 higher.

7 Conclusion

The various refractive index models give different results when applied to GaInAsP compounds; however, the difference between results from the modified single oscillator model and Sellmeier interpolation can be explained in terms of an absorption-edge effect, and we believe the modified single-oscillator model to be the best.

At present very few experimental results based on beam-angle measurements are available and it is not possible to select the best model on this basis.

Since the models give very different results for the group index, measurements of mode spacings for lasers with precisely known length may give further information. However, the variation with thickness of the effective group index must be taken into account. On the other hand, this effect can be used to estimate the thickness when a reliable refractive-index model is established. This latter technique will also apply to GaAs/AlGaAs lasers.

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