Mode dispersion, material dispersion and profile dispersion in graded-index single-mode fibres

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Abstract: A study of single-mode fibres shows that in the normal operating region the cladding has as much effect on the propagation constant as does the core. A detailed analysis shows that the complex expression for pulse dispersion can be arranged in three groups of terms which may be defined as composite material dispersion, waveguide dispersion and composite profile dispersion. The analysis has been applied to a fibre for which the material dispersion parameters are accurately known and it is found that zero total dispersion can be obtained at a wavelength which depends on the profile and the core diameter but may be selected within a range roughly 1·3–2 μm. However, there are severe requirements on the control of core diameter and profile.

1 Introduction

The bandwidth of step-index multimode fibres is limited primarily by the spread in group velocities of the various propagation modes to a value ~20 MHz km. This is comparable with the best coaxial cables, but further improvement can be made by selecting an appropriately graded refractive-index profile so as to minimise the group-velocity spread. Practical values which have been attained in this way are in the region of 1 ns/km, giving a bandwidth-length product of ~1 GHz km. An alternative approach is to reduce the number of modes to just one so that multimode propagation effects are eliminated entirely.

The principal limitations to bandwidth then become:

(*a*) material dispersion $(-\lambda/c)(d^2n/d\lambda^2)$ owing to the wavelength (λ) dependence of refractive index n

(*b*) waveguide dispersion $dv_g/d\lambda$ due to wavelength dependence of the group velocity v_g of the HE_{11} mode

(*c*) profile dispersion owing to variation of the refractive-index profile with wavelength.

Of these three components, material dispersion normally dominates at wavelengths in the region 0·85–0·9 μm. Its effect on bandwidth has, in the past, been calculated in terms of a plane wave propagating in the bulk material of the core with a phase constant $\beta = 2\pi n_1/\lambda = kn_1$. This approximation is acceptable for modes in a multimode fibre which are far from cutoff since the bulk of the power is then contained within the core. However, in single-mode fibres, as well as for modes near cutoff in a multimode fibre, a significant amount of power is carried by the cladding,[1] and therefore its influence must also be taken into account. In the following we introduce a new parameter which we have termed the 'composite material dispersion' which accounts for the combined pulse-dispersive effects of both core and cladding. It is an important factor not only because of its influence on the wavelength of zero material dispersion,[2] but also for its effect on the total dispersion.

In the true single-mode regime, i.e. for $V < 2{\cdot}4$ where

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} = ka(\text{n.a.}) \qquad (1)$$

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where a = core radius, $n_{1,2}$ = refractive index of core, cladding and n.a. = numerical aperture; the waveguide (i.e. intramode) dispersion is always finite. However, we have recently shown[3] that for V values as much as 25–30% above cutoff the loss of the LP_{11} mode can be made large by even modest bends and microbending. It follows that operation at a normalised frequency which gives zero waveguide dispersion in the HE_{11} mode can be achieved in practice. However a more pertinent condition is that which makes the total dispersion zero, i.e. when the waveguide dispersion is used to balance out other components of dispersion, and we see below how this may be attained.

If profile dispersion were zero then it would be possible to operate at a wavelength where the composite material dispersion exactly cancelled the waveguide dispersion, thus giving zero (to first-order) total dispersion. Unfortunately the profile dispersion, although small, is not negligible, especially in fibres of high numerical aperture, and must also be taken into account. As with material dispersion only the effect of the core has been considered in the past but in a single-mode fibre the properties of the cladding are also important, and we therefore define a second new parameter: 'composite profile dispersion'. We have previously indicated[4] that the sum of the three principal dispersion components can be made equal to zero.

Another factor of considerable importance in a single-mode fibre is the radial variation r of refractive index. Although it is not necessary to introduce some form of index gradient to improve the bandwidth, as in multimode fibres, some grading is inevitable, owing to diffusion during the fabrication process, and in single-mode fibres its effect will be emphasised, owing to the small core diameter. We therefore consider also the influence on pulse dispersion of a refractive-index gradient of the form

$$\begin{aligned} n^2(r) &= n_1^2[1 - 2\Delta'(r/a)^\alpha] && 0 \leqslant r \leqslant a \\ &= n_1^2(1 - 2\Delta') = n_2^2 && r > a \end{aligned} \qquad (2)$$

where $\Delta' = (n_1^2 - n_2^2)/2n_1^2 \simeq (n_1 - n_2)/n_1$.

The normal definition of refractive-index difference is Δ', but for mathematical convenience Δ is used in the following where

$$\Delta = (n_1^2 - n_2^2)/2n_2^2 \simeq (n_1 - n_2)/n_2 \qquad (3)$$

This paper thus considers the effect of composite material dispersion, waveguide dispersion and composite profile dispersion on the signal-carrying capacity of single-

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mode fibres having a radial variation of refractive index given by eqn. 2. It is found, for the case studied, that the total dispersion can be made to fall to zero anywhere in a range of wavelengths, roughly 1·3 μm–2·0 μm, by modifying the fibre parameters, for example, simply the core diameter. It is thus possible to operate at a wavelength of 1·55 μm where the fibre attenuation can be as low as 0·2 dB/km giving large bandwidths over repeater spacings of perhaps 100 or 200 km.

2 Propagation of the HE_{11} mode

The propagation constant β of the HE_{11} mode and its relation to both the field intensity and the field amplitude in a single-mode fibre will be considered first. It can be shown[1] that the normalised propagation constant β/k, which depends on the refractive indices of both core and cladding, may be expressed as

$$\beta/k = [n_2^2 + b(n_1^2 - n_2^2)]^{1/2} \tag{4}$$

where[1] $b = 1 - (U/V)^2$.

The parameter $U = U(V, \alpha)$ is obtained by solving, for given values of V and α, the characteristic equation of the HE_{11} mode, namely

$$\frac{WK_1(W)}{K_0(W)} + \left.\frac{dP_0(R)/dR}{P_0(R)}\right|_{R=1} = 0 \tag{5}$$

where $P_0(R)$ is a function defining the field in the fibre (e.g. see Reference 5) and where $K(\ )$ is the modified Hankel function; $R = r/a$ = normalised radial position and $W^2 = V^2 - U^2$.

The wavelength dependence of β/k for various α is illustrated by Fig. 1. The fibre considered in this example has a pure silica core of diameter $2a = 8\,\mu m$ and a borosilicate cladding. The upper and lower curves give as a comparison the refractive indices of core and cladding, respectively. As expected $n_2 k < \beta < n_1 k$ and at short wavelengths (large V) β/k approximates to that of the core (cf. multimode fibres), whereas at long wavelengths it approaches that of the cladding. However, the variation between these extremes is not nearly so obvious. For example, at the limit of single-mode operation ($V = 2{\cdot}4$ for $\alpha = \infty$) more than 80% of the total power is carried by the core. One might therefore expect the propagation constant to lie nearer the refractive index of the core but, on the contrary, Fig. 1 shows that in the single-mode regime β/k is closer to the refractive index of the cladding and does not reach the midway value $\beta/k = (n_1 + n_2)/2$ until $V > V_c$ for any α profile.

Fig. 2 shows the fraction of power carried in the core for the HE_{11} mode as a function of V for several values of α. The solid circles indicate the cutoff frequency V_c for each profile. It may be seen that at low V the power in the core decreases with falling α. This arises from the normal definition of V which refers only to the maximum value of n which occurs on the axis. As we have pointed out elsewhere,[6] a more realistic parameter is a term which may be defined as the guidance factor G which reflects the integrated effect of refractive index over the whole of the core. In fact, G is defined[6] in terms of the permittivity $\epsilon(R)$ as

$$G = \int_0^1 \left(\frac{\epsilon(R) - \epsilon_2}{\epsilon_1 - \epsilon_2}\right) R dR \tag{6}$$

where $\epsilon_{1,2}$ refer to axis, cladding. Using this equation the cutoff frequency can be defined, to a good approximation, for any profile as

$$V_c = 2{\cdot}40/(2G)^{1/2} \tag{7}$$

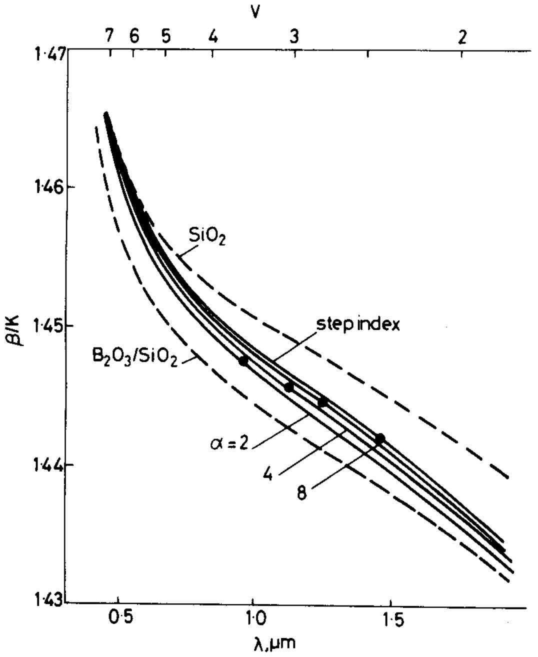


Fig. 1 *Propagation constant in various graded-index single-mode fibres having a silica core of 8 μm diameter and a borosilicate cladding*

---- propagation in the bulk core and cladding materials
• • • • LP_{11} mode cutoff

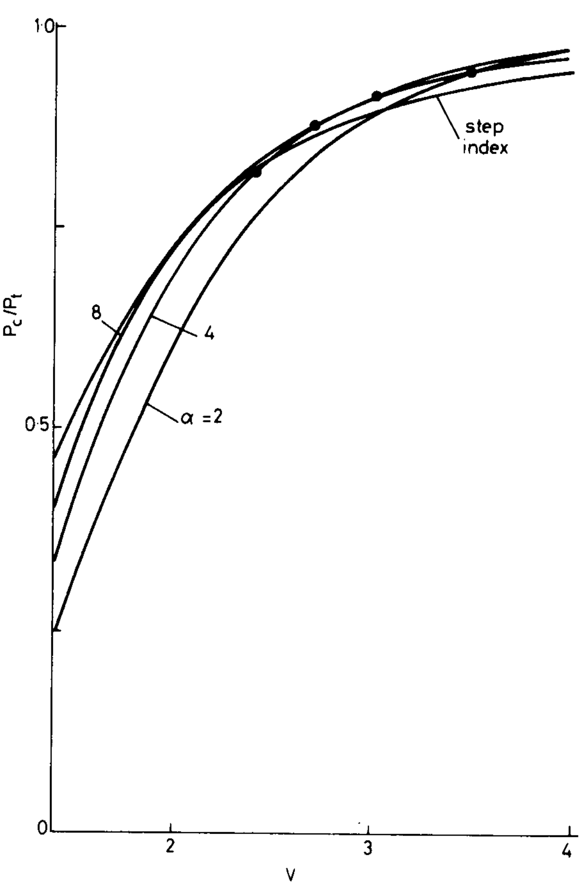


Fig. 2 *Fraction of power in the HE_{11} mode carried by the core for various α-profiles*

• • • • LP_{11} mode cutoff

Eqns. 6 and 7 are quite general and apply to any form of refractive-index distribution and show how the cutoff normalised frequency for single-mode operation can be estimated reasonably accurately from a knowledge of the index profile only. If we now take the particular case of the α-profile, then $G = \alpha/(2\alpha + 4)$ and $V_c = 2{\cdot}4(1 + 2/\alpha)^{1/2}$ which is a result obtained also by Okamato and Okoshi.[7] For a given ϵ_1 (or n_1) the guidance factor increases with rising α as more power is concentrated in the core. Above cutoff a crossover occurs so that profiles of lower α have relatively more power in the core. Detailed radial-intensity distributions for $V = 2{\cdot}4$ and $V = 3{\cdot}0$ are given in Reference 5.

An attempt to estimate the cutoff frequency for an arbitrary profile has also been made,[8] but the result is not as simple as eqn. 7 above.

It was remarked earlier that the propagation constant is not, as might have been expected, determined primarily by the fractional power carried in the core. This is shown by Fig. 3 where b is plotted as a function of P_c/P_t and the dependence is seen to be quite nonlinear. A further illustration can be obtained by calculating the ratio of the integrated field F_c over the cross-section of the core to the total integrated field F_t over the entire HE_{11} mode. For the simple case of a step-index fibre and the usual weak-guidance approximation,

$$\frac{F_c}{F_t} = \frac{\int_0^1 J_0(UR)RdR}{\int_0^1 J_0(UR)RdR + (J_0(W)/K_0(W))\int_1^\infty K_0(WR)RdR}$$

$$= 1 - \left(\frac{U}{V}\right)^2 = b \qquad (8)$$

and since

$$b = (\beta/k - n_2)/(n_1 - n_2) \qquad (9)$$

then

$$\beta/k = n_2 + (n_2 - n_1)(F_c/F_t) \qquad (10)$$

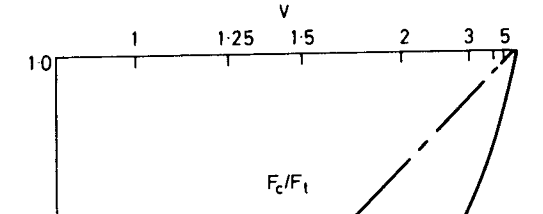


Fig. 3 *Variation of b parameter with fractional power in the core*

– – – – Linear relationship obtained when the abscissa is plotted in terms of the fractional integrated field F_c/F_t

Thus β depends linearly on the ratio of the integrated *fields*, not on the power ratio, and may be represented by the broken line in Fig. 3 if the abscissa is replaced by F_c/F_t. Thus even near cutoff, when the bulk of the power is carried by the core, the cladding has an effect on the propagation constant equal to that of the core.

The above calculation for a step-index fibre is only by way of illustration. In any other type of fibre where the refractive index is graded in some way then the guidance factor for a given n.a. will be smaller, and the effect of the cladding will be even more marked. The equivalent result can be obtained by using the concept of an effective core index.* It is essential, therefore, in single-mode fibres to include the cladding properties in calculating material dispersion, profile dispersion and other parameters.

3 Pulse dispersion in a single-mode fibre

The temporal broadening of a pulse per unit length of fibre, when an optical source of spectral width $\delta\lambda$ centred at λ is used, is given[9] by

$$T = \frac{2\pi}{c\lambda^2}\frac{d^2\beta}{dk^2}\delta\lambda \qquad (11)$$

where c is the speed of light in vacuum.

In practice, the index difference between the core and cladding is small and $b < 1$, so that $b\Delta \ll 1$ and β in eqn. 4 can be approximated (see Section 5) by

$$\beta = kn_2(1 + b\Delta) \qquad (12)$$

After a considerable amount of calculation of the first and second derivatives, eqn. 11 can be written simply as

$$T = \delta\lambda(T_{c.m.d.} + T_{w.d.} + T_{c.p.d.}) \qquad (13)$$

where

$$T_{c.m.d.} = \frac{\lambda}{c}\left[A(V)\frac{d^2n_1}{d\lambda^2} + \{1 - A(V)\}\frac{d^2n_2}{d\lambda^2}\right] \qquad (14)$$

$$A(V) = \frac{1}{2}\left[\frac{d(bV)}{dV} + b\right]$$

$$T_{w.d.} = \frac{n_2\Delta}{c\lambda}B(n)V\frac{d^2(bV)}{dV^2} \qquad (15)$$

$$B(n) = \left[1 - \frac{\lambda}{n_2}\frac{dn_2}{d\lambda}\right]^2$$

$$T_{c.p.d.} = -\frac{n_2}{c}C(n)D(V)\frac{d\Delta}{d\lambda} \qquad (16)$$

$$C(n) = 1 - \frac{\lambda}{n_2}\frac{dn_2}{d\lambda} - \frac{\lambda}{4\Delta}\frac{d\Delta}{d\lambda}$$

$$D(V) = V\frac{d^2(bV)}{dV^2} + \frac{d(bV)}{dV} - b$$

We define $T_{c.m.d.}$, $T_{w.d.}$, and $T_{c.p.d.}$ as the composite material dispersion, waveguide dispersion and composite profile dispersion, respectively, for unit length of fibre. Although these definitions are more complex than the approximate ones that have been used in the past and contain crossproduct terms, they have been grouped in such a

*Matsumura, H., and Gambling, W.A.: to be published way that when the dispersion parameter defining that particular component falls to zero, the composite dispersion is also zero. For example, profile dispersion arises from terms such as $d\Delta/d\lambda$, whereas eqn. 16 also contains terms of the form $\{d^2(bV)/dV^2\}$ and $\{d(bV)/dV\}$ which are generally considered as waveguide dispersion. Nevertheless it is seen that if $d\Delta/d\lambda$ is zero the composite profile dispersion is also zero. Similar arguments can be applied to the composite material dispersion and waveguide dispersion. The three dispersion components will now be considered in more detail.

First, however, the waveguide factors b, $d(bV)/dV$ and $Vd^2(bV)/dV^2$ are plotted as a function of V for several values of α in Fig. 4. The dark circles show the cutoff values of the single-mode region for each α. These curves will be used in the following discussion.

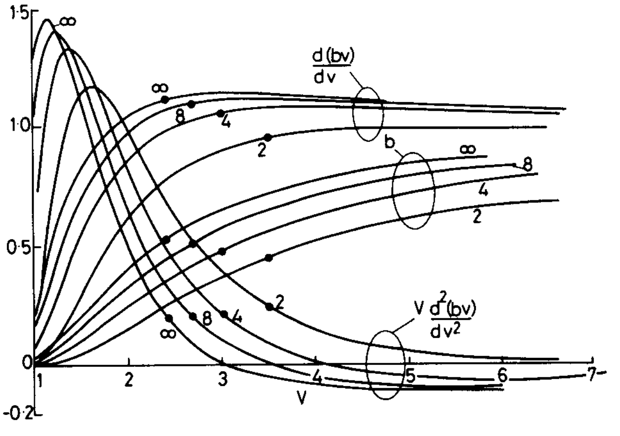


Fig. 4 *Variation with V of the three basic waveguide-mode parameters for various index profiles*

● ● ● ● LP_{11} mode cutoff

3.1 Composite material dispersion (c.m.d.)

The composite material dispersion given by eqn. 14 depends on the material dispersions of both core and cladding. It is also related to the waveguide parameters b and $d(bV)/dV$ by the function $A(V)$. Fig. 4 shows that, as V increases, both b and $d(bV)/dV$ approach asymptotically to 1. It follows, therefore, that in the multimode region the function $A(V)$ tends to 1 and the composite material dispersion reduces to

$$T_{c.m.d.} = \left(\frac{\lambda}{c}\right)\left(\frac{d^2 n_1}{d\lambda^2}\right) \tag{17}$$

This, as expected, is the expression obtained for material dispersion if only the dispersion of the core is considered.[2] In single-mode fibres, however, $A(V)$ is sufficiently less than 1 to make the dispersive properties of the cladding significant. For example, in a step-index fibre at cutoff, $V = 2{\cdot}4$, $A(V)$ is 0·83 and therefore the c.m.d. is given by

$${}_{\infty}T_{c.m.d.} = \frac{\lambda}{c}\left[0{\cdot}83\frac{d^2 n_1}{d\lambda^2} + 0{\cdot}17\frac{d^2 n_2}{d\lambda^2}\right] \tag{18}$$

For a parabolic-index fibre at cutoff ($V = 3{\cdot}5$) the c.m.d. is

$${}_{2}T_{c.m.d.} = \frac{\lambda}{c}\left[0{\cdot}69\frac{d^2 n_1}{d\lambda^2} + 0{\cdot}31\frac{d^2 n_2}{d\lambda^2}\right] \tag{19}$$

The composite material dispersion has been calculated for a fibre having a germania-doped silica core and a pure-silica cladding. These results are discussed in more detail in Section 4, but it is seen from Figs. 6*a* and 7*a* that, as expected, the c.m.d. lies between the material dispersion of the core and that of the cladding. The point at which the material dispersion falls to zero has shifted to shorter wavelengths compared with that obtained if the core alone is considered.

For a step-index fibre we now show that the weighting function, $A(V)_\infty$, is simply given by the ratio of the power in the core P_c to the total power P_t. The power ratio is given by

$$\frac{P_c}{P_t} = \left(\frac{W}{V}\right)^2\left[1 + \frac{J_0^2(U)}{J_1^2(U)}\right] \tag{20}$$

By use of the eigenvalue equation for a step-index fibre, the weighting function $A(V)$ becomes

$$A(V)_\infty = \frac{1}{2}\left[\frac{d(bV)}{dV} + b\right] = b\left[1 + \frac{J_0^2(U)}{J_1^2(U)}\right] \tag{21}$$

and from eqn. 4 it is seen that $b = W^2/V^2$, hence

$$A(V)_\infty = P_c/P_t \tag{22}$$

As seen from eqns. 18 and 19, which give the c.m.d. for step-index and parabolic-index fibres, respectively, the influence of the material dispersion of the cladding is greater in the case of the graded-index fibre. This can be explained in terms of field spreading and the guidance factor. In a graded-index fibre the number of doping ions in the core is less than for a step-index fibre; hence the degree of guidance is less. The field in a graded-index fibre therefore spreads further into the cladding and hence a greater proportion of the field is governed by the dispersion properties of the cladding.

3.2 Waveguide dispersion w.d.

The waveguide dispersion denotes the influence of the waveguide on pulse spreading and is defined[9] by the term $Vd^2(Vb)/dV^2$. However, our more exact definition, eqn. 15, contains an additional weighting factor $B(n)$ which depends on the material properties of the cladding.

Fig. 4 shows that the function $Vd^2(Vb)/dV^2$, and hence the waveguide dispersion, falls to zero at $V > 3$, the exact value depending on the index profile. The V value at which the waveguide dispersion is zero is therefore always outside the true single-mode regime, for example at $V = 3{\cdot}0$ in a step-index fibre compared with $V_c = 2{\cdot}4$. If necessary this problem can be overcome by operating in the 'quasi-single-mode' regime[3] for which the V value is above cutoff but the LP_{11} mode is suppressed by bending losses.

For the case when $(\lambda/n_2)dn_2/d\lambda \ll 1$, which is approximately true for the range of wavelengths under consideration, then

$$T_{w.d.} = (n_2\Delta/c\lambda)\, Vd^2(bV)/dV^2 \tag{23}$$

and is of the same form as that obtained when the dispersive properties of Δ are neglected.

3.3 Composite profile dispersion (c.p.d.)

The expression defined by eqn. 16 contains terms relating to the waveguide and material effects as well as $d\Delta/d\lambda$

which relates to the profile dispersion parameter so that, as indicated above, we have called it the composite profile dispersion.

If, as is normally the case, both $(\lambda/n_2)(dn_2/d\lambda)$ and $d\Delta/d\lambda$ are small, then the weighting factor $C(n)$ in eqn. 16 can be approximated to 1, and the c.p.d. then reduces to

$$T_{c.p.d.} = -\frac{n_2}{c}\frac{d\Delta}{d\lambda}\left[V\frac{d^2(bV)}{dV^2}+\frac{d(bV)}{dV}-b\right] \tag{24}$$

4 Numerical example

The practical consequences of the above conclusions can best be illustrated by reference to a specific example. The results obtained depend critically on the accuracy of the refractive-index and material-dispersion data of the core and cladding materials over the rather wide wavelength region of interest, namely 0·8–1·8 μm. Satisfactory results are available for silica[10] but not, in general, for other fibre materials. The most reliable figures for material dispersion we have obtained[11] are for a germanosilicate glass containing 11·1 m/o germania which, if used as the core in a fibre with a pure-silica cladding, corresponds to a numerical aperture of ~0·23. This is higher than that required in practice, but the results calculated for a single-mode fibre with these components have the virtue of reliability.

The variation of refractive index for core and cladding, and of numerical aperture, are seen in Fig. 5 as a function of wavelength. To illustrate the effect of a refractive-index gradient in the core, the extreme cases of a step-index and a parabolic-index fibre have been studied; and the composite material dispersion, waveguide dispersion, composite profile dispersion and total dispersion are shown in Figs. 6 and 7 as a function of wavelength for various core diameters.

The c.m.d., Figs. 6*a* and 7*a*, depends on the core diameter and wavelength as well as on the materials. Thus at small V (longer λ and/or smaller a) it lies closer to that of

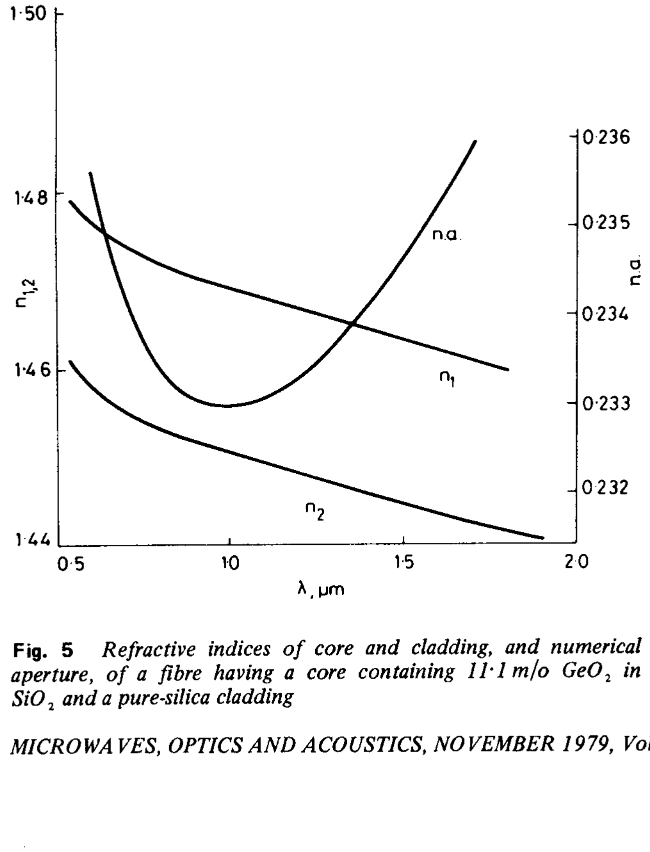


Fig. 5 *Refractive indices of core and cladding, and numerical aperture, of a fibre having a core containing 11·1 m/o GeO_2 in SiO_2 and a pure-silica cladding*

the cladding due to the increased field spreading. For the same reason the c.m.d. is larger in a step-index fibre, for a given core diameter, than in a parabolic-index fibre, see Fig. 2. The wavelength of zero material dispersion also depends on the core diameter as well as on the material combination.

The waveguide dispersion in a step-index fibre passes through zero at a wavelength which depends on the core diameter but always for the condition $V > 2{\cdot}4$. It increases for decreasing V up to a maximum at $V \sim 1{\cdot}2$. As with c.m.d., either positive or negative values can be obtained by choice of wavelength and/or core diameter. On the other hand, for $\alpha = 2$, the w.d. is positive over the wavelength range shown and also reaches a maximum at smaller V values. In general, a maximum w.d. is obtained at a V which depends on α. Conversely it has been shown[12] that for $\alpha \leqslant 2$ negative values never occur and as V is increased the curves become asymptotic to zero. This aspect will be discussed further when total dispersion is considered. Again it is clear that there is a major difference in behaviour of step-index and parabolic-index fibres.

In both cases, however, the composite profile dispersion, (Figs. 6*c* and 7*c*) is negative over the ranges shown and in general is small numerically compared with c.m.d. and w.d. However it becomes significant when the sum of the other two is also small and can change the wavelength of total dispersion by a significant amount.

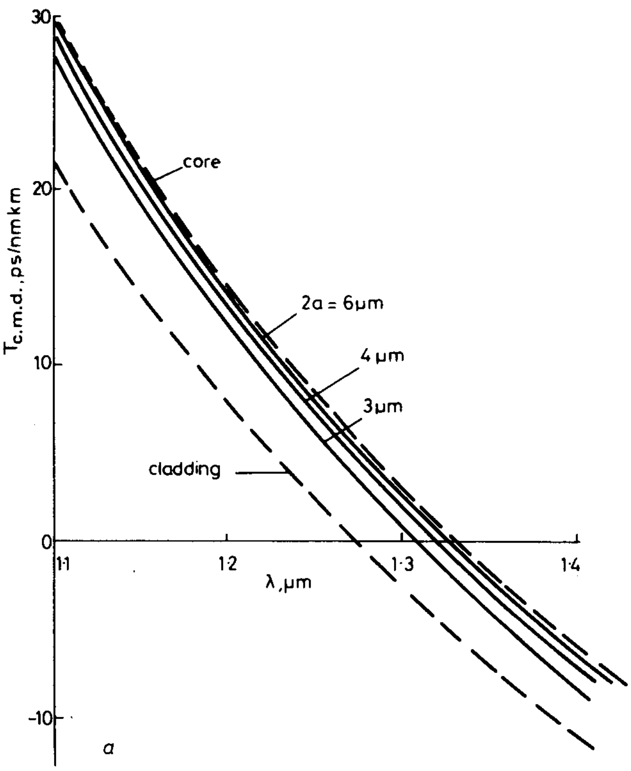
The total dispersion, calculated from the sum of the three components for $\alpha = \infty$ and 2, can be made zero (Figs. 6*d* and 7*d*) at any point over a wide range of wavelengths, e.g. from 1·33 μm to 1·65 μm for $\alpha = 2$, simply by changing the core diameter. Thus operation with zero dispersion is possible, by suitable choice of core diameter, at a wavelength of 1·55 μm where a transmission loss of 0·2 dB/km has been reported.[13] The necessary diameter will, of course, depend on the germania concentration in the core and on the particular index profile. There is therefore a prospect that considerable bandwidths may be available even over the potential-repeater spacings, as far as attenuation is concerned, of one of two hundred kilometres. Conversely such performance will require adequate control over core and cladding compositions, core diameter and refractive-index profile, since all three parameters are closely interrelated.

The core radius required to produce zero total dispersion at a given wavelength λ_0, Fig. 8, depends markedly on the profile, and at $\lambda = 1{\cdot}55\,\mu$m both fibres are in the single-mode region. At this wavelength there is a strong dependence of λ_0 on core diameter, for example with $\alpha = \infty$ a change of only 0·15 μm shifts λ_0 from 1·5 to 1·6 μm. Thus to maintain operation at 1·55 μm strict diameter control will be necessary. In the multimode regime, where quasi-single-mode operation is possible, the effect of core diameter is much less critical because there is a lower limit to the wavelength of zero dispersion.

This is due to the fact that, near the minimum wavelength at which zero total dispersion may be obtained, the V value can only be increased by increasing the core diameter, and $T_{w.d.}$ decreases only slowly with increasing V, as indicated by Fig. 4. In fact, for all fibres with $\alpha > 2$, the $T_{w.d.}$ falls to a minimum at $V \sim 6$; and as V is increased further it begins to rise again and is no longer sufficient to cancel the increasingly positive material dispersion at short wavelengths. There is hence a turning point in the curve of Fig. 8 for $\alpha = \infty$, and further increase in core diameter causes a rise in λ_0. For fibres with $\alpha \leqslant 2$, as V is increased,

$T_{w.d.}$ is asymptotic to zero, and there is no minimum and hence no turning point in the curve $\alpha = 2$ in Fig. 8.

Fig. 4 shows that towards small V values, $T_{w.d.}$ has a maximum at $V < 1{\cdot}65$, the precise value depending on α; so that at a fixed wavelength the same waveguide dispersion is obtained at two different core diameters. Therefore for a certain range of wavelengths there are two values of core diameter at which the total dispersion is zero for a given wavelength.



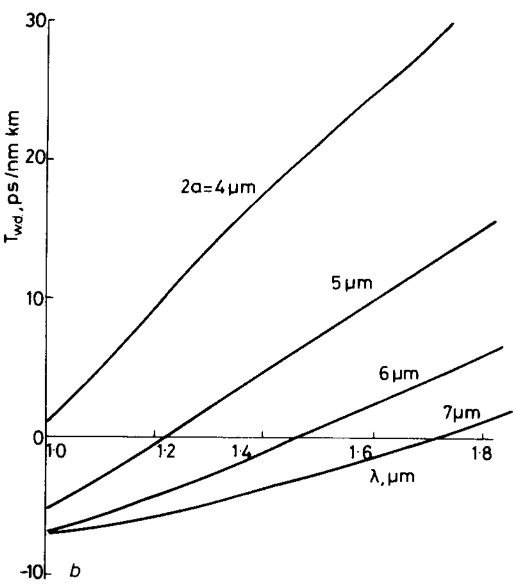


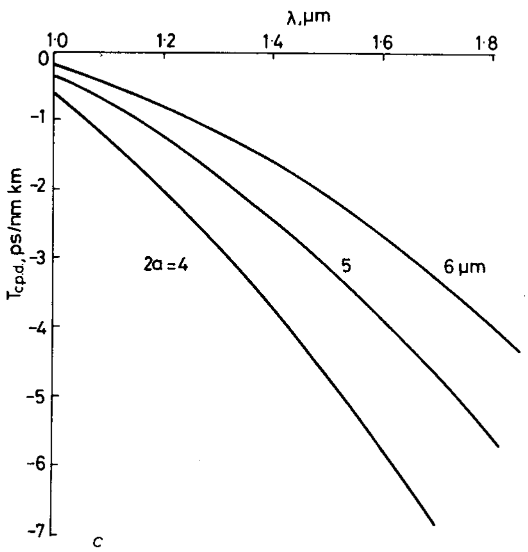
Fig. 6 *Dispersion in step-index single-mode fibre for core diameters shown*

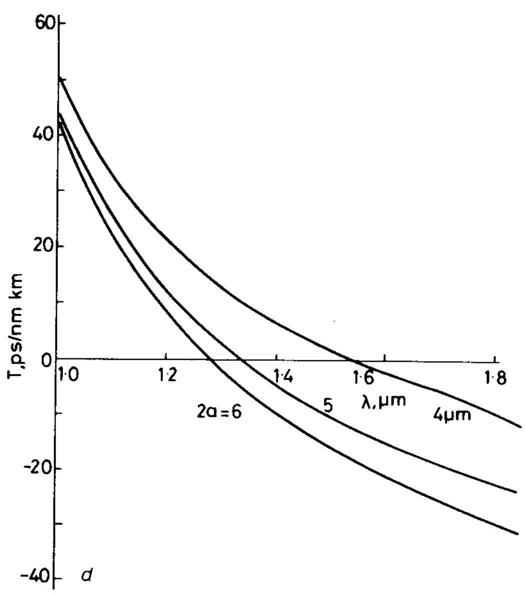
a Composite material dispersion
b waveguide dispersion
c Composite profile dispersion
d total dispersion

5 Conclusions

In single-mode fibres the cladding exerts a strong effect on propagation and must be fully taken into account when the dispersion is calculated. We have shown here that the full, and somewhat complex, expression for pulse dispersion can be arranged into three components, which may be termed composite material dispersion, waveguide dispersion and composite profile dispersion; each of which contains cross-product terms. The total dispersion obtained from these components may be reduced to zero at a wavelength which can be selected from a wide range by simply varying the core diameter. Unfortunately, for stable zero-dispersion operation at a wavelength near $1{\cdot}55\,\mu m$, fairly strict control of the core diameter is essential.

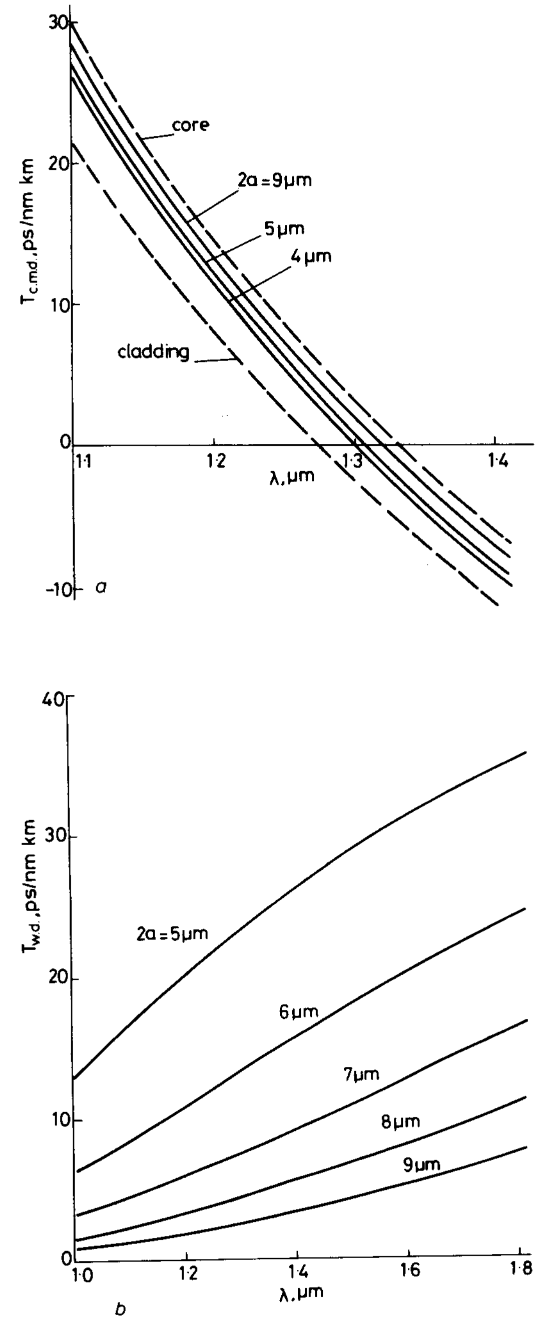
Some degree of grading in the refractive-index profile is almost inevitably introduced during the fabrication process of single-mode fibres and, for a given numerical aperture, the precise form of profile has a considerable effect on the wavelength of zero total dispersion (e.g. see Fig. 8). A compensating change can be made in the core diameter but it does mean that the profile as well as the core diameter must be carefully controlled.

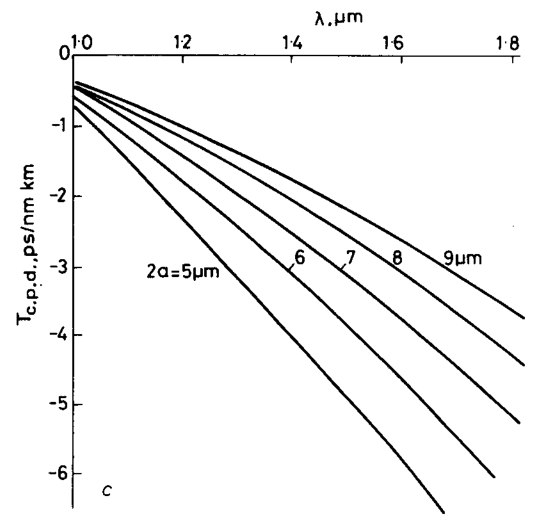




On the other hand, for operation at $T=0$ above cutoff of the LP_{11} mode, the dependence on core diameter is much less critical, and we have previously shown[3] that single-mode operation under this condition is possible. Indeed if fibres of any profile are operated near cutoff the profile shape has little effect on the total dispersion. Thus Fig. 9 shows T and its components plotted for a range of α values where the diameter of each fibre is adjusted at each wavelength to give operation at cutoff. It can be seen, for example, that the total dispersion is almost independent of α, and the curve for $\alpha=\infty$ is so close to that for $\alpha=8$ that it cannot be drawn separately. The total dispersion curves thus effectively form a single universal curve for the 11·1 m/o GeO_2/SiO_2 core and it may be used as follows: if the cutoff wavelength of a fibre having this axial composition is measured, its dispersion at this wavelength may be read directly off the $T=0$ curve without its profile being known. There are similar curves for other compositions.

Finally, although the analysis given here has been described in terms of the α-profile in particular, it is also applicable, including eqn. 13, to any form of refractive-index distribution. The general result is therefore equally valid. A further point is that terms of higher order in $b\Delta$ have been neglected in eqn. 12 whereas they could become important after the second differentiation. However, as indicated above, $\Delta \ll 1$. We have nevertheless calculated the change in λ_0 produced by including terms in $(b\Delta)^2$, and find it is 2 nm at a core radius of 2·5 μm, i.e. only ~0·1%. In practice other effects are likely to be much more important.





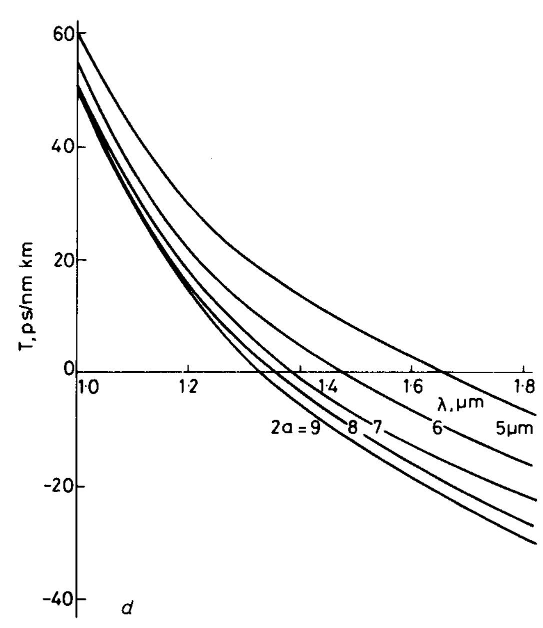


Fig. 7 *Dispersion in parabolic-index single-mode fibre for core diameters shown*

a Composite material dispersion
b Waveguide dispersion
c Composite profile dispersion
d Total dispersion

6 Acknowledgments

We wish to thank D.N. Payne for invaluable discussions and the provision of the dispersion data, the Pirelli General Cable Company Limited for the award of a Fellowship, and the UK Science Research Council for its support.

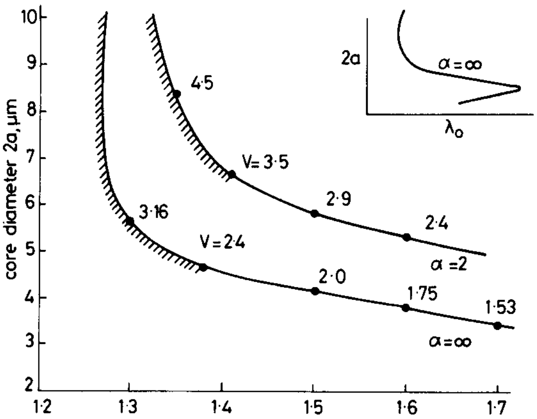


Fig. 8 *Core diameter required to give zero total dispersion T in both step-index and parabolic-index fibres*

The numbers on the curves are the values of V at the wavelengths shown, and the shaded portions denote multimode operation. The inset shows the generalised form of the curve for $\alpha = \infty$ over a wider wavelength range

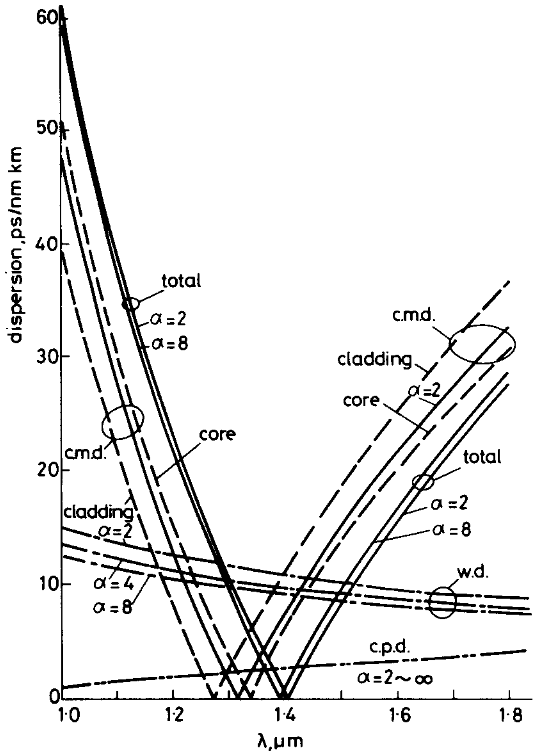


Fig. 9 *Total dispersion and its components for various profiles where the diameter of each fibre is chosen to give cutoff at each wavelength*

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