BIREFRINGENCE IN OPTICAL FIBRES WITH ELLIPTICAL CROSS-SECTION

Indexing terms: Birefringence, Optical fibres

A comparison is made between various approximations for the phase delay between orthogonally polarised modes in elliptical optical fibres. A more accurate analysis is presented which gives a lower value for the birefringence produced by a given ellipticity. The effect on fibre bandwidth is shown to be small compared with that resulting from stress birefringence.

The birefringence normally exhibited by single-mode optical fibres is the result of a difference $\delta\beta$ between the propagation constants of the two orthogonally polarised states of the fundamental mode. The difference in phase velocities causes the fibre to exhibit a linear retardation,[1] the value of which depends on the fibre length. This leads to a polarisation state which is, in general, elliptical but which varies periodically along the fibre with characteristic length L, where

$$L = \frac{2\pi}{\delta\beta} \tag{1}$$

Typical single-mode fibres are found to have a period of a few centimetres.[1] The origins of the birefringence are twofold:

(*a*) An ellipticity in the fibre core establishes preferred fast and slow axes of propagation and produces a retardance which depends on the degree of ellipticity.[2]

(*b*) The presence of an asymmetrical residual stress distribution within the fibre results in stress birefringence[3] and similarly contributes to the observed retardation.

The poorly defined polarisation state of light emerging from the fibre leads to difficulties when connecting to integrated-optical devices and to a loss of sensitivity in the Faraday-effect fibre current transducer.[4] These difficulties are compounded by the fact that power may be transferred from one polarisation to the other by bending of the fibre, and that the stress-birefringence effect is temperature-dependent.

One approach to the problem of ensuring a defined output polarisation direction is to deliberately induce a large degree of birefringence in the fibre by fabricating a highly elliptical core[3] or introducing an asymmetrical stress distribution.[5] By this means a preferred polarisation direction is established into which all power may be launched. If the beat length is short compared to the period of the spatial undulations of the fibre axis to be found in practice ($\approx$ 1 mm), transfer of power to the orthogonal mode will be precluded and the output will remain linearly polarised in the direction of the preferred axis. The converse approach is necessary to improve the performance of the fibre current transducer where the requirement is for a beat length L long enough ($\approx$ 50 m) to prevent significant polarisation variation in the length of fibre used.[6,7] A fibre having an exceptionally circular core and reduced stress levels is therefore necessary.

In both approaches the birefringence expected for a given core ellipticity is of considerable interest, particularly since it has been observed that little agreement exists between theoretically predicted values and those found in practice.[3,7] It is the intention of this contribution, therefore, to examine existing theoretical approximations describing propagation in elliptical fibres and to compare these with a more accurate analysis.

Theoretical background. Ramaswamy *et al.*[3] have adapted Marcatili's calculations for a rectangular dielectric waveguide[8] to compute $\delta\beta$. For small departures from a square cross-section, their result may be reduced to the form:

$$\frac{a\,\delta\beta}{e^2(2\Delta)^{3/2}} \approx \frac{3\pi^2 v^2}{(v+2)^4} \tag{2}$$

where a = core halfwidth, Δ is the relative refractive-index difference, v is the normalised frequency and e is the eccentricity $[=\{1-(\text{width/height})^2\}^{1/2}$ in the rectangular guide]. A somewhat more sophisticated analysis for $\delta\beta$ has been reported by Schlosser,[2] with the result for small eccentricities:

$$\frac{a\,\delta\beta}{e^2(2\Delta)^{3/2}} \approx \frac{u^4 w^3}{8v^5 J_1^2(u)}\left(\frac{K_0(w)}{K_1(w)} + \frac{1}{w}\right) \tag{3}$$

where u, w are the usual circular waveguide parameters and J, K are Bessel functions. Using coupled-mode theory, Marcuse[9] has found the following approximation:

$$\frac{a\,\delta\beta}{e^2(2\Delta)^{3/2}} \approx \frac{u^2 w^2}{8v^3} \tag{4}$$

Recently, Snyder and Young[10] have used another approximation to find for small eccentricities:

$$\frac{a\,\delta\beta}{e^2(2\Delta)^{3/2}} \approx \frac{u^2 w^2}{8v^3}\left[1 + \frac{uK_0^2(w)J_2(u)}{K_1^2(w)J_1(u)}\right] \tag{5}$$

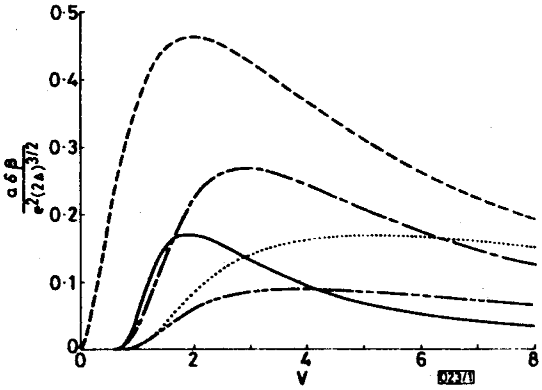

Fig. 1 *Comparison of various approximations for the normalised retardation $\delta\beta$ as a function of fibre v-value*

———— present work
- - - - - Reference 3
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The approximations eqns. 2–4 above have been rearranged to demonstrate that each yields the same normalisation parameter $[a\,\delta\beta/e^2(2\Delta)^{3/2}]$ as found in Reference 10. The predictions of eqns. 2–5 are plotted in Fig. 1, and it may be seen that they differ widely both for the value of $\delta\beta$ produced by a given ellipticity and in its variation with normalised frequency v.

Calculation of $\delta\beta$ for elliptical fibres: The exact theory of dielectric waveguides with elliptical cross-section is given in Reference 11, with approximations claimed to be suitable for optical fibres in Reference 12. Making the usual weakly guiding assumption ($\Delta \ll 1$), together with the assumption of small ellipticity ($e^2 \ll 1$), simplifies the analysis considerably. Yeh's[11] eqn. 13 for the eigenvalues of the $_oHE_{11}$ mode may then be written as:

$$\frac{v^4}{w^4} - \left[\frac{Se_1'}{Se_1} + \frac{u^2}{w^2}\frac{Gek_1'}{Gek_1}(1-2\Delta)\right]\left[\frac{Ce_1'}{Ce_1} + \frac{u^2}{w^2}\frac{Fek_1'}{Fek_1}\right] = 0 \tag{6}$$

where Se_1, Ce_1, Gek_1 and Fek_1 represent Mathieu functions, with arguments omitted for brevity. Similarly, the eigenvalue equation for the $_eHE_{11}$ mode of orthogonal polarisation may be written as

$$\frac{v^4}{w^4} - \left[\frac{Ce_1'}{Ce_1} + \frac{u^2}{w^2}\frac{Fek_1'}{Fek_1}(1-2\Delta)\right]\left[\frac{Se_1'}{Se_1} + \frac{u^2}{w^2}\frac{Gek_1'}{Gek_1}\right] = 0 \tag{7}$$

Using the standard expressions for Mathieu functions and retaining terms to order Δ and e^2, we obtain eigenvalue equations for the two modes in terms of Bessel functions. If the latter equations are expressed as Taylor-series expansions around the weakly guiding circular-fibre eigenvalue equation,[13] then the result for $\delta\beta$ is:

$$\frac{a\,\delta\beta}{e^2(2\Delta)^{3/2}} = \frac{u^2w^2}{8v^5}\left\{\left(\frac{J_0(u)}{J_1(u)}\right)^3 \frac{(u^2-w^2)w^2}{u} + \left(\frac{J_0(u)}{J_1(u)}\right)^2\left(\frac{(w^4+u^4)}{u^2}\right) + \left(\frac{J_0(u)}{J_1(u)}\right)2u(4+w^2) - (8+w^2-u^2)\right\} \qquad (8)$$

Once again the same normalisation parameter is obtained as in the previous approximations eqns. 2–5.

Discussion: The results obtained from eqn. 8 are given in Fig. 1. It may be seen that the present treatment yields values substantially lower than those of Marcatili,[8] and indeed for $v > 4$ gives a figure lower than all other predictions. In physical terms this implies that the birefringence produced by a given core ellipticity is here found to be less than previously predicted, by a considerable margin in some cases.

It may be noted that the degree of circularity required to produce a low-birefringence fibre for current-transducer applications is highest near $v = 2$, which, unfortunately, is close to the preferred operating point for a single-mode fibre. A move to lower v or into the multimode region of large v considerably relaxes the tolerance on circularity. The latter effect has been observed experimentally[14] for a fibre having $v = 125$, in which it was possible to launch only the lowest-order mode and maintain the state of polarisation over a length as great as 400 m.

Further indications for the fabrication of low-retardance fibres may be found by examination of the normalisation parameter. We see that a greater ellipticity can be tolerated in fibres having large cores and small values of index difference Δ. However, the circularity required to produce a fibre with a periodic length L of 50 m (i.e. $\delta\beta = 0{\cdot}13$ rad/m) remains exceptionally high. A fibre designed to operate at 0·633 μm and $v = 2{\cdot}4$, having a core diameter of 6 μm, requires the difference between major and minor axes of the core to be only 0·7%. In addition, it will be necessary to minimise stress-birefringence effects.

The implications of the present analysis for telecommunications may be seen by computing the derivative of the $\delta\beta$ curve to yield the difference $\delta\tau = (1/c)\{d(\delta\beta)/dk\}$ in group delay/unit length between the orthogonally polarised modes. The normalised result is shown in Fig. 2. A typical fibre having $v = 2{\cdot}4$,

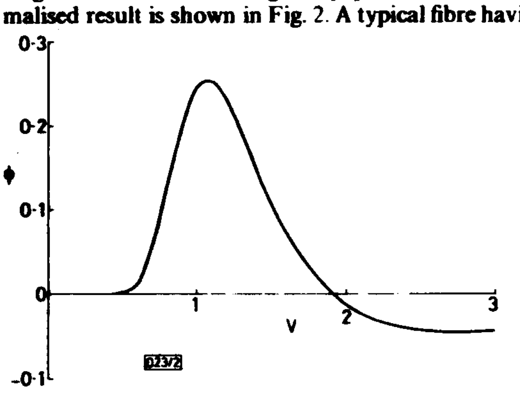

Fig. 2 *Normalised group-delay difference Φ between orthogonally polarised modes*

Time delay $\delta\tau$ between pulses emerging from 1 km of fibre is given by

$$\delta\tau = \frac{1}{c}\frac{d}{dk}(\delta\beta) = \frac{1}{c}n_1e^2(2\Delta)^2\Phi$$

$\Delta = 4 \times 10^{-3}$ and a difference between major and minor axes of 5% would give a pulse separation of 1·3 ps/km, a very small value. Note, however, that if the stress-induced birefringence is such as to produce a periodic length L of 10 cm at 1·3 μm, the pulse separation $\delta\tau = \lambda/cL$ (in the absence of mode coupling) is expected to be 43 ps/km, a dispersion which may well be significant on long-distance routes. On the other hand, a reduction in birefringence to give a periodic length of 10 m would render the effect negligible.

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