Measurement of radiation loss in curved single-mode fibres

W.A. Gambling, H. Matsumura, C.M. Ragdale and R.A. Sammut

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Abstract: Experimental and theoretical studies on the radiation loss from curved single-mode fibres show that it includes contributions due to transition loss at discontinuities between straight and curved sections of fibre, as well as the bend loss due to uniform curvature. These two contributions have been measured separately, and there is good agreement with the theoretical predictions. The transition loss at large bend radii can be considerably greater than the uniform bend loss. It is found that the transition loss incurred by a change of fibre curvature from R to $-R$ is about 8 times bigger than that from R to ∞. The loss due to fibre undulation has also been measured and in this case the results can be completely explained using only the pure bending loss theory.

1 Introduction

The recent fabrication of a low-loss single-mode fibre with an attenuation of only 0·5 dB/km at 1·27 μm[1] indicates a promising future for single-mode fibres in long-distance optical communication. Bandwidths of tens of gigahertz over tens of kilometres are possible, in principle, at 0·9 μm, and much larger values,[2] limited only by mode dispersion, at 1·2 μm. Despite these potential advantages, comparatively little effort has been expended on an investigation of single-mode fibres, and many practical problems have still to be solved. Of these the effect of radiation loss due to bends may well be most important.

Recent studies of propagation in curved fibres have shown that the radiation emitted in the transverse direction at the beginning of a bend is not continuous, but can appear in the form of discrete divergent rays.[3] Measurement of the variation of radiated intensity from a single-mode fibre in the plane of the bend confirms[4] this result. The emitted intensity distribution is oscillatory at first, the peaks corresponding to the ray radiation of Reference 3. However, the radiation loss does not suddenly increase at the point of curvature because, we believe, the mechanical stiffness of a fibre usually prevents an abrupt change of the bending radius. This is discussed below.

With increasing distance along the curved fibre, the beams broaden and decrease in amplitude, so that the radiation gradually becomes more uniform and characteristic of that of a stable curved mode. In the following, we refer to the oscillatory section as the transition region and the section following the transition region, where the radiation is nonoscillatory, is called the pure bend, or stable-mode region. Two kinds of radiation loss, therefore, must be considered in bent fibres, namely a transition loss and a pure bending loss. The transition loss is due to mode conversion as the mode of the straight fibre transforms to that in a curved fibre.[5] On the other hand, the pure bending loss occurs because with increasing distance from the centre of curvature the local phase velocity increases. When the phase velocity equals the velocity of light, guidance ceases and energy radiates away from the fibre.

The transition loss[6] and the pure bending loss[7, 8] have been treated theoretically. By comparing these theoretical results with the experimental values of the total bending loss, it has been suggested[4] that the transition loss at large bending radii can be considerably greater than the pure bending loss for short lengths of curved fibre.

Experimental studies of propagation in a transition region have been reported in References 4, 9 and 10 but only qualitative evaluations are given. We therefore consider here more carefully the transition loss from a curved single-mode fibre, in order, (*a*) to obtain quantitative experimental values for the transition and pure bending losses separately (*b*) to assess the effects of other types of bend that may be present in a practical fibre installation.

2 Transition and pure bending losses in a curved fibre

Before describing the experiments in detail, we first outline the method of measurement. To determine the curvature loss experimentally, part of a fixed length of single-mode fibre was bent into a curve of constant radius and

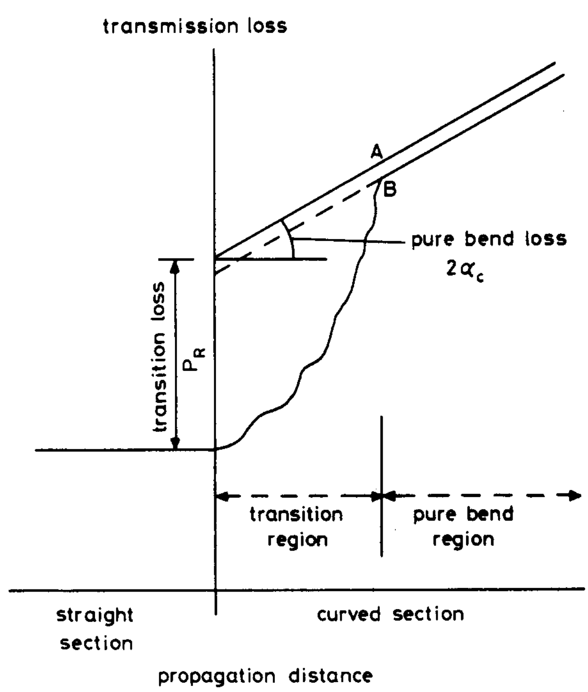

Fig. 1 *Schematic diagram of the variation of transmission loss with distance at the junction between straight and curved sections of single-mode fibre. Transmission loss of the straight fibre is assumed to be zero*

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The authors are with the Department of Electronics, University of Southampton, Southampton SO9 5NH, England

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connected to the source and detector by straight sections of fibre. The transmitted power was then measured so that losses due to uniform curvature, as well as those due to changes in curvature, were included. Miyagi and Yip[6] have considered the mode shift of the HE_{11} mode in a single-mode fibre by using the first-order perturbation theory and calculate the transition loss $2P_R$ due to coupling between the guided modes of the straight fibre and the radiation field at the beginning and end of the curved section of fibre. They have shown that the total transmission loss Γ_t is given by

$$\Gamma_t = 2P_R + 2\alpha_c L \qquad (1)$$

where L is the length of the curved fibre and $2\alpha_c$ the pure bending loss per unit length.[7,8] On this basis the transmission loss can be illustrated schematically by the straight line A in Fig. 1.

However, in practice this does not happen because, first, the power coupled to the radiation field is not instantaneously lost, as evidenced by the ray radiation, and secondly, an abrupt change in radius of curvature cannot be obtained[11] owing to the mechanical stiffness of the fibre. Thus, experimentally, the transition region is always of finite length, and the power lost from the core initially increases slowly because of the gradual curvature and then more rapidly as the radiation field is all lost. Therefore intuitively one might expect the transmission loss to be of the form expressed by curve B in Fig. 1. In the transition region, the transmission loss increases in an oscillatory fashion because of the forward and backward mode coupling between the HE_{11} mode and the unwanted leaky and cladding modes.[4,9–13] Beyond the transition region, a pure bending regime is obtained and the transition loss can be obtained by an extrapolation of the linear pure bending loss curve to the ordinate. As is illustrated in Fig. 1, the transition loss may be less than that for case A because of the gradual change in curvature[14] (see Section 2.2).

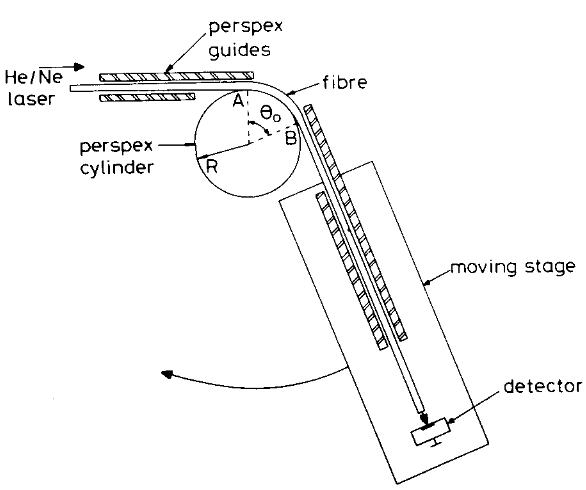
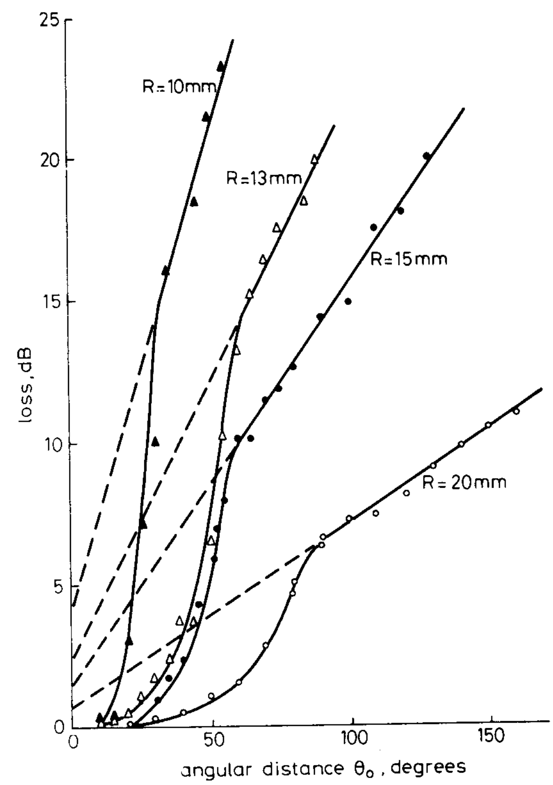


Fig. 2 *Experimental arrangement for measuring transmission loss over an angular distance θ_0 of curved fibre*

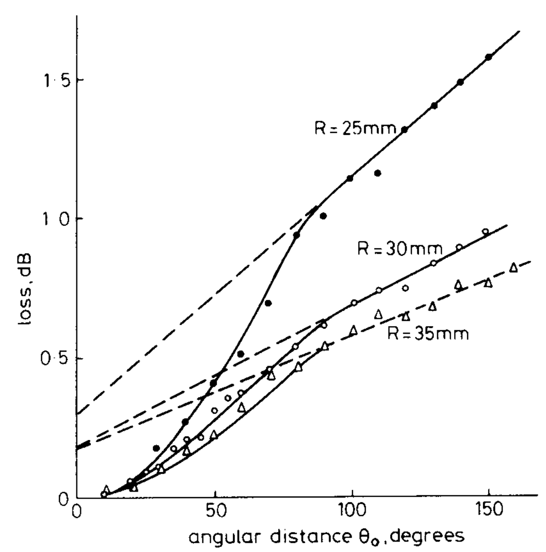
2.1 Experimental results

It can be seen from Fig. 1 that both the transition loss and the pure bending loss may be obtained by measuring the transmitted power as a function of the length of curved fibre. The very simple experimental arrangement of Fig. 2 was employed with a He/Ne laser as the excitation source of the HE_{11} mode. The fibre was progressively wrapped on the drum of given diameter $2R$ by rotating the moving stage carrying the output detector. An important point to note is that the fibre is supported in the tangential directions at points A and B on the drum by Perspex guides so as to produce as sharp a change of curvature as possible without causing additional stress to the fibre. The transitions, therefore, occur at two points, A and B.

Figs. 3a and b show the measured transmission loss as a function of angular distance θ_0 for various bending radii. In the transition region, the transmission loss is oscillatory as expected, but for convenience this is ignored and smooth curves are fitted to the data. The results follow exactly the



a



b

Fig. 3 *Measured transmission loss as a function of angular distance along a curved fibre*

Fibre parameters $V = 2{\cdot}4$, $a = 3{\cdot}9\ \mu m$, $NA = 0{\cdot}06$
a radii of curvature between 10 mm and 20 mm
b radii of curvature between 25 mm and 35 mm

predictions of Fig. 1. The transmission loss rises only gradually at the commencement of the bend, but with increasing distance along the curved fibre it increases more rapidly, and finally becomes a linear function of length. The dotted lines in Figs. 3 have been obtained via the least-squares method from the experimental values over the linear part of the curve. The slope represents the pure bending loss per unit length, and its intersection with the vertical axis at $\theta_0 = 0$ is the transition loss. As expected, the transition loss and the pure bending loss both increase with decreasing radius of curvature. The length of the transition region is a function of the radius of curvature of the fibre, and Figs. 3 indicate clearly that it decreases with increase of bending at least for the smaller bend radii. In other words, the pure bending region develops more quickly with sharp bends.

This observation is in qualitative agreement with theory. Thus, mode coupling[12] between the HE_{11} guided mode and the LP_{11} leaky mode produces an energy exchange between them so that the level of power guided by the core oscillates, as described above, in the transition region. However, with increase in distance along the curved fibre, both the leaky-mode power and the exchanged power decrease in amplitude until the equilibrium, pure bend, condition is attained. The leaky mode might be expected to radiate more rapidly at small radii of curvature, and indeed this theory predicts[12] that the transition-region length increases with radius of curvature of the fibre, in accordance with Figs. 3.

Section 2.2 shows that the length of the transition region also depends on the rate of change of curvature at the junction between the straight and curved sections of fibre. In these experiments, see Fig. 2, the fibre was constrained by guides to obtain a sharp bend without causing an additional stress effect. This was confirmed by remeasuring the transmission loss with the Perspex guides adjusted more precisely in the tangential direction and fixed more closely to the drum. The results show that the transition loss and the pure-bending loss are almost the same in each case (and therefore that the transition-loss parameter δ, see Section 2.2, is $\simeq 1$), but that the variation within the transition region is somewhat different. With a sharper change in curvature, the loss increases more rapidly in the transition region, as may be expected.

Finally, the measured transition and pure-bending losses are given in Fig. 4 as a function of the radius of curvature of the fibre. In addition, the theoretical results of Miyagi and Yip[6] for the transition loss, and those of Kuester and Chang[15] for the pure-bending loss, are indicated by the solid lines. Generally speaking, the experimental and theoretical results are in good agreement. It should be noted that the transition loss $2P_R$ is given in dB, but the pure bending loss $2\alpha_c$ is expressed as dB/10 cm. It is very important to observe from Fig. 4 that the transition loss due to changes in curvature can be considerably greater than the pure bending loss, and can play a significant role in determining fibre curvature losses, especially for short lengths of curved fibre and at large bend radii. Because the bending radius of curvature in a practical situation is relatively large, the transition loss is always important. This is one of the physical mechanisms contributing to micro-bending loss.

2.2 Effect of continuous transition

As indicated above, the power radiated away from the fibre core at the beginning of a bend does not appear suddenly, but increases gradually with distance. This is illustrated by the near-field cladding mode patterns in Fig. 5. These were obtained in a single-mode fibre with $V = 2{\cdot}4$, $a = 2{\cdot}6\,\mu m$, $NA = 0{\cdot}09$ and cladding diameter $136\,\mu m$. The fibre was coated with a layer of soft silicone rubber that produced two effects. First, it acted as a guiding layer for the cladding modes so that their rate of leakage was small, and secondly it ensured that the change of fibre curvature was gradual, and not sharp. At a bend, the power radiated away from the core can excite cladding modes[4,9,16] because of

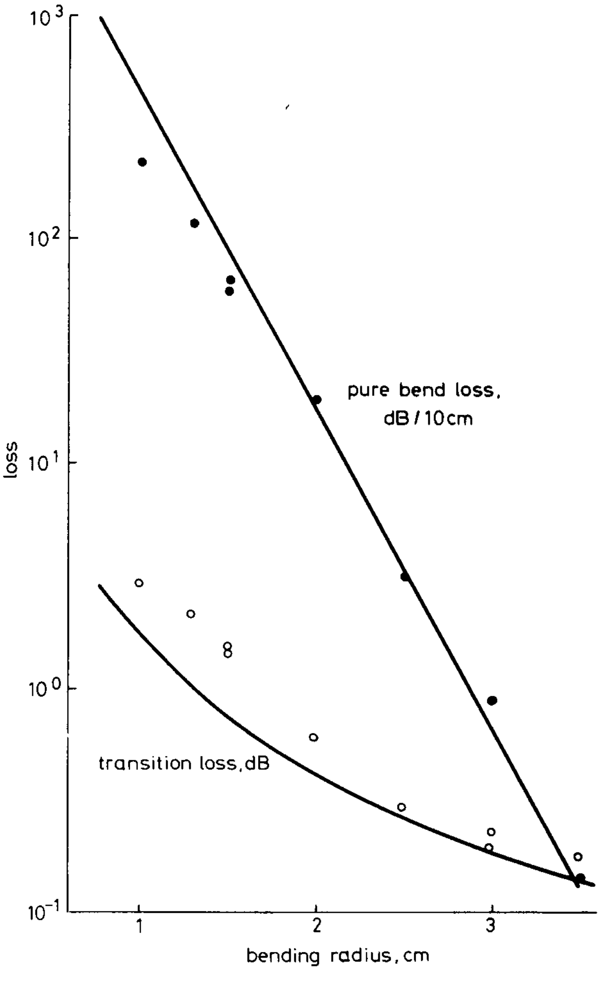


Fig. 4 *Curvature loss in a single-mode fibre. Solid lines represent theoretical calculations as indicated in the text. Fibre parameters are the same as those for Fig. 3*

○ transition loss
● pure-bend loss for 10 cm length

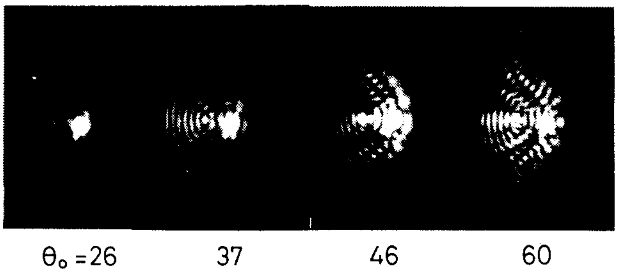


Fig. 5 *Near-field patterns of the cladding mode in a single-mode fibre of 15 mm radius of curvature at various angular distances from the onset of the bend*

Fibre parameters are $V = 2{\cdot}4$, $a = 2{\cdot}6\,\mu m$, $NA = 0{\cdot}09$ and the cladding diameter is $136\,\mu m$

the finite cladding thickness. It can be seen from Fig. 5 that the power in the cladding, i.e. the power lost by leakage from the core, increases gradually with distance. The mode pattern itself is stable and can be explained in terms of parabolic co-ordinates as described in detail in Reference 17. This process of steadily building up the mode in a curved fibre is consistent with the supposition that the transition at a bend is continuous rather than sudden.

Calculations of the transition loss in a region of varying curvature may be carried out by using coupled mode equations, but the analysis is very complicated. However, Petermann[18,19] has introduced a very simple formula for the microbending loss under these conditions. Assuming that the coupling between the HE_{11} mode and the forward travelling radiation waves predominates and that the power coupled to the radiation field is instantaneously lost, then the transition loss $2P_R$ may be derived[18] as

$$2P_R = \tfrac{1}{2}(kn_1\omega_{02})^2 \left| \int_0^L \frac{\exp(-j\Omega z)}{R(z)}\, dz \right|^2 \qquad (2)$$

where $R(z)$ is the axial z variation of the radius of curvature, k the wave number in free space, $\Omega = (\omega_{01}^2 kn_1)^{-1}$ is the difference in propagation constants of the HE_{11} and the radiation modes and ω_{01}, ω_{02} are the effective spot sizes defined in Reference 18. Originally eqn. 2 was derived for a random curvature distribution, but it is possible to show that it also applies, to a good approximation, to both continuous and sudden changes of curvature. The justification for this statement is too detailed to give here and will be published separately.* Thus, eqn. 2 can be used to calculate the transition loss under conditions of varying curvature.

As an example, we now calculate the transition loss between a straight section of fibre and a portion of constant radius of curvature R_0 where the intermediate portion has a curvature of the form

$$R(z) = R_0 \sin^{-2}\left(\frac{bz}{2}\right) \qquad (3)$$

where b is a constant. From eqns. 2 and 3, the average value of the transition loss ($2\bar{P}_R$) is obtained as

$$2\bar{P}_R = \tfrac{1}{2}\left(\frac{kn_1\omega_{02}b^2\cos(\Omega\pi/2b)}{R_0\Omega(b^2-\Omega^2)}\right)^2 \qquad (4)$$

This result may be compared with that for a step change in curvature ($b = \infty$) which is given by[18]

$$2\bar{P}_R(b=\infty) = \frac{1}{2R_0^2}(kn_1\omega_{02})^2(kn_1\omega_{01}^2)^2 \qquad (5)$$

which is approximately the same as the result obtained by Miyagi and Yip.[5] It may be seen from eqn. 4 that if the change in curvature takes place very gradually (small values of b) the transition loss is small, tending to zero as $b \to 0$, and rises to the maximum of $\bar{P}_R(b=\infty)$ for a step function change in R, i.e. for $b = \infty$. It is possible, therefore, to define a transition-loss factor δ, which will depend on the rate of change of curvature. Thus

$$\delta = \frac{2\bar{P}_R}{2\bar{P}_R(b=\infty)} = \left(\frac{\cos(\pi x/2)}{1-x^2}\right)^2 \qquad (6)$$

*GAMBLING, W.A., MATSUMURA, H., and RAGDALE, C.M.: 'Transition loss in a single-mode optical fibre' (to be published)

where

$$x = \Omega/b \qquad (7)$$

Fig. 6 shows δ as a function of x, and indicates that the transition loss depends on the length of the continuous transition region, with δ decreasing as x increases. For $a = 4\,\mu m$, $\lambda = 0{\cdot}633\,\mu m$, $V = 2{\cdot}0$ and $\pi/b = 1$ mm, we get $\delta = 0{\cdot}36$.

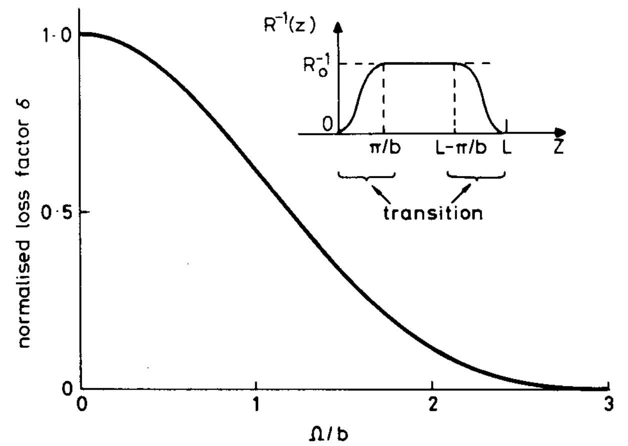


Fig. 6 *Normalised transition loss as a function of normalised length of the continuous transition region*

Variation of inverse radius of curvature between the straight and constantly curved sections of fibre is of the form $R_0^{-1}\sin^2(bz/2)$

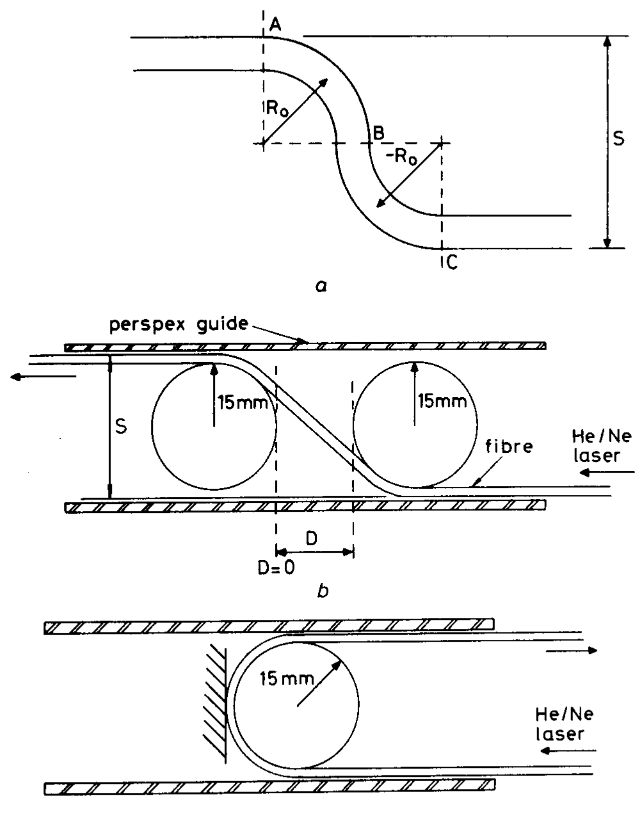


Fig. 7 *Bending due to 'kinks' in fibre*

a Offset in fibre produced by two 90° bends
b Experimental arrangement for measurements in the continuous transition region
c Experimental arrangement for measuring the transition loss between straight and curved sections

3 Bending due to 'kinks' in a fibre

We now extend the results obtained for a simple change in curvature to the situation representing a cable laid in a cable duct. The waveguide bend illustrated in Fig. 7*a* represents a typical kink in a practical fibre. The bend may consist of two sections of uniform curvature connected to each other and to straight waveguide sections as indicated by the dashed lines in Fig. 7*a*. The bends need not be through angles of 90° as in the Figure, and a more general case is shown in Fig. 7*b*. For the same waveguide displacement S (see Fig. 7), the transition loss is now produced by two separate bends rather than the step change at B in Fig. 7*a*. Furthermore, the pure-bending loss itself decreases because of the shorter length of the curved section. We will consider the effect on the transmission in this system of changing the length D which is defined in Fig. 7*b*. The first result that may be obtained from this experiment is the minimum value of D to give negligible transmission loss for a constant fibre displacement. The second is a comparison of the transition loss in a fibre, the radius of curvature of which changes from R_0 to $-R_0$ as in Fig. 7*a* (case 1) with that from a constant bending radius R_0 to a straight fibre (case 2). The transition loss $P_r(R_1:R_2)$ of a fibre due to a change of radius of curvature from R_1 to R_2 is given by

$$P_R(R_1:R_2) = \frac{Q}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)^2 \tag{8}$$

where the proportionality constant Q may be obtained from eqn. 5 and also from Reference 6. Therefore, the transition loss in case 1 in which the radius of curvature changes from $R_1 = R_0$ to $R_2 = -R_0$ should be 4 times the transition loss of case 2 in which $R_1 = R_0, R_2 = \infty$.

The transmission loss was first measured by bending the fibre around two drums each of radius 15 mm, as in Fig. 7*b*. The latter were so aligned that the displacement of the movable drum was always exactly parallel to the straight parts of the fibre. Two Perspex guides approximately 20 cm long were placed on either side of the drums and almost touching them to ensure a sharp change of curvature of the fibre. The fibre used in the experiment had parameters[20] $V = 2{\cdot}81$, $a = 4{\cdot}33\,\mu$m and $NA = 0{\cdot}065$. Although the normalised frequency V is greater than 2·4, only the HE_{11} mode was observed. This is due to the very high microbending loss of the LP_{11} mode.[21]

Starting at maximum separation, the loss of the fibre was measured as a function of the distance D between the two drums. For each set of readings, the space between the plates and the drums was kept constant so that changes in the effects of stress were minimised. The results are plotted in Fig. 8 as open circles. The transmission loss for a change of radius from R_0 to $-R_0$ is given at $D = 0$ (case 1). It is clear that the curvature loss decreases with increase in D and eventually becomes negligibly small, in this case at approximately $D = 40$ mm. Thus, it is very important to avoid too sharp a kink in any fibre installation.

The transmission loss due to change of radius of curvature from R_0 to ∞ was also measured (case 2) by bending the fibre 180° around a drum of radius 15 mm (Fig. 7*c*). A sharp change of curvature was again obtained by using the Perspex guides with the same spacing as before and the loss obtained was measured several times as shown in Fig. 8 by the solid circles.

Both measurements include the same amount of uniform bending loss ($\theta_0 = 180°$). However, the transition losses are given by

$$P_R = 2P_R(R_0:\infty) + P_R(R_0:-R_0) \qquad \text{for case 1}$$

and

$$P_R = 2P_R(R_0:\infty) \qquad \text{for case 2}$$

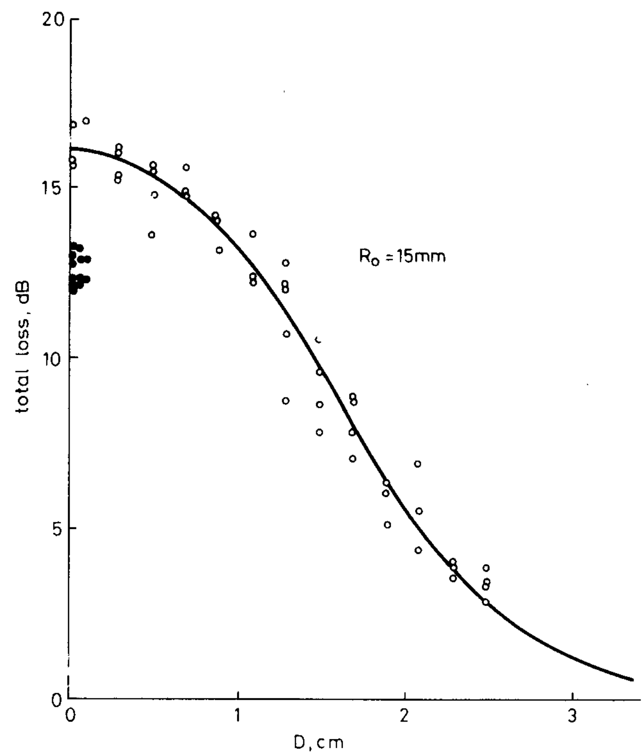


Fig. 8 *Total loss ○ measured for the configuration of Fig. 7b as a function of cylinder separation D. Solid circles ● correspond to the measurements in Fig. 7c*

The difference between the two measured losses gives the transition loss of a fibre whose radius changes from R_0 to $-R_0$. For the first case, the total loss is given when $D = 0$ in Fig. 8. This is seen to be about 16·2 dB. In the second case, the average value of the loss is 12·7 dB. Therefore $P_T(R_0:\ -R_0)$ is 3·5 dB. The transition loss has been calculated for case 2 (i.e. $2P_R(R_0:\ \infty)$), and is found to be 0·722 dB. Therefore, in this experiment we obtained

$$\frac{P_R(R_0:-R_0)}{P_R(R_0:\infty)} = 8{\cdot}8$$

which is about twice that predicted theoretically. This discrepancy may well be due to an additional stress effect and the fact that the bend transitions are not sharp.

4 Radiation loss due to sinusoidal bending

We have shown that the transmission loss for the fibre displacement shown in Fig. 7*b* is a function of the separation D of the two drums, and that the loss decreases rapidly with increasing D. However, in a practical situation there will be many such displacements and the total effect can be obtained by adding the individual losses providing the transition regions do not overlap. However, another type of modulation is one where the change of curvature is continuous rather than sudden and we have therefore considered the integrated transmission loss of several displacements in the form of sinusoidal bending.

The experimental arrangement is given in the inset to Fig. 9. Two long aluminium plates, on which several cylinders of 2·2 mm diameter were mounted, were placed parallel to each other and with a given separation S. The distance τ between the cylinders on each plate could be varied. The fibre was laid in a serpentine path constrained only by the cylinders and by its own stiffness. Although it was not possible to measure accurately the actual shape of the curve, we assume it is given by the sinusoidal formula

$$y = \frac{S}{2}\sin(\pi\eta/\tau) \qquad (9)$$

The transmission loss was measured by changing the periodic distance τ for a fibre with $V = 2{\cdot}38$, $a = 4{\cdot}1\,\mu m$ and $NA = 0{\cdot}06$. The results for 22 cylinders are shown in Fig. 9. The solid and open circles are for $S = 2{\cdot}2$ mm and $S = 32{\cdot}1$ mm, respectively. As is clearly shown, the loss depends strongly on the amplitude of the sinusoidal curve and the periodical length.

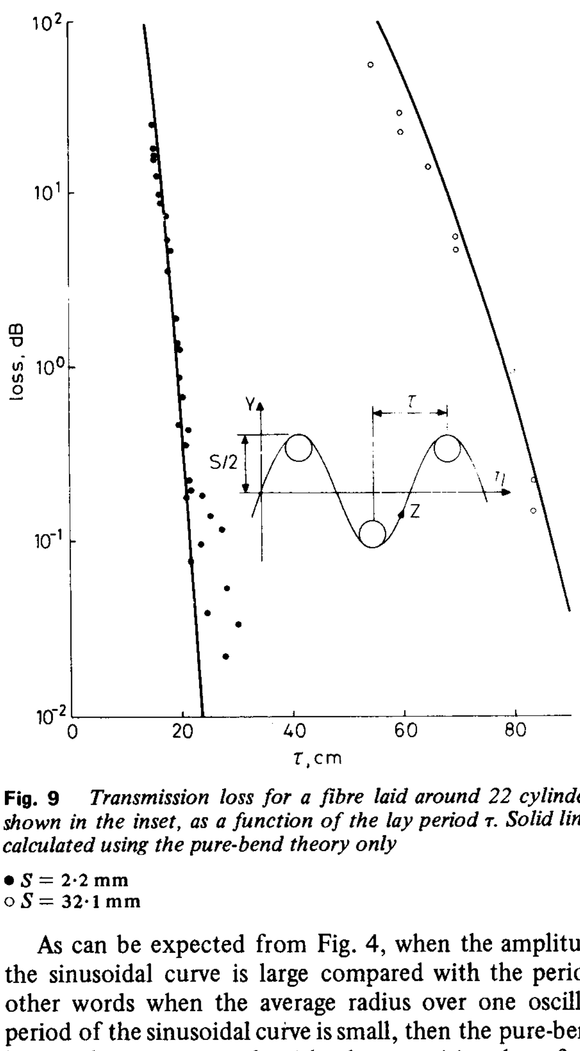


Fig. 9 *Transmission loss for a fibre laid around 22 cylinders, as shown in the inset, as a function of the lay period τ. Solid lines are calculated using the pure-bend theory only*

● $S = 2{\cdot}2$ mm
○ $S = 32{\cdot}1$ mm

As can be expected from Fig. 4, when the amplitude of the sinusoidal curve is large compared with the period, in other words when the average radius over one oscillation period of the sinusoidal curve is small, then the pure-bending loss is large compared with the transition loss for our experimental conditions. It might be expected, therefore, that in a continuous transition region the total loss is due mainly to pure bending. The pure-bend loss can be calculated theoretically by the step approximation method, which gives

$$2\alpha_c = \frac{1}{2}\int_0^L \left(\frac{\pi}{R(z)aW^3}\right)^{1/2} \left(\frac{U}{VK_1(W)}\right)^2$$

$$\exp\left\{-\frac{2W^3}{3V^2}\left(1-\left(\frac{n_2}{n_1}\right)^2\right)\frac{R(z)}{a}\right\}dz \qquad (10)$$

where

$$V^2 = U^2 + W^2$$

and U, W have their usual meanings. $K_1(W)$ is the modified Hankel function and $n_{1,2}$ the refractive indices in the core and the cladding, respectively. The radius of curvature in a sinusoidal fibre is given by

$$R(\eta) = \left(1+\left(\frac{dy}{d\eta}\right)^2\right)^{3/2} \Big/ \frac{d^2y}{d\eta^2} \qquad (11)$$

where

$$z = \int_0^\eta \left(1+\left(\frac{dy}{d\eta}\right)^2\right)^{1/2} d\eta \qquad (12)$$

Numerical calculations have been performed, and the results, shown in Fig. 9 by the solid curves, are in very good agreement with experiment. We can therefore conclude that when the amplitude of the sinusoidal deformation of the fibre is comparatively high and the periodical length is small, the transmission loss can be estimated from eqn. 10.

6 Conclusions

The transmission loss due to various forms of curvature in a single-mode fibre has been studied theoretically and experimentally. In a fibre having both straight and uniformly curved portions, the radiation loss has two components. One is the transition loss due to mode conversion at the junctions between the straight and curved portions, and the other is the pure bending loss due to uniform curvature of the fibre. An important observation is that the transition loss at large bend radii is considerably greater than the pure bending loss for short lengths of curved fibre. The transmission loss observed experimentally is oscillatory at the commencement of the bend where it is dominated by mode conversion and increases gradually. With increasing distance along the curved fibre, the loss becomes more uniform and finally increases linearly in accordance with the pure-bend theory. The rate of variation of loss in the oscillatory, or transition, region depends on the shape of the junction between the straight and curved sections. The transition and pure-bending losses have been measured separately and the results are in good agreement with theory.

In practice, there will not be a sudden increase in mode conversion at the onset of a bend because the radius of curvature cannot be changed sharply owing to the mechanical stiffness of the fibre. The effect of a continuous change of curvature has been analysed and can be expressed in terms of a transition-loss factor δ that is related to the spot size and rate of change of curvature. The actual loss is a factor δ times the loss for a step change of curvature where $0 \leqslant \delta \leqslant 1$,

A kink in the fibre can be represented by two uniform circular bends of opposite curvatures and can produce a high transition loss. Theoretically, the transition loss of a fibre due to change of radius of curvature from R to $-R$ is 4 times the transition loss for the case where the radius changes from R to ∞, but the experimental measurement gives a factor of 8. For less sharp kinks, the loss can be negligibly small.

A simple formula has been derived to describe the loss due to a sinusoidal deformation that indicates that there is a strong dependence on amplitude and periodicity. The experimental measurements follow closely the predictions of the modified bend-loss theory.

From these results it can be seen that in the prediction of transmission loss in a single-mode fibre both pure bend loss and transition loss must be taken into account. A detailed consideration of these effects for practical fibres and configurations is being undertaken.

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