OPTIMUM OPERATING WAVELENGTH FOR CHROMATIC EQUALISATION IN MULTIMODE OPTICAL FIBRES

Indexing terms: Optical dispersion, Optical fibres

Complete chromatic equalisation is not possible in a graded-index fibre owing to the variation of material properties across the core. We show that it is still possible to choose an operating wavelength which effectively eliminates first-order material dispersion.

Introduction: The transmission capacity of a multimode optical fibre may be maximised by equalising the transit-time differences which exist (*a*) between modes and (*b*) between the various spectral components launched by the source. The former may be achieved by choosing a near-parabolic refractive-index profile[1] and the latter by operation at a wavelength where the group-index is constant,[2] known as the wavelength of zero chromatic dispersion λ_0.

In a recent publication[3] it was shown that the optimum wavelength λ_0 depends on the glass composition, particularly for fibres containing germania. Thus it would appear that in a graded-index fibre where the dopant concentration varies typically from a maximum on-axis to zero in the cladding, complete chromatic equalisation is not possible since a group of modes propagating in a region of low dopant concentration requires a different operating wavelength for equalisation than does a group confined near the core centre. In this letter we show that in such a situation there exists an optimum wavelength at which material dispersion effects may be minimised and that this lies at a weighted mean between the wavelengths most appropriate for the core centre and cladding. Furthermore, operation at this wavelength results in residual first-order chromatic dispersion which in all practical cases is small compared to second-order material dispersion effects.[4,5]

Theory: The transit time $\tau_{\mu\nu}$ of mode (μ, ν) at wavelength λ may be related to that at the mean source wavelength λ_s by expanding $\tau_{\mu\nu}$ as a Taylor series about λ_s

$$\tau_{\mu\nu}(\lambda) = \tau_{\mu\nu}(\lambda_s) + (\lambda - \lambda_s)\,\tau'_{\mu\nu}(\lambda_s) + \frac{(\lambda - \lambda_s)^2}{2}\,\tau''_{\mu\nu}(\lambda_s) + \ldots \qquad (1)$$

where the primes denote differentiation w.r.t. λ. We are interested here in a mean operating wavelength λ_s close to the zero of the chromatic dispersion parameter $\tau'_{\mu\nu}$ such that the second-order term above provides the ultimate limitation on bandwidth.

It has been shown[6] that a knowledge of the r.m.s. pulse width σ is sufficient to predict the fibre bandwidth

$$\sigma^2 = \frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2 \qquad (2)$$

where the ms are the moments of the total fibre impulse response[1]

$$m_n = \sum_{\mu\nu}\left[p_{\mu\nu}\int_0^\infty \tau^n_{\mu\nu}(\lambda)\,S(\lambda)\,d\lambda\right] \qquad (3)$$

Here $p_{\mu\nu}$ is the modal power distribution function, assumed independent of λ, and $S(\lambda)$ is the spectral distribution of the source, which for convenience may be normalised such that

$$\int_0^\infty S(\lambda)\,d\lambda = 1.$$

The r.m.s. pulse width is separable into intermodal terms resulting from delay differences between modes, and intramodal terms which include averages of the pulse broadening within each mode as well as the purely material effect

$$\sigma^2 = \sigma^2_{intermodal} + \sigma^2_{intramodal} \qquad (4)$$

Using eqns. 1–4 and retaining terms up to fourth order we find

$$\sigma^2_{intramodal} = \sigma_s^2\langle\tau'^2\rangle + \eta_3\langle\tau'\,\tau''\rangle + \frac{\eta_4}{3}\langle\tau'\,\tau'''\rangle + \tfrac{1}{4}[\eta_4\langle\tau''^2\rangle - \sigma_s^4\langle\tau''\rangle^2] \qquad (5)$$

where $\langle\ \rangle$ denotes an average over the power distribution $p_{\mu\nu}$.

The r.m.s. source spectral width σ_s and higher-order moments η_k are defined as $(k = 3, 4)$

$$\sigma_s^2 = \int_0^\infty (\lambda - \lambda_s)^2\,S(\lambda)\,d\lambda; \qquad \eta_k = \int_0^\infty (\lambda - \lambda_s)^k\,S(\lambda)\,d\lambda \qquad (6)$$

It is possible to eliminate the leading term of eqn. 5 by operating at the wavelength of zero material dispersion only if λ_0 does not vary for the range of compositions used to grade the core refractive index. For the fibres considered here, however, this is no longer the case, although we will show that minimisation is possible. Similar remarks apply to terms 2 and 3, which together with the final term represent residual higher-order chromatic dispersion; moreover, term 2 vanishes

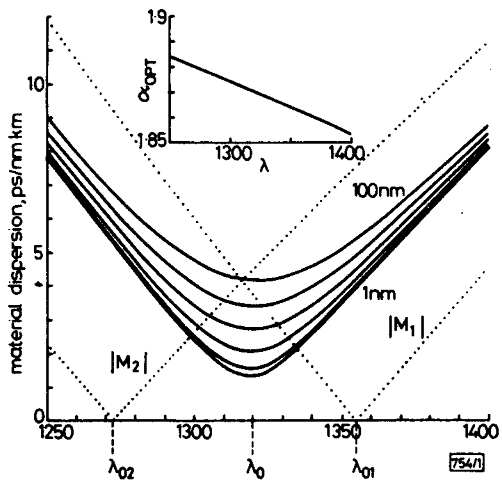

Fig. 1 *M and α as functions of wavelength*

Dotted lines: Material dispersion parameter $|M_1|$ measured for a 13m/o GeO_2/SiO_2 composition and $|M_2|$ calculated for silica
Solid lines: Effective material-dispersion parameter calculated for graded-index fibre having 13m/o GeO_2 at core centre and silica cladding. Curves shown for source spectral width $2\sigma_s$ of 1, 20, 40, 60, 80 and 100 nm
Inset: Measured wavelength dependence of optimum $\alpha = 2 - 2P$ for germania-doped fibres

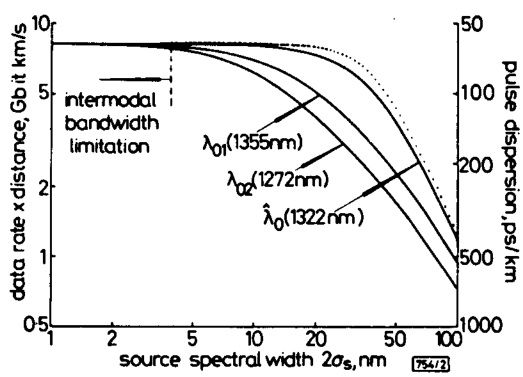

Fig. 2 *Dependence of r.m.s. pulse dispersion 2σ and data-rate × distance product on r.m.s. source spectral width $2\sigma_s$ for fibre of Fig. 1*

Curves indicate the effect of operating at wavelengths λ_{01}, λ_{02} and $\hat{\lambda}_0$ (see Fig. 1). Dotted curve shows result for $M_1 = M_2$
Note: f.w.h.m. spectral width = 1·18 × $(2\sigma_s)$.

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for sources symmetric about the mean. The final term is the second-order effect investigated by Kapron.[4]

A similar expression to eqn. 5 may be derived for[1] $\sigma^2_{intermodal}$. However, it is found that the inclusion of higher-order moments does not significantly affect the result given in Reference 1, and that for a fully-optimised profile the residual r.m.s. intermodal contribution is $n_1 \Delta^2/(20c\sqrt{3})$, where n_1 is the refractive index at core centre, Δ is the relative core/cladding index difference and c is the speed of light in vacuo.

For the remainder of this contribution we restrict attention to sources with a Gaussian spectral intensity distribution, for which $\eta_3 = 0$ and $\eta_4 = 3\sigma_s^4$.

Results: Using the WKB approximation[1] for $\tau_{\mu\nu}$ and assuming all modes equally excited ($p_{\mu\nu} = 1$) the averages in eqn. 5 may be computed, retaining terms to order Δ^3. We taken a fibre having a germania concentration of 13m/o at the core centre and a silica cladding (n.a. $\sim 0{\cdot}25$) and assume that at each wavelength the optimal index profile is achieved. Both the material dispersion parameter M for various germania compositions and the value of α describing the optimal profile have been measured as a function of wavelength in our laboratory.[3,7] The results are given in Fig. 1, together with $|M|$ calculated for silica.[2] Also shown is the computed effective material-dispersion parameter for the graded-index fibre, expressed as $2\sigma_{intramodal}/2\sigma_s$, for values of $2\sigma_s$ from 1 to 100nm. The lowest curve thus indicates the limitation imposed by the first-order term of eqn. 5 and clearly shows that although this achieves a minimum at a wavelength $\hat{\lambda}_0$ between those of the constituent materials, it cannot be eliminated. The remaining curves are increasingly dominated by the second-order term in source spectral width[4] but continue to exhibit a minimum close to $\hat{\lambda}_0$.

The relative magnitudes of the residual first and second-order material dispersion effects may be seen more easily in Fig. 2 where the optimised[1] intermodal r.m.s. pulse width for this fibre is included, according to eqn. 4. The l.h. axis indicates the data rate X distance product f given by[6] $f\sigma = 1/4$. Operation at the optimum wavelength $\hat{\lambda}_0$ produces a bandwidth capability which differs only marginally from that predicted by consideration of the second-order limitation alone. However, operation at the wavelengths λ_{01} or λ_{02} corresponding to the zeros of material dispersion of the core centre and the cladding is dominated by first-order material dispersion, and results in a curtailment of bandwidth.

Although the exact expression for intramodal pulse broadening is complex, a useful approximation is given by

$$\left(\frac{\sigma_{intramodal}}{\sigma_s}\right)^2 = \left(\frac{2M_1 + M_2}{3} - \frac{\Delta n_1 P}{3c\lambda}\right)^2 + \tfrac{1}{8}\left(\frac{dM_1}{d\lambda}\right)(2\sigma_s)^2 + O\left(\frac{\Delta^2 M_1}{\lambda c}\right) \quad (7)$$

where P is the profile dispersion parameter and M_1 and M_2 are respectively the material dispersion parameters of core centre and cladding. The expression is normally dominated by the first term on the r.h.s.; the zero of this term corresponds approximately to the optimum wavelength $\hat{\lambda}_0$. For the small values of P usually found, we see that this wavelength is given simply by the zero of the weighted mean $(2M_1 + M_2)$, i.e. roughly 2/3 of the way between λ_{02} and λ_{01}. Note, however, that this wavelength will depend on the modal power distribution within the fibre. The second term in eqn. 7 is the r.m.s. form of the second-order contribution[4] which dominates in practice at the optimum wavelength $\hat{\lambda}_0$. The remaining term gives the order of the residual first-order effect at the optimum wavelength.

Conclusions: In a graded-index fibre there exists an operating wavelength at which first-order material dispersion effects may be minimised. This wavelength lies approximately 2/3 of the way between the wavelengths of zero material dispersion for cladding and core centre, an effect which should be considered when interpreting material dispersion measurements[3] made on graded-index fibres. Operation at the optimum wavelength results in residual first-order chromatic dispersion which is small relative to the combined effects of second-order material dispersion and intermodal delay differences. Thus the bandwidth limitation can in practice be made similar to that of a fibre having uniform material dispersion properties throughout the core.

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