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LOSS CALCULATIONS IN WEAKLY-GUIDING OPTICAL DIELECTRIC WAVEGUIDES

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The application of perturbation theory to a three-layer weakly-guiding slab waveguide composed of lossy dielectric media yields a simple formula for the attenuation coefficient α of a guided mode: $\alpha = (\Sigma_{i=1}^{3} \alpha_i P_i)/(\Sigma_{i=1}^{3} P_i)$, where α_i, P_i are respectively the loss coefficient and model power in region i (i = 1, 2, 3). It is shown that this result can also be obtained from arguments based purely on geometric optics. The result is easily extended to apply to circularly-symmetric optical fibres where it yields confirmation of an earlier approximation for the power ratios $P_i/(\Sigma_{i=1}^{3} P_i)$.

1. Introduction

The theory of optical dielectric waveguides currently finds applications in a number of fields related to optical communications, viz. integrated optics [1], injection lasers [2], waveguide lasers [3], and optical fibres [4]. In all these applications the calculation of waveguide attenuation for a guided mode forms an important topic. In principle, solution of the eigenvalue equation (dispersion relation) for the waveguide structure of interest yields the attenuation coefficient from the imaginary part of the longitudinal propagation constant. However, in most cases such a solution must be performed numerically [5,6] and little physical insight is gained into the roles played by various mechanisms for attenuation. Hence approximations are usually employed in order to obtain estimates of the relative contributions of such mechanisms; a useful review of such approximations has been given by Reisinger [7].

To take a concrete example, consider the slab waveguide structure illustrated in fig. 1, consisting of a central core layer of complex refractive index $n_1 + iK_1$, between cladding layers of index $n_2 + iK_2$ and $n_3 + iK_3$. The quantities K_i are sometimes termed extinction coefficients and are related to the usual absorption coefficients α_i by $\alpha_i = 2K_i k$, where k is the wavenumber ($2\pi/\lambda$). There are two principal ranges of numerical values for n_i, K_i for which useful approximate results may be obtained for the attenuation of a

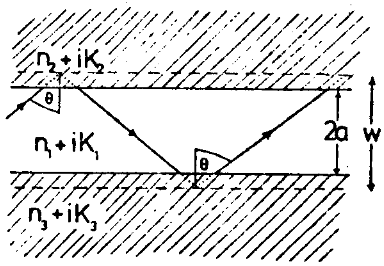

Fig. 1. The lossy slab waveguide with a modified zig-zag ray path including the effects of the Goo-Haenchen shifts at each reflection.

guided wave in this structure:

(i) $K_1 = 0$; $K_i^2 \gg n_1^2 + n_i^2$ (i = 2, 3): This corresponds to the usual dielectric waveguide with metallic walls, which has been adequately discussed in the relevant literature [3,7–11]. However it is worth noting that the same expression for attenuation coefficient may be derived either from perturbation theory [8–11] (the conventional microwave frequency method) or from geometric optics [12,13].

(ii) $n_1 > n_2, n_3$; $(n_1 - n_i) \ll n_1$ (i = 2, 3); $K_i \ll n_i$ (i = 1, 2, 3): This corresponds to a weakly-guiding dielectric guide with relatively low-loss dielectric media. For this case perturbation theory [4,7,14] yields the simple result for attenuation coefficient:

$$\alpha = \left(\sum_{i=1}^{3} \alpha_i P_i\right) \Big/ \left(\sum_{i=1}^{3} P_i\right) \qquad (1)$$

where P_i is the proportion of the total power in region i, which is easily obtained from the modal solutions of the guide [14,15]. Note that in the weakly-guiding situation considered here the TE and TM modes are degenerate. We will show below that eq. (1) can be derived from arguments based purely on geometric optics.

2. Geometric optics derivation of modal attenuation

Fig. 1 shows the path of a ray in the slab waveguide, sometimes referred to as the modified zig-zag ray model [1,16]. If the waveguide is of width $2a$, it can be shown [16] that for all calculations involving energy transfer and power flow the ray appears to propagate in a guide with an effective width w, as shown in fig. 1, where, in terms of the angle of incidence θ, [16]

$$w = 2a + \frac{1}{k(n_1^2 \sin^2\theta - n_2^2)^{1/2}} + \frac{1}{k(n_1^2 \sin^2\theta - n_3^2)^{1/2}} . \tag{2}$$

The "end-corrections" involved in this expression are those resulting from the Goos-Haenchen shift occurring at each reflection on the dielectric interfaces [16–18]. Now the modal attenuation coefficient α may be defined as the relative power loss per unit length from the guide core. The mechanisms for this loss are (i) absorption in the core region, and (ii) transmission to the cladding layers [7,18] which occurs since total internal reflection is impossible at an interface between lossy dielectric media. If R_{12}, R_{13} are the power reflection coefficients at the appropriate interfaces [19], the proportion of power lost in an axial length $2w \tan\theta$ is $(1 - R_{12}R_{13})$. Hence

$$\alpha = \left(\frac{1 - R_{12}R_{13}}{2w \tan\theta}\right) + \alpha_1 . \tag{3}$$

Invoking our assumptions of weak guidance and $K_i \ll n_i$ ($i = 1, 2, 3$), the expressions for R_{12}, R_{13} simplify considerably to yield [18,20] (TE/TM degenerate):

$$R_{1i} \approx 1 - \frac{4n_1 n_i^2 \cos\theta\,(K_i/n_i - K_1/n_1)}{(n_1^2 - n_i^2)(n_1^2 \sin^2\theta - n_i^2)^{1/2}}, \quad (i = 2, 3) \tag{4}$$

If we change to normalized variables [21] $v = ak(n_1^2 - n_i^2)^{1/2}$ ‡, $b = (n_1^2 \sin^2\theta - n_2^2)/(n_1^2 - n_2^2)$, $c = (n_2^2 - n_3^2)/(n_1^2 - n_2^2)$ ‡, with the aid of eqs. (2) and (4) eq. (3) may be re-written:

$$\alpha = \frac{1}{w}\left[\frac{(1-b)(\alpha_2 - \alpha_1)}{b^{1/2}} + \frac{(1-b)(\alpha_3 - \alpha_1)}{(b+c)^{1/2}(1+c)}\right] + \alpha_1$$

$$= \alpha_1\eta_1 + \alpha_2\eta_2 + \alpha_3\eta_3 , \tag{5}$$

where

$$\eta_1 = \frac{1}{w}\left[2v + b^{1/2} + \frac{(b+c)^{1/2}}{(1+c)}\right], \quad \eta_2 = \frac{(1-b)}{b^{1/2}w}, \tag{6a, b}$$

$$\eta_3 = \frac{(1-b)}{(b+c)^{1/2}(1+c)w}, \quad w = 2v + \frac{1}{b^{1/2}} + \frac{1}{(b+c)^{1/2}}, \tag{6c, d}$$

Comparison of eq. (6) with the expressions for modal power P_i in each region i [14,15] shows immediately that

$$\eta_i = P_i \Big/ \sum_{i=1}^{3} P_i . \tag{7}$$

Hence eq. (5) reproduces the result (1) by an argument based entirely on the zig-zag ray model.

Eq. (6d) gives the expression for the normalised mode width of an asymmetric weakly-guiding slab guide, graphical representations of which will be found if ref. [21]. For the sake of completeness we present in fig. 2 plots of the core power ratio η_1 versus v from

‡ Note that the definition of v used here is one half that in ref. [21]; this is to conform with the definition of v in the circularly symmetric guide considered later.

‡ The asymmetry factor is here termed c rather than a (as in ref. [21]) in order to distinguish it from the guide half-width a.

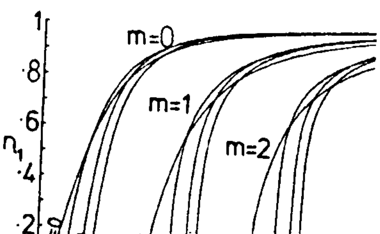

Fig. 2. Ratio of power in the core to total power, η_1, from eq. (6a) plotted versus normalized frequency v for the lowest-order modes of a slab waveguide of various degrees of asymmetry characterised by c.

(6a) for the lowest order modes. A range of values of the asymmetry factor, c, was used in the calculation, as indicated in the figure.

3. Extension to circularly-symmetry guides

For circular optical fibres the zig-zag ray model has also been applied to determine the modal attenuation coefficient. For the case of a lossless core ($K_1 = 0$), and ignoring the Goos–Haenchen shifts (which are small in many cases for multimode fibres), Gambling et al. [22] calculated the attenuation for meridional rays. More recently, Pask and Snyder [23] have used the approximation of weak guidance and relatively low loss to calculate the attenuation including skew rays. If attenuation is restricted to the important case of a multimode optical fibre whose diameter is much larger than the wavelength of the guided radiation, then the contribution of the Goos–Haenchen shifts to the effective width w is negligibly small and one may use the approximation $w \approx 2a$. Following a similar argument to that used above for the slab guide, the attenuation coefficient of a zig-zag ray may be found by summing the loss due to absorption in the core and that due to partial reflection at the core-cladding interfaces. The result, expressed in normalised variables, is [18,20,23]:

$$\alpha \approx \frac{(1-b)(\alpha_2-\alpha_1)}{(bv^2+\nu^2)^{1/2}} + \alpha_1 = \alpha_1\eta_1 + \alpha_2\eta_2 , \qquad (8)$$

where

$$\eta_1 = 1 - \eta_2 , \quad \eta_2 = \frac{1-\mathrm{b}}{(bv^2+\nu^2)^{1/2}} \qquad (9a, b)$$

and ν ia the azimuthal mode number.

In the derivation of eq. (8) it has been assumed that the Goos-Haechen shifts are negligibly small, i.e. the fibre is strongly overmoded. Regions 1 and 2 in this case are identified with the core and cladding regions, respectively. Once again the η's represent the power ratios:

$$\eta_i = P_i / \sum_{i=1}^{2} P_i \ . \qquad (10)$$

This may be verified by using the expression for power ratios resulting fromt he weakly-guiding LP mode theory [24,25]:

$$\frac{P_2}{P_1+P_2} = (1-b)\left[1 - \frac{K_\nu^2(vb^{1/2})}{K_{\nu-1}(vb^{1/2})K_{\nu+1}(vb^{1/2})}\right], \qquad (11)$$

where the K's are modified Bessel functions. Comparison of eqs. (9-11) yields a verification of the asymptotic relation [4,24]:

$$\frac{K_\nu^2(vb^{1/2})}{K_{\nu-1}(vb^{1/2})K_{\nu+1}(vb^{1/2})} \approx 1 - (bv^2+\nu^2)^{-1/2} . \qquad (12)$$

A more accurate derivation of the attenuation coefficient along the lines indicated above, but retaining the expressions for Goos-Haenchen shifts would presumable furnish an improved approximation for the lhs of eq. (12). Plots of η_1 versus v for the lowest order modes of the fibre have been given by Gloge [24] and Snyder [25].

4. Conclusion

It has been demonstrated that the simple formula (1) for the attenuation coefficient of a weakly-guiding asymmetric slab waveguide composed of relatively low-loss dielectric media may be derived from arguments based purely on geometrical optics, viz. Snell's and Fresnel's laws, proved the Goos-Haenchen shift is included. Extension of the argument to weakly-guiding optical fibres with lossy cores and claddings provides a verification of an approximation for the ratio of power in the cladding to total power of the mode.

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