SIMPLE CHARACTERISATION FACTOR FOR PRACTICAL SINGLE-MODE FIBRES

Indexing term: Optical fibres

The spot size of the HE_{11} mode in a practical single-mode fibre mainly depends on the numerical aperture and not on the normalised frequency. An alternative definition of spot size in terms of a matching Gaussian beam is much simpler to use.

Introduction: One of the most important parameters of single-mode fibres is the spot size of the HE_{11} mode, which largely determines the microbending loss, the coupling efficiency from a Gaussian source (indeed, from any source), as well as the losses caused by longitudinal, transverse and angular misalignments at joints. Strictly speaking, the spot size is a function of the core radius a, wavelength λ, normalised frequency $V=(2\pi a/\lambda)(n_1^2-n_2^2)^{1/2}$ and numerical aperture n.a. $=(n_1^2-n_2^2)^{1/2}$, where n_1, n_2 are the refractive indices of the core and cladding, respectively. In fact, extensive computations[1] for single-mode fibres of the efficiency with which a Gaussian beam can be launched, and of the loss caused at joints by spatial and angular misalignment, have shown that, in practice, the numerical aperture is the only important factor. We illustrate this by calculating the spot size as a function of n.a. and V. We show, also, that a new definition of spot size of the HE_{11} mode in terms of a Gaussian beam[2,3] is a close approximation to the more complex definition of Petermann,[4] but both may differ appreciably from the $1/e$ width of the mode.

Spot size: The spot size w_0 of the HE_{11} mode in a single-mode fibre is defined[4] as

$$w_0^2=\frac{\int_0^\infty r^3\,E^2\,dr}{\int_0^\infty r\,E^2\,dr} \tag{1}$$

where E is the field and r the radial co-ordinate. Using the

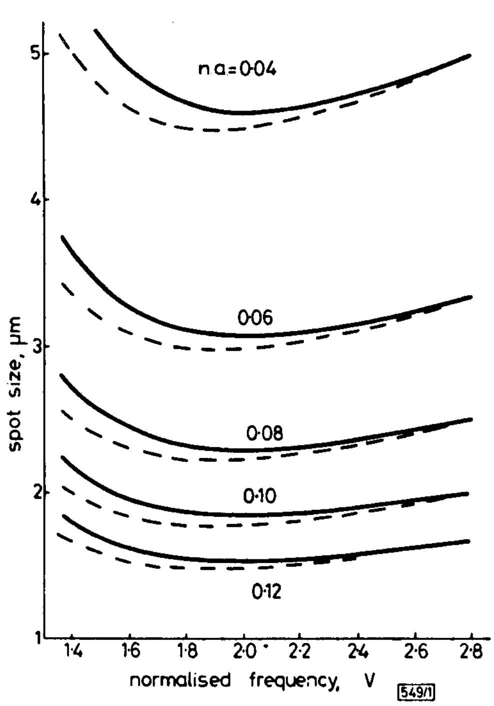

Fig. 1 *Spot size of HE_{11} mode as function of V for various n.a.*

The solid lines are calculated from eqn. 3 and the broken curves are for matching Gaussian beams

field equations for the weakly guiding fibre,[5] it can be shown that

$$w_0^2=a^2\,\frac{\int_0^1 R^3\,\frac{J_0^2(UR)}{J_0^2(U)}\,dR+\int_1^\infty R^3\,\frac{K_0^2(WR)}{K_0^2(W)}\,dR}{\int_0^1 R\,\frac{J_0^2(UR)}{J_0^2(U)}\,dR+\int_1^\infty R\,\frac{K_0^2(WR)}{K_0^2(W)}\,dR} \tag{2}$$

where $V^2=U^2+W^2$, and U, W are the arguments of the Bessel (J_0) and modified Hankel (K_0) functions. Since

$$\frac{UJ_1(U)}{J_0(U)}=\frac{WK_1(W)}{K_0(W)}$$

eqn. 2 can be reduced to

$$w_0=a\left[\frac{2}{3}\left(\frac{J_0(U)}{UJ_1(U)}+\tfrac{1}{2}+W^{-2}-U^{-2}\right)\right]^{\frac{1}{2}} \tag{3}$$

By using eqn. 3, the spot size has been calculated for a range of V and n.a. at $\lambda=0{\cdot}633\,\mu m$ (at other wavelengths, the curves are of similar form, although the numerical value of spot size is slightly different). Fig. 1 shows that w_0 is a strong function of n.a., but, for a fixed n.a., the variation with V is small. Indeed, for the range of V likely to be used in practical single-mode fibres, say, V between 2·0 and 2·4, the variation of w_0 is only 2%. The minimum spot size occurs for any n.a. at about $V=2{\cdot}03$, and, as the n.a. increases, the spot size is reduced, as may be expected. The interesting fact is that, for $1{\cdot}8<V<2{\cdot}4$, the spot size is almost independent of V, and therefore of core radius and wavelength, and depends mainly on n.a., i.e. on n_1 and n_2.

Alternative definition of spot size: In practice, the definition of spot size given by eqn. 1 is somewhat cumbersome and it has been noted[1,3,6] that an alternative definition is much simpler and more convenient to use. When a Gaussian beam is matched to a fibre, the launching efficiency is greater than 99% for $V>2{\cdot}0$, showing that the Gaussian beam forms a good match to the HE_{11} mode. Thus the new definition of spot size is that of the input Gaussian beam which gives the highest launching efficiency, and it is compared with the normal value in Fig. 1 (broken curves). Particularly for the V values normally used, the agreement between the two sets of curves is very close (within 3%), showing that the new definition is perfectly acceptable for calculations of spot size and thus of microbending loss, as well as offset, tilt and mismatch at joints. The minimum spot size from the Gaussian fit is at $V=1{\cdot}9$, rather than 2·03 from eqn. 1, but the curves are so insensitive to changes in V that this difference is unimportant for most applications.

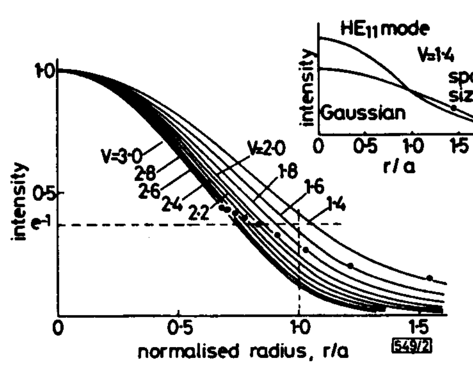

Fig. 2 *Near-field HE_{11} mode distribution for various V*

The solid circles are the spot sizes calculated from eqn. 3. The inset compares, for the (atypical) value $V=1{\cdot}4$, the HE_{11} mode distribution with that of the Gaussian beam giving maximum launching efficiency

It should be noted that the values obtained in this way differ from the width to $1/e$ intensity of the HE_{11} mode except, as described below, for a small range of V. In Fig. 2

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are plotted the near-field intensity distributions of the HE_{11} mode for various normalised frequencies. The points shown on the curves represent the spot sizes calculated from eqn. 3, and, as we have seen, these lie close to the spot size of the Gaussian beam giving the best launching efficiency. However, the $1/e$ intensity widths of the HE_{11} mode lie along the horizontal broken line and correspond to the other definitions only for V near 2·2. Particularly towards small values of V, the difference becomes quite marked and arises because the HE_{11}-mode distribution progressively departs from a true Gaussian form, so that, for example, the maximum launching efficiency and jointing loss, depends on numerical aperture The difference in distribution is illustrated in the inset to Fig. 2.

Conclusions: The mode spot size in a single-mode fibre, which is a critical parameter controlling microbending loss, launching efficiency and jointing loss, depends on numerical aperture and is little affected by V values between 2·0 and 2·4. An alternative definition in terms of a matching Gaussian beam is sufficiently accurate over the same range (within 1·5%) and is much simpler to use.

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