an analysis of stimulated raman scattering in guided systems

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ABSTRACTS

In this paper we compare the performance of a capillary waveguide and un unguided cell in terms of the pump power necessary to reach the threshold for Stimulated Raman Scattering (SRS) and the Raman exponential gain. The main characteristics of the capillary waveguide and their role in the SRS process are discussed. The case of a non diffraction limited beam is also considered and a general expression is derived for the overall enhancement provided by the capillary.

RESUMO

Neste trabalho é feita uma comparação entre um guia de onda dielétrico e uma célula sem guiagem em termos da potência necessária para se alcançar o limiar de geração Raman e do ganho exponencial. As principais características dos capilares e o papel que representam no espalhamento Raman estimulado (ERE) são discutidas. O caso de um feixe não limitado por difração é também considerado e uma expressão geral é apresentada para o incremento no ganho exponencial devido à utilização de um sistema guiado.

1. INTRODUÇÃO

Stimulated Raman Scattering (SRS) provides a simple means for extending the characteristics of lasers in the visible region to the ultraviolet or infrared region of the spectrum. For example, the tuning range of the dye laser radiation can be extended into regions of the spectrum where dye laser operation is inefficient or difficult to achieve. Similarly the high repetition rate and high pulse power available with visible lasers can be transferred to different wavelengths.

However the SRS effect is a third order non linear process which generally implies that a high pulse intensity must be provided in order to reach threshold and make the whole process efficient. The problem becomes greater when one goes to the infrared region where the Raman gain coefficient for most of the materials is very much reduced. The most widely used arrangement involves tight focussing of the pump beam into the Raman medium and the Stokes radiation being generated on a single pass through the medium. In practice this arrangement gives poor control over the various processes which can occur, such as multiple Raman shifts, backward Raman scattering and generation of high order Stokes and anti-Stokes radiation through four-wave-mixing processes. The result of such an uncontrolled mixture of processes can be poor conversion efficiency and a large angular spread of the generated radiation. Besides, in this tight focussing arrangement the Raman exponential gain depends only on the pump laser characteristics, such as pump pulse power, wavelength and spatial quality of the pump beam.

Radiation in the near-and far-infrared is of great interest nowadays due to their applications in communication and photochemistry. However, in order to obtain efficient Raman conversion in these regions of the spectrum one needs a laser of high power with a very good beam quality, since these parameters will compensate for the small Raman gain coefficient in the infrared region of the spectrum. Nevertheless, if one uses a guiding structure containing the Raman medium, a new parameter is involved: the cell geometry.

Unlike the usual tight focussing system, the guiding structure can prevent the beam from diffracting by reflecting it back into the Raman medium repeatedly. The overall Raman exponential gain now depends on the laser pulse power, the Raman gain coefficient and the cell geometry, i.e. the length of the waveguide and its internal diameter. By using this device we can make the spot size much smaller, increasing the beam intensity in the Raman medium while still maintaining a long interaction length, since the diffraction losses are less important and the interaction length is defined basically by the attenuation of the propagating radiation.

2. RAMAN THRESHOLD FOR UNGUIDED MEDIUM

The Raman exponential gain along the axis for an unguided system with a length L can be written as,

$$G_u = g\int_{-L/2}^{L/2} \frac{2P}{\pi w^2(z)}\, dz \qquad (1)$$

where g is the Raman gain coefficient, P is the pump pulse peak power and w(z) the pump beam spot size at a position z inside the cell (Figure 1). By using the expression for the spot size,[1]

$$w(z)^2 = w_0^2\,[1+(2z/b)^2] \qquad (2)$$

$$b = 2\pi w_0^2/\lambda \qquad (3)$$

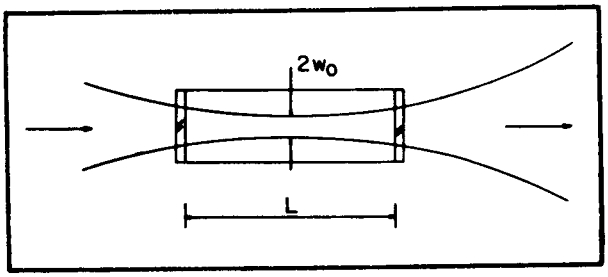

Fig. 1 — Scheme of SRS generation in an unguided medium.

where w_0 is the beam waist formed in the center of the cell, we have:

$$G_u = g(4P/\lambda).\tan^{-1}(L/b) \qquad (4)$$

However, in the general case when a tight focussing configuration is considered, expression (1) does not correctly describe the gain experienced by the Stokes wave, since in this case the Stokes radiation has a gain which is averaged over the pump intensity profile and consequently the overall Raman exponential gain is smaller than the one predicted by expression (1).

The Raman exponential gain for this case is given by[2]

$$G = [(\lambda_s/\lambda_p).\tan^{-1}(L/b)].$$
$$.\sqrt{4Pg/\lambda_s}.\,(\sqrt{4Pg/\lambda_s} - 2) \qquad (5)$$

For the case of a Raman amplifier expression (1) can still be used since then, tight focussing is not used and the confocal parameter is usually a few times the cell length. The Stokes average gain can be well approximated by the maximum gain since the Stokes spot size can be made small relative to the pump spot size.

Let us assume G_{th} is the exponential gain necessary to reach the SRS threshold. The term "threshold" used in this section and throughout this paper does not indicate the usual condition in a resonator when the gain provided by the active medium should balance the round trip losses for a given oscillator mode. For the stimulated Raman scattering process the term describes a situation in which the output power for the generated Stokes pulse has reached a level which is just enough to start to deplete the pump pulse. The Stokes radiation pulse power grows exponentially from the noise background level or an input signal and when its energy is strong enough to extract an appreciable part of the pump pulse energy, we say that the Stokes component has reached threshold. For a pump pulse width of the order of 10 nanoseconds the Stokes energy at threshold is about 1 to 10 microjoules per pulse. This happens for a Raman exponential gain (G_{th}) of between 20 and 30.

The pump pulse needed to reach this Raman exponential gain is defined as the Raman threshold power for this pumping system,

$$P_{th} = (\lambda_s/4g).$$
$$\left\{1 + [1 + G_{th}\lambda_p/(\lambda_s \mathrm{tg}^{-1}(L/b))]^{1/2}\right\}^2 \qquad (6)$$

The minimum value for P_{th} is reached in the tight focussing situation, i.e., when the beam waist size is made very small so that the confocal parameter is much shorter than the cell length.

$$P_{th}|_{min} = (\lambda_s/4g).$$

$$.[1 + (1 + 2G_{th}\lambda_p/\pi\lambda_s)^{1/2}]^2 \qquad (7)$$

As can be observed from expression (7) the value for the pump pulse power necessary to reach Raman threshold depends only on the pump laser wavelength and the Raman medium gain coefficient. The dependence on the beam waist size disappears when one assumes tight focussing.

3. RAMAN THRESHOLD FOR A GUIDED MEDIUM

From expression (6) one can see that there is no optimum value for the pump beam waist which minimizes the pump power threshold, since the whole expression decreases monotonically with $tg^{-1}(L/b)$, consequently any value for the beam waist size which produces $b \ll L$ will lead to the minimum threshold pump power. This happens because an increase in the pump intensity due to a reduction in the beam waist size is compensated by an increase in the diffraction of the beam which decreases the interaction length. The overall result is that P_{th} does not depend on the pump focussing geometry any more.

However, when one uses a capillary waveguide as the Raman cell the pump beam can be focused to a small waist size without reducting the interaction length, since the confinement of the radiation in the guided medium reduces the diffraction. However, due to the fact that the refractive index of the gas inside the guide is smaller than the refractive index of the guide walls, the propagating radiation has losses, unlike the lossless propagation which occurs in guided based on total internal reflection. The attenuation coefficient is given by[3]

$$\alpha_{nm} = (\mu_{nm}/2\pi)^2 . \frac{\lambda^2}{a^3} . \frac{\frac{1}{2}(\eta^2+1)}{\sqrt{\eta^2-1}} \qquad (8)$$

where μ_{nm} is the m_{th} root of the equation,

$$J_{-1}(\mu_{nm}) = 0$$

and *n* and *m* are integers that characterize the propagating mode, ***a*** is the capillary radius and η is the capillary wall refractive index. This attenuation coefficient is for a straight capillary, but if the system is curved, an extra term should be added to the attenuation coefficient given in expression (8).[3] Experimentally, however we observed that this bend effect is a minor problem compared with the transmission losses and the coupling losses. In Table I we have the seven

TABLE I

Mode Description	μ_{nm}
1. EH_{11}	2.405
2. $EH_{-11}+EH_{31}$	5.313
3. EH_{12}	5.520
4. $EH_{-21}+EH_{41}$	6.380
5. $EH_{-31}+EH_{51}$	7.588
6. $EH_{-12}+EH_{32}$	8.417
7. $EH_{-22}+EH_{42}$	9.761

first capillary propagating modes in order of increasing μ_{nm}.

From the attenuation coefficient we have that a large value of μ_{nm} will produce a highly attenuated mode. Thus we must try to keep μ_{nm} as small as possible. From Table I we can see that the hybrid mode EH_{11} has the smallest value for μ_{nm}, therefore this will be the preferred mode for the beam inside the capillary since it will produce the smallest attenuation of all the propagating modes. The condition for optimizing the coupling between a TEM_{00} mode and the EH_{11} mode is given by[4] That is, beam waist of the input radiation should be 2/3 of the capillary bore radius. With this condition satisfied calculations[4] predict only 2% loss in the coupling $TEM_{00} \rightarrow EH_{11}$.

The exponential gain for a guided structure can be written as

$$Gg = g\int_0^L \frac{P.\exp(-2\alpha z).dz}{\pi w_0^2} \qquad (9)$$

where all the quantities have the same meaning as defined previously for the unguided cell and w_0 is the beam waist size formed at the capillary entrance, which satisfies the condition for minimun coupling losses (Figure 2). Solving the integral in expression (9) we have.

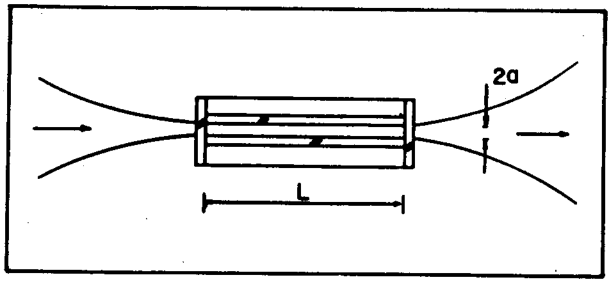

Fig. 2 — Scheme of SRS generation in a capillary waveguide.

$$G_g = \frac{P}{\pi w_0^2} \cdot g \cdot L_{eff} \qquad (10)$$

where

$$L_{eff} = [1 - \exp(-2\alpha L)]/2\alpha \qquad (11)$$

where L is the actual capillary length and L_{eff} the equivalent capillary length over which the pump radiation intensity can be regarded as having a constant value.

From expression (10) we may observe that unlike the tight focussing arrangement, the capillary waveguide Raman exponential gain depends strongly on the cell characteristics, like cell length, bore diameter and the cell material refractive index.

The pump pulse threshold power can be obtained from expresion 10 by setting $G_g = G_{th}$ in that expression:

$$P_{th} = \frac{G_{th} \cdot \pi w_0^2}{g L_{eff}} \qquad (12)$$

In this case, unlike the unguided case, the minimum pump pulse threshold is not reached for a very small value of w_0. Let us assume a fixed length of cell and since w_0 is related to ***a***, a small value of w_0 will lead to a very small capillary radius and consequently an increase in the beam propagating losses. As the attenuation losses are proportional to w_0^{-3} they will overcome the increase in the pump intensity due to the spot size reduction and the whole expression(12) will follow a w_0^{-1} behaviour:

$$P_{th} \Big|_{\lim w_0 \to 0} \propto 1/w_0 \qquad (13)$$

At the other extreme, a large beam waist does not reduce the pump pulse power threshold in either the unguided or guided case, because the beam intensity drops down and the whole Raman exponential gain becomes very small. The expression (12) can be written in this situation as proportional to w_0^2:

$$P_{th} \Big|_{\lim w_0 \to \infty} \propto w_0^2 \qquad (14)$$

Thus we expect that for a capillary waveguide of a given length we will have a specific value of pump beam waist and capillary radius which give a minimum SRS threshold power. This value can be obtained from the expression (11)and it is found to be a function of the pump wavelength and the capillary characteristics. The exact value of the optimum capillary radius is given through the implicit equation,

$$\exp(\alpha L) = (1 + 3\alpha L) \qquad (15)$$

Figure 3 shows the pump pulse power threshold variation with the beam waist size for both arrangements, and we notice the minimum value reached for the guided structure and the enormous reduction in the P_{th} value when compared with the unguided system.

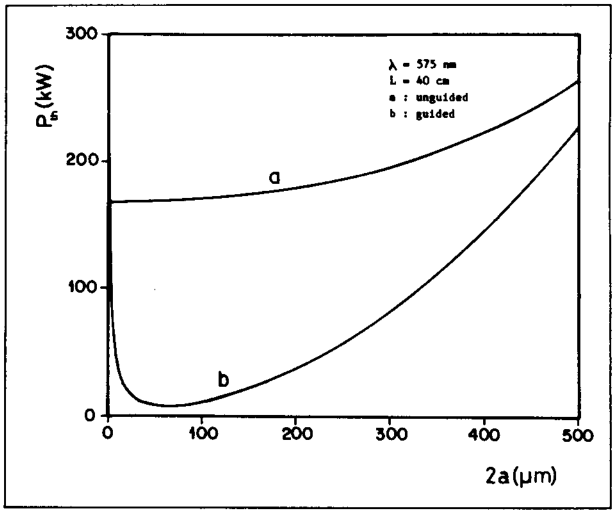

Fig. 3 — SRS threshold power for a guided and an unguided media.

4. SRS WITH A NON DIFFRACTION-LIMITED BEAM

Let us now consider a beam whose divergence δ' is ***M*** times that for a diffraction limited beam with a beam waist size w_0,

$$\delta' = M.\lambda/\pi w_0 \qquad (16)$$

Since the confocal parameter is a measure of how much the beam diffracts we can write:

$$b' = b/M \qquad (17)$$

where ***b*** is the confocal parameter for a diffraction limited beam. The expression for the unguided Raman exponential gain then becomes,[5]

$$G_u = [(\lambda_s/M\lambda_p).tg^{-1}(L/b')]. \sqrt{4Pg/(\lambda_s)}. \cdot (\sqrt{4Pg/\lambda_s} - 2) \qquad (18)$$

where

$$b' = b/M$$

As we may notice, the Raman exponential gain for a tight focussing configuration still depends only on the Raman medium and the pump laser characteristics, i.e. pulse power, wavelength and number of times the beam is diffraction-limited.

$$L \gg b'$$

$$G_u' = [\pi\lambda_s/(2M\lambda_p)].\ \sqrt{4Pg/(\lambda_s)}.$$
$$(\sqrt{4PG/\lambda_s} - 2) \qquad (19)$$

For the guided structure the non diffraction limited beam effect will appear in the attenuation coefficient, since it will produce a large beam diffraction angle inside the capillary and consequently will increase the leakage losses, and we have[5]

$$\alpha' = (\mu n m/2\pi)^2 \frac{\lambda'^2}{a^3} \cdot \frac{\tfrac{1}{2}(\eta^2+1)}{\sqrt{\eta^2+1}} \qquad (20)$$

and

$$\lambda' = M\lambda$$

The effect of the non diffraction limited beam in both systems is to "increase" the radiation wavelength and its effect can be taken into account in any previous expression by performing the transformation:

$$\lambda \rightarrow M\lambda$$

It is important to point out that there is no real change in the radiation wavelength, and what the transformation does is to assume that the increase in the beam diffraction is not due to any peculiarity of the beam spatial profile but is due to a virtual increase in the radiation wavelength of a beam with a perfect Gaussian profile.

The enhancement of the Raman exponential gain produced by the guided structure relative to the gain in the unguided situation can be defined in a very general way as

$$S = G_g'/G_u' \qquad (21)$$

where G_g' and G_u' are the Raman exponential gains for the guided and unguided systems respectively for a non diffraction limited beam and a general value of beam waist size. By substituting expressions (10), (18) and (20) into expression (21) we have

$$S = PgL_{eff}'M\lambda_p/[\pi w_0^2 \lambda_s \tan^{-1}$$
$$(L/b')\ \sqrt{4Pg/\lambda_s}\ (\sqrt{4Pg/\lambda_s} - 2)] \qquad (22)$$

where

$$L_{eff}' = [1\text{-}\exp(-2\alpha' L)]/2\alpha'$$

$$b' = b/M$$

In Figure 4 we show the plot of the ***S*** values against the capillary radius for a wavelength of 532 nm and several values of ***M***. As we may observe from these diagrams the optimum value for the capillary radius increases when the value of ***M*** is increased and the ***S*** peak value decreases simultaneously.

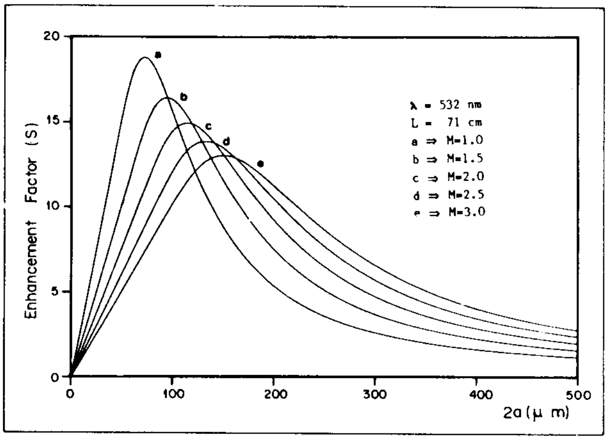

Fig. 4 — Enhancement factor against capillary diameter for a beam 1, 1.5, 2.0, 2.5 and 3 times diffraction limited.

For a large beam waist size the Raman exponential gain can be approximated by the Stokes gains along the axis given by expression (4), and the factor S_0 can be written as,

$$S_0 = \lambda_p M b' L_{eff}'/(4\pi w_0^2 L)$$

Additionally the effective length can be approximated by the actual capillary length since a large spot size implies a large capillary bore diameter and consequently a low transmission loss (expression (8)),

$$S_0 = \lambda_p M b'/(4\pi w_0^2) \qquad (23)$$

and as

$$b' \propto 1/M$$

the final expression for the S_0 factor does not depend on the beam diffraction any longer.

$$S_0 = \lambda_p . b/(4\pi w_0^2) \qquad (24)$$

2. CONCLUSION

The capillary waveguide provides a very convenient way of reducing the threshold pump pulse power since it can keep the interaction length relatively long while the pump pulse intensity is increased through the reduction of the capillary bore diameter. Besides this main feature of producing a large Raman exponential gain the capillary brings out another advantage which is the mode selection due to the spatial filter effect. This mode selection is particularly important when one has a large amount of available energy in the pump pulse but the beam intensity spatial distribution over the cross section does not resemble a Gaussian one. In this case the attenuation for the higher order capillary modes will select basically the EH_{11} mode and the output beam will have a Gaussian-like shape. The Raman conversion efficiency for this situation is not expected to be very high since the effective input beam pulse energy is very small. However, from the enhancement factor S defined in the text we can see for example, that the capillary waveguide can provide a maximum enhancement in the Raman exponential gain by a factor of almost fourteen times for a beam 2.5 times diffraction, limited when a 132 micron bore diameter capillary is used. An important point to note is that the enhancement provided by the capillary is improved for a non diffraction limited beam when the capillary bore diameter is not the optimum value, i.e. for a non diffraction limited beam it is even more worthwhile to use a capillary waveguide than for a diffraction limited pump beam. For example, for a 350 micron bore diameter the enhancement factor is 2.0 for a diffraction limited beam, and 4.4 for a 2.5 times diffraction limited beam. Figure 5 shows a photograph taken of the beams coming out of the capillary waveguide used as a Raman amplifier. There are two anti-Stokes and 3 Stokes components, the pump beam has a wavelength of 532 nm and it is non diffraction limited and the input radiations have a ring-like shape yet a reasonably symmetrical beam was generated as one can observe from the Stokes spot shape. The wavelength for the third Stokes component is in the 1.5 μm region and its pulse energy is ~ 500 microjoules in a 20 nanosecond pulse, approximately the same energy as the pump pulse seen in the photograph. This experiment will be described in more deitail in a later publication in this magazine.

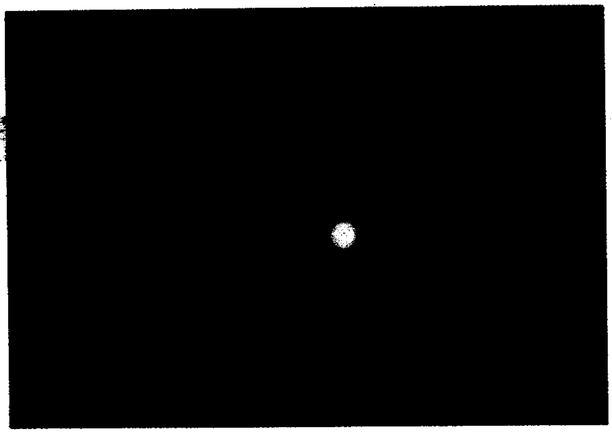

Fig. 5 — Photograph of the beams coming out the capillary waveguide. The wavelength of the pump radiation is 532 nm. Two anti-Stokes components can be observed at the left side of the pump spot and two Stokes components at the right side.

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