layer which is contributing to most of the current. This enables us to determine an 'effective lifetime' against the doping level defined at $x = 1/\alpha$, as plotted in Fig. 2. Although Fig. 2 is not a true plot of 'lifetime against doping level', since excess carriers exist and recombine at all values of x, and N_D is a function of x, the curve does show conclusively that the lifetime τ_p is position dependent, i.e. a function of N_D. In fact, the results show that τ_p must be made position dependent to obtain a quantitatively correct numerical solution using the same $\tau_p(x)$ function at all wavelengths.

It is interesting to note from these results that the value of surface recombination velocity, used as a fitting parameter, has only a very small effect on the deduced values of carrier lifetime.

Conclusions: The experimentally determined low values (τ_p is of the order of 1 ns) of minority-carrier lifetime in shallow diffused n^+ regions explains adequately the fall-off of collection efficiency at short visible wavelengths of the shallow ($x_j \leqslant 1$ µm) n^+-p photodiodes. Collection efficiency losses in the blue end of the visible spectrum are apparently mainly due to heavy recombination in the shallow n^+-region and not to surface recombination, as generally believed. Surface recombination losses due to large surface recombination velocities (10^5 cm s^{-1}) may be present, but such losses are still small compared with typical bulk recombination loss in the n^+-region.

The lifetime values are of significance not only in determining the performance of photodiodes, but also in studying transistor gain h_{FE}, as limited by emitter injection efficiency. Note that the values obtained for the 4 µm diffused junction in this study (0·4 to 40 ns) compare with the values deduced from h_{FE} measurements in Reference 1 (0·53 to 2·1 ns for a junction depth $X_j = 0{\cdot}6$ µm, 14 ns for $x_j = 2{\cdot}5$ µm and 65 ns for $x_j = 8$ µm). The results are also compatible with those given in Reference 2.

S. G. CHAMBERLAIN *18th July 1977*
D. J. ROULSTON
S. P. DESAI

Electrical Engineering Department
University of Waterloo
Waterloo, Ont., Canada N2L 3G1

References

1 CHAMBERLAIN, N. G., and ROULSTON, D. J.: 'Determination of minority carrier lifetime of bipolar transistors from low-current h_{FE} fall-off', *IEEE Trans.*, 1976, **ED-23**, pp. 1346–1348
2 ILES, P. A., and SOCLOF, S. I.: 'Effect of impurity doping concentration on solar cell output'. Presented at 11th IEEE photovoltaic spec. conference, Scottsdale, AZ, 1975
3 HAUSER, J. R., and DUNBAR, P. M.: 'Performance limitations of silicon solar cells', *IEEE Trans.*, 1977, **ED-24**, pp. 305–321
4 ROULSTON, D. J., CHAMBERLAIN, S. G., and SEHGAL, J.: 'Simplified computer-aided analysis of double diffused transistors including two dimensional high-level effects', *ibid.*, 1972, **ED-19**, pp. 809–820
5 VAN OVERSTRAETEN, R. J., DE MAN, H. J., and MERTENS, R. R.: 'Transport equations in heavy doped silicon', *ibid.*, 1973, **ED-20**, pp. 290–298

INTENSITY MODULATION OF LIGHT TRANSMITTED IN CURVED OPTICAL FIBRES

Indexing terms: Optical fibres, Optical waveguide theory

Coupled-mode theory is used to explain the intensity modulation of transmitted light which results from small changes in the radius of curvature. It is shown that the mode coupling induced by bends in small-V fibres introduces an uncertainty into light-transmission measurements.

Introduction: In recent experiments, Nelson, Kleinman and Wecht[1] have measured the intensity modulation, of light transmitted through curved optical fibres, due to small-amplitude mechanical vibrations. As a byproduct of these measurements, it was noted that varying the radius of curvature caused oscillations in the transmitted intensity, i.e. a fibre could sometimes transmit more light when bent in a smaller loop. In fact, a plot of transmitted intensity against radius of curvature showed that an irregular oscillation with a frequency of the order of 50 cycles per loop was superimposed on a much slower variation at about 5 cycles per loop.

Further experiments showed that the high-frequency oscillation could be completely damped by immersing the fibre in a high-refractive-index liquid, leading Nelson *et al.* to conclude that it could be attributed to curvature-induced coupling between core and cladding modes of the fibre. However, no change was seen in the low-frequency oscillation and no comment was made on its possible origin.

The purpose of this letter is to show that the slower oscillation is also the result of curvature-induced coupling and to point out that, together with recent observations of discrete radiation from curved fibres,[2] these results emphasise the care that must be taken in interpreting transmission measurements in short-range fibre applications. A limitation of the coupled-mode theory of radiation from curved fibres[3] is also discussed.

Theory: In the experiment, the transmitted light is measured after one complete loop of a fibre with $V \simeq 2{\cdot}6$. When the radius of curvature is varied by ΔR, the length of fibre at that radius is therefore varied by $2\pi\Delta R$, and it will be shown that it is the latter change, rather than just the change in radius, which produces the oscillation.

It has previously been found that, in a small-V fibre, there is a transient region near discontinuities in the fibre curvature where mode coupling results in an oscillation of the position of peak light intensity.[3] This occurs because the fundamental (LP_{01}) mode and the LP_{11} mode, to which it is predominantly coupled, have different field distributions. The overall field distribution therefore varies according to the proportion of the total power contained in each mode. Moreover, since the two modes are not equally confined to the core, the transmitted power within the core will also oscillate as light travels around the bend. This is the source of the experimentally observed intensity modulation.

To see this, we calculate the power within the core P_c as a function of distance from the beginning of the bend. Using the coupled-mode equations given in Reference 3 (and originally derived in Reference 4), we find that, if all the light is initially in the fundamental mode,[4]

$$P_c = \eta_1 - (C_{12}d/\pi)^2(1 - \cos(2\pi l/d))(\eta_2 - \eta_1) \quad (1)$$

where al is the length of the curved section of fibre, a is the core radius, η_p gives the proportion of mode-p's power which is confined to the core[5] and

$$d = 2\pi/\text{Im}\,[(C_{11} - C_{22})^2 + 8C_{12}^2]^{\frac{1}{2}} \; * \quad (2)$$

The coefficients C_{pq} are defined by eqn. 2 of Reference 3.

Results and discussion: For purposes of comparison with the experimental results, Fig. 1 shows the power within the core plotted as a function of radius of curvature in the range $R_1 \leqslant R \leqslant 2R_1$, where $R_1 = 1{\cdot}8$ cm. All other parameters are taken from Reference 1 and are given in the Figure.

The broken curve shows the result when l is left unchanged at a value of $2\pi R_1/a$, while R is varied. There is clearly very little variation in P_c over the entire range. The solid curve gives the result when the power is calculated at the end of one complete loop at each radius ($l = 2\pi R/a$) and shows a very distinct oscillation. When translated into units of axial distance, la, the period of oscillation of this curve lies between 3·714 and 3·875 cm, agreeing very closely with the measured period of about 3·8 cm.

This theoretical curve differs from the experimental curve[1] in detail in that (a) we have not attempted to duplicate the experimental launching conditions, so that there is a phase difference between the curves, (b) the theoretical results do not include any loss effect, since, unlike Reference 3, the

* The factor 2 was omitted from the definition of d given in Reference 3

LP_{11} mode is now above cut-off—the loss could only be taken into account by considering coupling to the radiation field (see Appendix)—and (*c*) the measured modulation has a greater amplitude than that calculated here. This could be attributable to the difference in launching conditions, but would also occur if the measured output were detected from an area smaller than the whole fibre core, thus enhancing the modulation effect.

Setting aside these differences in detail, the similarity in behaviour of the theoretical and experimental results and the very close agreement between the two values for the period of oscillation suggest that the observed intensity modulation is, in fact, the result of coupling between the core modes of the curved fibre.

Together with earlier observations of nonuniform radiation from curved, small-V fibres,[2] this shows that, in short-range applications where there is insufficient time for an equilibrium field distribution to be established, the measured output intensity may be very sensitive to the precise length of the fibre and to relatively small changes in curvature.

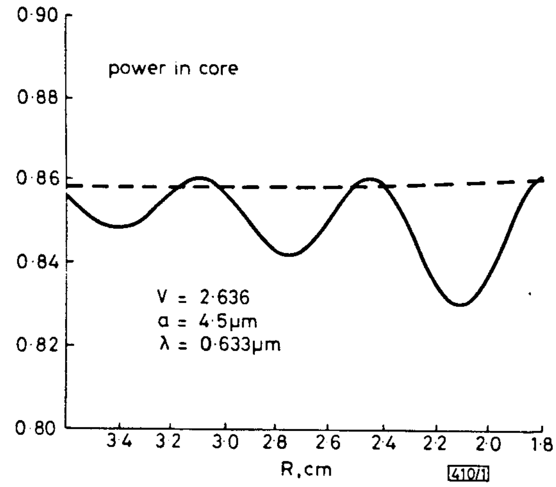

Fig. 1 *Power transmitted within core as function of radius of curvature*

The solid curve shows the power after one complete loop at the radius in question. The broken curve gives the output after a fixed length of fibre (see text)

Appendix: Loss calculation using coupled-mode theory: The coupled-mode theory is an extremely useful and simple one for describing the behaviour of fields in curved small-V fibres. However, a limitation of the theory should be pointed out.

In a previous paper,[3] the method was used, not only to calculate the field distribution, but also, by using the inherent attenuation coefficient of a leaky mode, to calculate the radiation loss from the fibre. While the results presented in that paper were correct, subsequent analysis has shown that the value of the loss calculated in this way is extremely sensitive to the imaginary part of the eigenvalue for the leaky mode and to the outer radius chosen to determine the guided part of the leaky mode's power.

Thus, while the theory provides the only explanation for the experimental observations of both nonuniform radiation and intensity modulation due to changes in radius, it is not to be recommended for the calculation of bending loss. For this purpose, the modified uniform attenuation coefficient calculated by Marcuse,[6] combined with the mode conversion loss found by Miyagi and Yip,[7] is probably the most appropriate method at the moment. The latter will, however, overestimate the loss in short-range applications, since it assumes that all power coupled to the radiation field is instantaneously lost.

Acknowledgment: This work was supported by the UK Science Research Council.

R. A. SAMMUT

Department of Electronics
University of Southampton
Southampton SO9 5NH, England

29th July 1977

References

1 NELSON, D. F., KLEINMAN, D. A., and WECHT, K. W.: 'Vibration-induced modulation of fibreguide transmission'. Paper TuE7, Technical Digest of the OSA topical meeting on optical fiber transmission, Williamsburg, 1977

2 GAMBLING, W. A., PAYNE, D. N., and MATSUMURA, H.: 'Radiation from curved single-mode fibres', *Electron. Lett.*, 1976, **12**, pp. 567–569

3 SAMMUT, R. A.: 'Discrete radiation from curved single-mode fibres', *ibid.*, 1977, **13**, pp. 418–419; notes that References 6 and 7 in this letter should be interchanged

4 GAMBLING, W. A., PAYNE, D. N., and MATSUMURA, H.: 'Propagation in curved multimode cladded fibres'. AGARD symposium on electromagnetic wave propagation involving irregular surfaces and inhomogeneous media, The Hague, 1974

5 SNYDER, A. W.: 'Asymptotic expressions for eigenfunctions and eigenvalues of a dielectric or optical waveguide', *IEEE Trans.*, 1969, **MTT-17**, pp. 1130–1138

6 MARCUSE, D.: 'Field deformation and loss caused by curvature of optical fibres', *J. Opt. Soc. Am.*, 1976, **66**, pp. 311–320

7 MIYAGI, M., and YIP, G. L.: 'Mode conversion and radiation losses in a step-index optical fibre due to bending', *Opt. Quant. Electron.*, 1977, **9**, pp. 51–60

SUBJECTIVE QUALITY OF TELEVISION PICTURES IMPAIRED BY LINEAR WAVEFORM DISTORTION

Indexing terms: Television interference, Television systems

Existing subjective results relating to the effect of undistorted echo in television have been applied to determine the psychophysical relationship between subjective picture quality and the objective measure of linear waveform distortion, termed the 'K-rating'. Values of the coefficients G and K_M are given, which, starting with a known value of K, permit estimates to be made of mean picture quality and of subjective impairment in summable (imp) units.

Interest in the use of a scale of summable subjective impairment units, along the lines originally suggested by Lewis,[1] is continuously increasing for studies of the performance of television transmission systems.[2,3] The relationship between the summable subjective impairment, or I, scale and objective impairment magnitude has been established for many types of impairment,[4] but a generally accepted relationship has so far been lacking for the general case of linear waveform distortion.

So far as objective measurement of waveform distortion is concerned, there now appears to be a fairly general acceptance of the principle of measuring the response to a sine-squared pulse-and-bar test waveform.[5] In a number of countries, including the United Kingdom, a unified method of expressing results, popularly termed 'K-rating', has been successfully employed for many years. Although the rating factor K, as defined hitherto, is known to represent a standard that is not completely constant in terms of subjective quality, any satisfactory revision of the K-rating method must await further exploration of its characteristics. Fuller discussion is to be found elsewhere.[6] Meanwhile, K, as defined, for example,[7] in CCIR Recommendation 451, provides the most satisfactory scale on which to express linear waveform distortion in objective terms.

Standardisation of the experimental arrangements is an essential requirement before any subjective study of the effect of television impairment can be claimed to yield a reliable result. The CCIR has recommended some of the necessary conditions,[8] although the list is not yet complete. A relationship between I and K, based on the results of standardised subjective experiments for long-delayed echo,[9] was first proposed by Wolf.[10] He used a 2 µs positive echo as a 'transfer standard'. However, this is unnecessarily stringent because,