References

1 OXLEY, C. H., HOWARD, A. M., and PURCELL, J. J.: 'Design and performance of TRAPATT devices, oscillators and amplifiers', *Solid-State & Electron. Dev.*, 1976, **1**, (1), pp. 24–30

2 HOWARD, A. M., GORDON, S. M. R., and PURCELL, J. J.: 'Design and fabrication of X-band silicon TRAPATT diodes'. IEE Colloquium on trapatt diodes and circuits, 4th Feb., 1977, Digest 1977/6

3 GRACE, M. I.: 'Broadband high-efficiency-mode amplifiers at S-band'. Proceedings of European Microwave Conference, 1971

4 EVANS, W. J.: 'Circuits for high-efficiency avalanche diode oscillators', *IEEE Trans.*, 1969, **MTT-17**, pp. 1060–1067

5 ROSEN, A., REYNOLDS, J. E., LIU, S. L., and THERIAULT, G. E.: 'Wideband class-C TRAPATT amplifiers', *RCA Rev.*, 1972, **33**, pp. 729–736

DISCRETE RADIATION FROM CURVED SINGLE-MODE FIBRES

Indexing terms: Optical fibres, Optical-waveguide theory

The energy loss and field distribution in a curved single-mode fibre are calculated in terms of the coupling between the fundamental (LP_{01}) mode and the radiation field. In agreement with recent experimental results, it is found that, initially, light travels in a zigzag path around the bend, and radiation is emitted in discrete beams.

Introduction: Theoretical treatments of bending loss from optical fibres have hitherto assumed a uniform energy loss and constant (or no) field deformation around the bend.[1,2] By contrast, experiments using single-mode fibres[3,4] suggest that there is a transition region at the beginning of the bend where radiation is emitted in the form of discrete beams and where the field distribution resembles that of a Gaussian beam travelling in a zigzag down the fibre.[5]

In this letter, an analysis of bending loss in single-mode fibres is presented which accounts for the experimental observations very simply. Energy loss is determined by calculating the coupling induced by the bend between the fundamental (LP_{01}) mode and the radiation field (approximated by the lowest-order, or LP_{11}, leaky mode). The field deformation is then accounted for directly by power transfer between the two modes, and radiation peaks coincide with points around the bend where the rate of power transfer from LP_{01} to LP_{11} is greatest.

Theoretical approach: By combining the discrete, bound modes of a straight dielectric waveguide with the continuous spectrum of radiation modes, one can form a complete set of functions in terms of which an arbitrary electromagnetic field can be expanded. But, in general, i.e. in any system other than the perfect, straight guide, the modes in such an expansion are not power orthogonal.[6] Energy is coupled among the modes as they propagate.

The modal expansion ceases to be worth while if extensive mode coupling occurs, because the amplitudes of the individual modes in the expansion are then difficult to calculate. However, in problems where only weak coupling occurs, analysis in terms of the modes of the straight fibre is very useful, as it enables one to replace what may be a very complicated structure by a much simpler one with a superimposed coupling mechanism.[7]

Thus, in this letter, bending loss is studied not by examining the modes of the curved fibre itself, but by considering the bend as simply a mechanism for coupling together modes of the straight fibre. Losses occur naturally as a result of coupling to the radiation field. Moreover, the calculation is greatly simplified by using a leaky-mode approximation to the radiation field.[6]

Consider, then, a waveguide with a core of radius a and permittivity ε_1 immersed in an infinite cladding of permittivity ε_2. The radius of curvature of the fibre is R and electromagnetic fields in the waveguide are described by using cylindrical co-ordinates (r, θ, z).

Neglecting a small, saddle-point contribution to the radiation field,[8] the electromagnetic field of the curved waveguide can be written as a sum of discrete bound and leaky modes of the straight guide.[6] Substituting this sum into Maxwell's equations for the curved waveguide[5] and using the generalised orthogonality conditions for leaky modes[6] leads to the system of coupled equations

$$\left.\begin{aligned} \frac{\partial E_p}{\partial z} &= -\sum_q C_{pq} H_q \\ \frac{\partial H_p}{\partial z} &= -\sum_q D_{pq} E_q \end{aligned}\right\} \qquad \ldots \ldots \quad (1)$$

where

$$\left.\begin{aligned} C_{pq} &= i\omega\varepsilon_1 \int_{S_\infty} \xi \boldsymbol{e}_p . \boldsymbol{e}_q \, dA - \frac{\varepsilon_1}{i\omega\mu} \int_{S_\infty} \nabla_t \left(\frac{\xi}{\varepsilon} \nabla_t . \boldsymbol{e}_q \right) . \boldsymbol{e}_p \, dA \\ D_{pq} &= i\omega \frac{\mu}{\varepsilon_1} \int_{S_\infty} \varepsilon \xi \boldsymbol{h}_p . \boldsymbol{h}_q \, dA - \frac{1}{i\omega\varepsilon_1} \int_{S_\infty} \nabla_t (\xi \nabla_t . \boldsymbol{h}_q) . \boldsymbol{h}_p \, dA \end{aligned}\right\} \quad \ldots \quad (2)$$

E_p and H_p are the amplitudes of the electric and magnetic fields of mode p, μ is the permeability, ω is the angular frequency of the light, S_∞ is a surface defined in Reference 6 and $\xi = 1 + (r/R)\cos\theta$.

The function ξ enters Maxwell's equations, because, in the curved fibre, an incremental length is given by

$$(ds)^2 = (dr)^2 + (rd\theta)^2 + (\xi dz)^2.$$

Its effect is to couple modes with azimuthal order $\nu = l$ to modes with $\nu = l \pm 1$, the coupling coefficients being given by eqn. 2. The system of equations (1) is, in general, difficult to solve. However, Gambling *et al.*[5] have shown that for weakly guiding fibre, if backward-travelling waves are neglected and the field incident at $z = 0$ is assumed to be an LP_{01} mode, a good approximation is obtained by considering only coupling to the LP_{11} mode. This remains true when the LP_{11} mode becomes 'leaky'. The analytic continuation of solutions given in Reference 5 can therefore be used to calculate the field distribution in the curved, single-mode fibre, although, in this case, the coupling is not between two bound modes, but between a bound and a leaky mode.

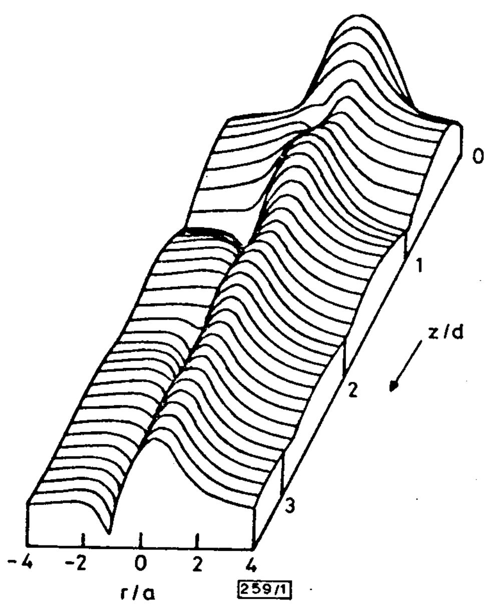

Fig. 1 *Field intensity in plane of curvature (plotted logarithmically)*

The fibre axis lies along the line $r = 0$, and negative values of r correspond to points nearer the centre of curvature. In this example, $v = 2{\cdot}35$, $R/a = 1000$, $\delta = 1 - \varepsilon_2/\varepsilon_1 = 0{\cdot}0015$ and $d/a \simeq 113$

Results and discussion: Based on the above analysis, Fig. 1 shows how the field of the incident LP_{01} mode is modified as it enters the bend. Initially, the intensity distribution is that of an LP_{01} mode. As it travels around the bend, energy which was originally carried entirely in the LP_{01} mode is then coupled back and forth between the LP_{01} and LP_{11}

modes, resulting in an oscillation in the position of peak intensity. The period of oscillation, which follows from the analysis of eqns. 1 and 2, is given by

$$d = \pi/\mathrm{Im}[(C_{11}-C_{22})^2+8C_{12}^2]^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where Im denotes the imaginary part, modes 1 and 2 are LP_{01} and LP_{11}, respectively, and C_{11}, C_{12}, C_{22} are defined by eqn. 2.

However, since the LP_{11} mode is leaky, the oscillation is damped and, as Fig. 1 shows, an equilibrium situation is soon reached with the steady-state field distribution being offset at a fixed distance from the fibre axis (in agreement with References 1 and 2).

In the specific case illustrated here, $d \simeq 113a$, corresponding to an angular distance around the bend of approximately 6·4°, and equilibrium is reached after a distance of 500 to 700a. For comparison, measurements made on a fibre with similar characteristics and surrounded by index-matching fluid[3] show that six or seven discrete beams of radiation are visible at the beginning of the bend and that the angular separation between successive beams is approximately 5·2°.

The agreement between these results supports the argument that it is the oscillation of the energy peak which leads to emission of the apparently discrete beams. This is confirmed by calculation of the energy loss.

The mechanism for loss is the coupling of energy from the LP_{01} mode to the leaky LP_{11} mode and its subsequent radiation. The power lost after a distance z is calculated by subtracting from the incident power both the total power in the LP_{01} mode and the power in the guided portion[7] of the LP_{11} mode (in this case, power flowing within a radius of approximately 25a).

Fig. 2 shows the radiation loss P calculated in this way, using the same parameters as in Fig. 1. Two significant features can be seen. The first is that, although the LP_{11} mode has a constant attenuation coefficient, radiation from the curved fibre is, initially, not uniform. Since power loss varies with the amplitude of the LP_{11} mode, the rate of loss oscillates with the field distribution until equilibrium is established. This is more clearly shown in the inset curve, where the rate of power loss $\Delta P/P$ is plotted as a function of distance around the bend. Peaks in this curve coincide with points of maximum energy transfer from LP_{01} to LP_{11} and correspond to the discrete beams of radiation found experimentally.[3]

The second point to note is that, while the uniform, equilibrium loss predicted by this method agrees closely with that calculated by Marcuse,[1] the loss in the transient region is considerably higher than the equilibrium value. In the example considered, approximately 34% of the incident power is lost within an angular distance of 1° from the beginning of the bend, whereas, in the equilibrium region, the loss/degree has fallen to a uniform value of approximately 24%.

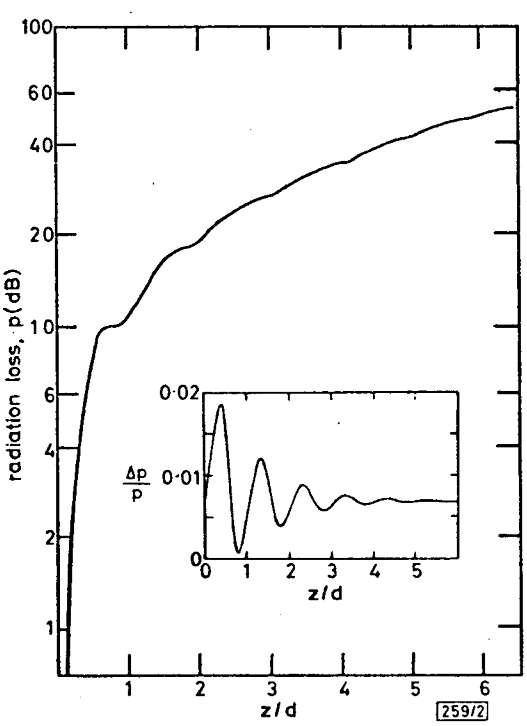

Fig. 2 *Radiation loss P and (inset) its rate of change as function of distance around bend*

Peaks in the inset curve correspond to discrete beams of radiation emitted from the fibre. r, R, δ and d are as in Fig. 1

Acknowledgments: I am grateful to Dr. H. Matsumura for access to his experimental results, and, together with D. N. Payne and Prof. W. A. Gambling, for many helpful discussions. This work was supported by CSIRO (Australia) and the UK Science Research Council.

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References

1 MARCUSE, D.: 'Field deformation and loss caused by curvature of optical fibers', *J. Opt. Soc. Am.*, 1976, **66**, pp. 311–320; and references cited therein
2 MIYAGI, M., and YIP, G. L.: 'Field deformation and polarization change in a step-index optical fibre due to bending', *Opt. Quantum Electron.*, 1976, **8**, pp. 335–341
3 GAMBLING, W. A., PAYNE, D. N., and MATSUMURA, H.: 'Radiation from curved single-mode fibres', *Electron. Lett.*, 1976, **12**, pp. 567–569
4 NEUMANN, E-G., and RUDOLPH, H-D.: 'Radiation from bends in dielectric rod transmission lines', *IEEE Trans.*, 1975, **MTT–23**, pp. 142–149
5 GAMBLING, W. A., PAYNE, D. N., and MATSUMURA, H.: 'Propagation in curved multimode cladded fibres'. AGARD symposium on electromagnetic wave propagation involving irregular surfaces and inhomogeneous media, The Hague, 1974
6 ABRAM, R. A., and REES, G. J.: 'Mode conversion in an imperfect waveguide', *J. Phys. A.*, 1973, **6**, pp. 1693–1708; and references cited therein
7 SAMMUT, R. A., and SNYDER, A. W.: 'Leaky modes on a dielectric waveguide: orthogonality and excitation', *Appl. Opt.*, 1976, **15**, pp. 1040–1044, 2953
8 SAMMUT, R. A.: 'A comparison of leaky-mode and leaky-ray analyses of circular optical fibers', *J. Opt. Soc. Am.*, 1976, **66**, pp. 370–371
9 MIYAGI, M., and YIP, G. L.: 'Mode conversion and radiation losses in a step-index optical fiber due to bending', *Opt. Quantum Electron.*, 1977, **9**, pp. 51–60

AVALANCHE-NOISE DEPENDENCE ON AVALANCHE-PHOTODIODE STRUCTURES

Indexing terms: Avalanche diodes, Electron-device noise, Photodiodes

Excess avalanche-photodiode noise is calculated by considering photogenerated diffusion currents and photogenerated carrier distributions. Computed results reveal the effects of junction depth, depletion-layer thickness and incident light direction on avalanche noise for various optical-absorption coefficients.

It is known that excess avalanche-photodiode (a.p.d.) noise depends on the incident light wavelength.[1,2] This effect occurs when the composition of electrons and holes initiating avalanche multiplication changes with the light wavelength variation[3] and ionisation rates for electrons and holes are not equal, as intimated by McIntyre's theory.[4] The initial current composition is affected by the photogenerated diffusion current flowing both from a surface layer and a substrate, as well as by the distribution of photocarrier generation rate in a depletion layer.

This letter reports exact avalanche-noise calculations, without neglecting the photogenerated diffusion-current contribution, from which the effects of junction depth and incident light direction on avalanche noise can be elucidated. The initial carriers undergoing the avalanche process are assumed to be composed of diffused carriers, photocarriers generated within an avalanche region and drift carriers, photogenerated in a depletion layer and injecting into an avalanche region. Photocarriers are assumed to be generated with an exponential function. The a.p.d. model used in this analysis is an n^+-p-π-p^+ structure, which is an optimum constitution to minimise avalanche noise in Si.[5] A high-field