EFFECT OF DIP IN THE REFRACTIVE INDEX ON THE CUT-OFF FREQUENCY OF A SINGLE-MODE FIBRE

Indexing terms: Optical fibres, Refractive index

The normalised frequency for single-mode cut-off in an optical fibre has been calculated as a function of the dip in the refractive index at the centre of the core. For a particular class of profiles, dip widths up to 40% have a negligible effect on the propagation characteristics.

Introduction: A particular virtue of the homogeneous chemical vapour deposition method is that it greatly simplifies the fabrication of single-mode optical fibres.[1] However, it is difficult to produce a perfectly stepped refractive-index distribution because of diffusion at the core–cladding boundary and preferential evaporation of the more volatile component during the preform collapsing stage. Core–cladding diffusion produces a rounding of the profile at the outer edges, and the resulting effect on the cut-off conditions has been discussed elsewhere.[2] Evaporation usually causes a depression of the refractive index at the centre of the core, and the magnitude of the change in the frequency range for single-mode operation is considered here, by calculating the cut-off frequency of the second mode.

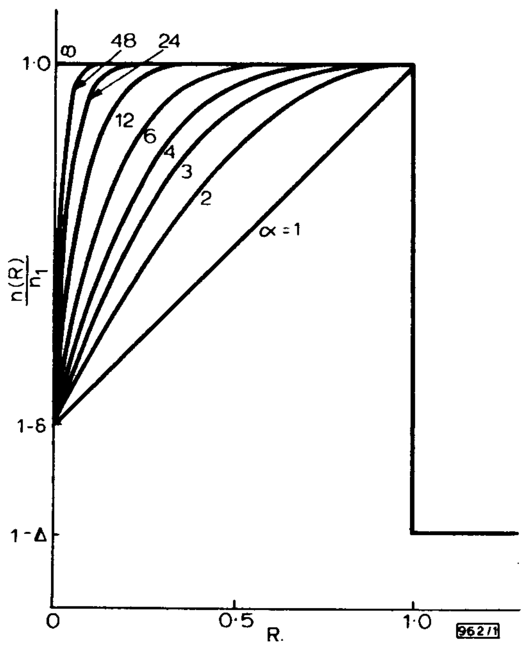

Fig. 1 *Refractive-index profile as function of normalised radius for various values of α*

Theory: The dip is represented by a radial r variation of relative permittivity in a fibre of core diameter $2a$ given by

$$\varepsilon(R) = \begin{cases} \varepsilon_1\{1-2\delta(1-R)^\alpha\} & 0 \leqslant R \leqslant 1 \\ \varepsilon_1(1-2\Delta) = n_2^2 & R \geqslant 1 \end{cases} \quad (1)$$

where $R = r/a$, $\varepsilon_1 = n_1^2$ is the relative permittivity at the edge of the core, $\Delta = (\varepsilon_1 - \varepsilon_2)/2\varepsilon_1$ is a measure of the relative difference in relative permittivity between the edge of the core and the cladding and equals the relative refractive-index difference when $\Delta \ll 1$, δ is the equivalent relative difference between the edge and the centre of the core and denotes the dip height and α is a parameter between 1 and ∞ which defines the dip width. The resulting profile distributions are shown in Fig. 1.

The single-mode régime is terminated by the TE_{01} mode cut-off frequency V_c, and, following the method of Reference 2, the following conditions may be deduced:

$$\frac{d^2 e_\theta}{dR^2} + \frac{1}{R}\frac{de_\theta}{dR} + \{V_c^2[1-\gamma(1-R)^\alpha] - R^{-2}\}e_\theta = 0 \quad (2)$$

and, at $R = 1$,

$$\frac{de_\theta}{dR} + e_\theta = 0 \quad (3)$$

where $e_\theta(R)$ is the R-dependent factor in the azimuthal θ component of the electric-field vector and $\gamma = \delta/\Delta$ is the relative depth of the dip in refractive index at the core centre.

To solve eqn. 2, a series-expansion method[2] is used. Thus we write

$$e_\theta(R) = \sum_{n=0}^{\infty} a_n R^{n+1} \qquad a_0 \neq 0 \quad (4)$$

and obtain the coefficients a_n by substituting eqn. 4 into eqn. 2 and equating terms in like powers of R to give the recurrence relations

$$\left.\begin{aligned}
&2.4a_2 + V_c^2(1-\gamma)a_0 = 0\\
&3.5a_3 + V_c^2(1-\gamma)a_1 + V_c^2\gamma\sum_{m=1}^{1}(-1)^{m+1}\binom{\alpha}{m}a_{3-m-2} = 0\\
&4.6a_4 + V_c^2(1-\gamma)a_2 + V_c^2\gamma\sum_{m=1}^{2}(-1)^{m+1}\binom{\alpha}{m}a_{4-m-2} = 0\\
&\cdots\cdots\cdots\\
&(\alpha+2)(\alpha+4)a_{\alpha+2} + V_c^2(1-\gamma)a_\alpha + V_c^2\gamma\sum_{m=1}^{\alpha}(-1)^{m+1}\binom{\alpha}{m}a_{\alpha-m} = 0\\
&(\alpha+3)(\alpha+5)a_{\alpha+3} + V_c^2(1-\gamma)a_{\alpha+1} + V_c^2\gamma\sum_{m=1}^{\alpha}(-1)^{m+1}\binom{\alpha}{m}a_{\alpha-m+1} = 0
\end{aligned}\right\} \quad (5)$$

where $a_1 = 0$ and

$$\binom{\alpha}{m} = \frac{\alpha!}{m!(\alpha-m)!} \quad (6)$$

From the boundary condition of eqn. 3 we obtain

$$\sum_{n=0}^{\infty}(n+2)a_n = 0 \quad (7)$$

Finally, substitution of eqns. 5 and 6 into eqn. 7 results in an algebraic equation for V_c which may be solved by the Newton–Raphson method.

Results: For $\alpha = \infty$, corresponding to a step-index fibre as in Fig. 1, eqn. 7 can be rewritten as the Bessel function of order zero, i.e. $J_0(V_c) = 0$, so that $V_c = 2{\cdot}4$, which is the value for a perfect step-index fibre. The variation of V_c for other values of α is shown in Fig. 2 for a range of dip heights γ. It can be seen that, with departure from a stepped profile, i.e. increased width of dip (smaller α) and increased depth (larger γ), the cut-off frequency V_c increases. Fig. 2 indicates that V_c is effectively unchanged from that for a step-index fibre

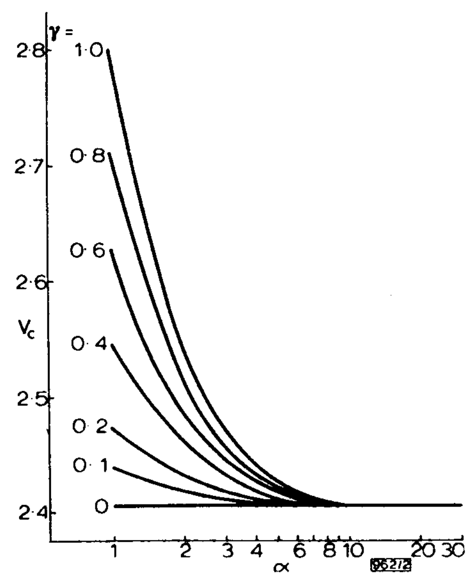

Fig. 2 *Variation of cut-off frequency as function of α for various values of γ = δ/Δ*

for $\alpha \geq 10$, and from Fig. 1 this corresponds to dip widths up to 40%, even at $\gamma = 1$, where the axial refractive index has fallen to that of the cladding, equivalent to the loss of all the dopants from the core centre. If $\gamma < 1$, there is an even greater tolerance on dip width. Thus for a dip in relative refractive-index difference of 40% a dip width of 85% ($\alpha = 3$ in Fig. 1) produces a change in V_c of only 1%. It appears, therefore, that relatively large depressions in refractive index at the centre of the core can be tolerated before V_c changes appreciably. Since the TE_{01} mode has a zero in intensity at $R = 0$, this result is perhaps not unexpected. Calculations of normalised propagation constant have been carried out[3] for a stepped-ring-profile fibre in which the 'dip' is of uniform depth Δ, i.e. $\gamma = 1$, and of width 25%. In this case also the change in V_c is negligible.

Recently, we reported[4] a method for the direct determination of V, core diameter and index difference in single-mode fibres. The fibres studied had a dip in the core centre, the depth of which was difficult to measure, but from scanning electron micrographs of an etched fibre end the width was estimated at not more than 40%. From the results given here the dip in these fibres would not be expected to change the cut-off behaviour noticeably from that of step-index fibres.

Conclusions: Relatively large changes in the refractive index about the core centre of a single-mode fibre can be tolerated before an appreciable change in V_c occurs.

Acknowledgments: Grateful acknowledgment is made to the Pirelli General Cable Company for the award of research fellowships, and to the UK Science Research Council for supporting the work.

W. A. GAMBLING
D. N. PAYNE
H. MATSUMURA

15th February 1977

Department of Electronics
University of Southampton
Southampton SO9 5NH, England

References

1 GAMBLING, W. A., PAYNE, D. N., HAMMOND, C. R., and NORMAN, S. R.: 'Optical fibres based on phosphosilicate glass', *Proc. IEE* 1976, **123**, (6), pp. 570–576
2 MATSUMURA, H., *et al.*: 'Cut-off normalized frequency in radially inhomogeneous single-mode fibre' (to be published)
3 STOLEN, R. H.: 'Modes in fibre optical waveguides with ring index profile', *Appl. Opt.*, 1975, **14**, pp. 1533–1537
4 GAMBLING, W. A., PAYNE, D. N., and MATSUMURA, H.: 'Determination of core diameter and refractive-index difference of single-mode fibres by observation of the far-field pattern', *MOA*, 1976, **1**, pp. 13–17