# MEASUREMENT OF NORMALISED FREQUENCY IN SINGLE-MODE OPTICAL FIBRES

*Indexing term: Optical waveguides*

It is shown that methods of measuring the normalised frequency in single-mode fibres through observation of mode cut-off wavelengths give erroneous results. This is because the various loss mechanisms which exist in a practical fibre can have an appreciable effect near cut-off.

*Introduction:* Recently, we described[1] a method of determining unambiguously, from simple measurements on the far-field radiation pattern, not only the normalised frequency of a single-mode fibre, but also the core diameter $2a$ and refractive-index difference $\Delta$ between core and cladding. At the same time, we noted that the output radiation field consisted substantially of the lowest-order mode, even when operating above the cut-off frequency $V_c$ of the $LP_{11}$ mode. This implies that the normalised frequency $V$ cannot be found by using a tunable source and noting the wavelength at which the higher-order mode disappears.

Subsequently, Katsuyama *et al.*[2] described a method of estimating $V$ (but not $a$ and $\Delta$) from the loss due to bending of a curved fibre. They suggest that the measured bending loss should fall sharply as the wavelength is increased through the cut-off value for a mode. For example, $V = 2{\cdot}4$ is the cut-off point for the $LP_{11}$ mode and should therefore correspond to a sharp change in bend loss. However, the experiments of Katsuyama *et al.* (as well as Fig. 2*b* of this letter) fail to show this sudden change.

We can explain the above anomalies and show below that all techniques which involve the observation of (any) mode cut-off are prone to error. .

*Attenuation near cut-off due to imperfections:* In a perfect fibre having no absorption or scattering, a mode propagates with zero loss at all frequencies down to cut-off. However, in practice, as well as absorption and scattering, there are additional attenuation mechanisms, such as microbending and cladding imperfections. The effects of these are greatly accentuated as a mode approaches its cut-off, because of the decrease in power localised within the core and the increased tendency of such a weakly guided mode to radiate at minor deviations from perfect straightness.

As an example, consider conditions just above cut-off for the $LP_{11}$ mode in a single-mode fibre with microbends. We assume the deviations are such that the loss of the $LP_{01}$ mode can be neglected; it does not, in any case, change rapidly near $V = 2{\cdot}4$. On the other hand, as $V$ approaches

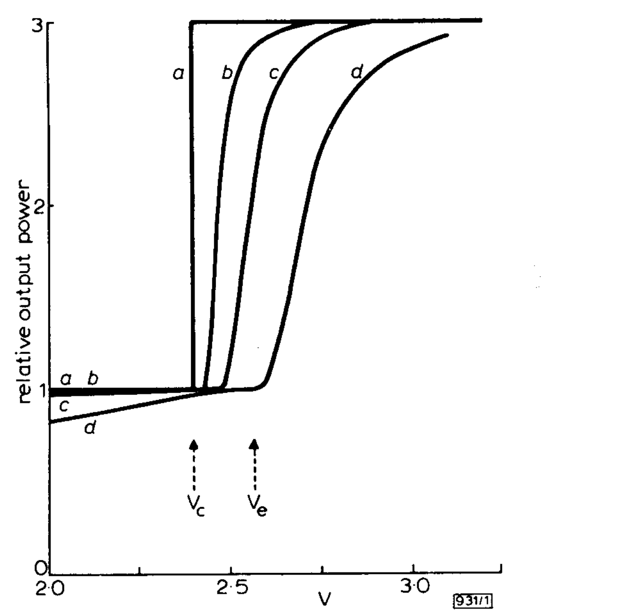


**Fig. 1** *Variation of relative output power with V, assuming equal excitation of the propagating modes at the input for a numerical aperture of 0·1 and $n_2$ = 1·4585*

Curves (*a*) to (*d*) are for the values of normalised coupling factor $C_0 a^2 L = 0$, $10^{-10}$, $10^{-9}$, $10^{-8}$, respectively

2·4, the $LP_{11}$ mode couples with the radiation modes and experiences a loss $\alpha_0$ per unit length:[3]

$$\alpha_0 = \sum_{\nu} \int_{-n_2 k}^{n_2 k} |K_{1\nu}|^2 |F(\beta_1 - \beta)|^2 |\beta| d\beta$$

$$= \sum_{\nu} \int_{-n_2 k}^{n_2 k} |K_{1\nu}|^2 \frac{C_0 |\beta|}{(\beta_1 - \beta)^{4+2p}} d\beta \quad (1)$$

where

$\beta_1, K_{1\nu}(\beta)$ = propagation constant, coupling coefficient with the radiation mode, respectively, for the $LP_{11}$ mode

$\beta, \nu$ = propagation constant, azimuthal mode number, respectively, for the radiation mode

$n_1, n_2$ = refractive indices of core, cladding

$k = 2\pi/\lambda$

and in the power spectrum for fibre distortion (microbending) values of $p = 0$, 1 or 2 may be used.[4] The summation is taken only over $\nu = 0, 2$ because the strongest coupling occurs between adjacent modes.

If all modes above cut-off are equally excited at the fibre input (as with an incoherent source), because of the double degeneracy of the $LP_{11}$ compared with the $LP_{01}$ mode, the relative output power $P$ for length $L$ is

$$P = 1 + 2\exp(-\alpha_0 L) \quad \text{for} \quad V > 2{\cdot}4 \quad . \quad . \quad . \quad (2)$$

Using eqns. 1 and 2, $P$ has been evaluated for typical fibre parameters and normalised coupling[3] coefficients and with $p = 1$. Fig. 1 shows clearly that the effect of microbending is to move to a higher frequency the point at which the output power increases due to transmission of the $LP_{11}$ mode. Thus the effective cut-off frequency $V_e$ may be appreciably higher than the theoretical value $V_c$; e.g. for a coupling coefficient $C_0 a^2 L = 10^{-8}$ the output radiation pattern is indistinguishable from that of a single mode up to $V = 2{\cdot}6$. Because of the lower excitation efficiency of the $LP_{11}$ mode, the effect is accentuated when a laser source is used, and we have observed experimentally values up to $V_e = 2{\cdot}8$ for $L = 1$ m. $V_e$ is increased with longer lengths and for $p > 1$. Thus the observation of mode cut-off is an inaccurate method of determining $V$.

*Attenuation due to bending:* The method of determining $V$, whereby the loss of a curved fibre is measured, inevitably determines the ratio of the attenuation in a curved fibre with microbends to that in the straight fibre with microbends. The effect of curvature can be taken into account[5] by expressing the loss $\alpha_1$ of the $LP_{11}$ mode at a bend radius $R$ as

$$\alpha_1 = \frac{\pi^{0\cdot5} U^2 \exp[(-2/3)(W^3 \beta_1^2)(R/a)]}{a W^{1\cdot5} V^2 (R/a)^{0\cdot5} K_0(W) K_1(W)} \quad . \quad . \quad (3)$$

where the symbols have their usual meanings. Thus the ratio $\alpha$ of the losses in the curved and straight configurations in the presence of microbending is

$$\alpha = -10\log_{10}\left[\frac{1 + 2\exp(-\alpha_0 L)\exp(-\alpha_1 L)}{1 + 2\exp(-\alpha_0 L)}\right] \quad (4)$$

Fig. 2*a* shows $\alpha$ as a function of $V$ for various bend radii and a normalised coupling coefficient $C_0 a^2 L = 10^{-8}$. It is clear that the attenuation does not exhibit a sharp drop at $V = 2{\cdot}4$, but has a gradual slope. The apparent cut-off frequency $V_e$ inferred from such an experiment differs appreciably from the true value $V_c$, the magnitude of the error depending only on the degree of microbending present and not on the bend loss introduced. This is shown by the curves for increasing $R/a$, which differ in height and width depending on the bend radius, but have a common low-frequency edge.

The results of an experimental study of the effects of curvature and microbends in Fig. 2*b* show the relative attenuations

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of a 1·2 m length of phosphosilicate single-mode fibre with bends of radius 1·1, 2 and 3 cm. The shape of the curves is similar to that reported in Reference 2 and in excellent agreement with the theory and Fig. 2*a*. They confirm that

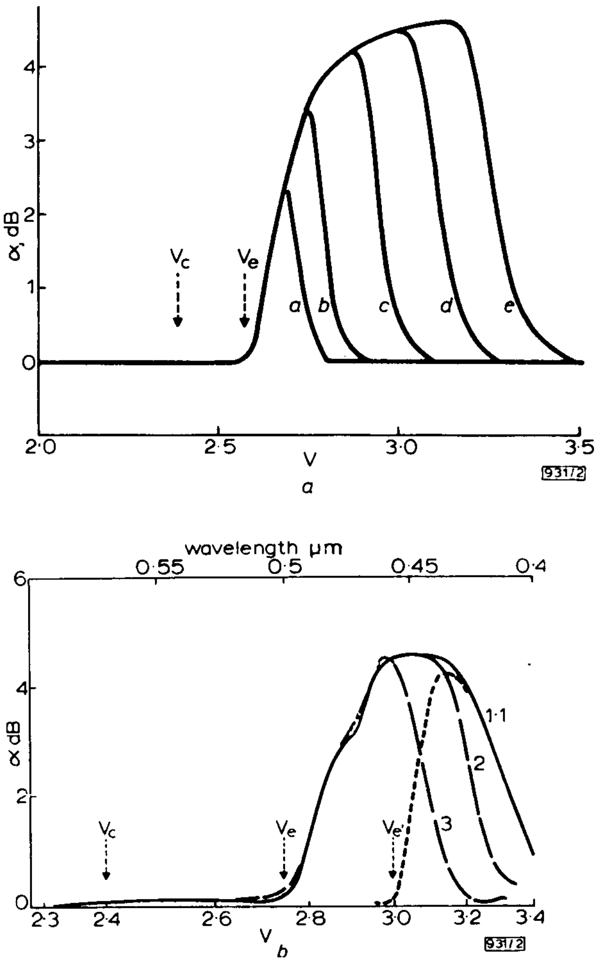


**Fig. 2**

*a* Calculated relative loss α as a function of *V*. Curves (*a*) to (*e*) refer to normalised curvatures $R/a$ of $3{\cdot}5\times10^4$, $2{\cdot}5\times10^4$, $1{\cdot}5\times10^4$, $1{\cdot}0\times10^4$ and $0{\cdot}75\times10^4$, respectively. The normalised length and coupling coefficient are $L/a = 10^6$ and $C_0 a^2 L = 10^{-8}$

*b* Measured relative loss α as a function of *V* in a phosphosilicate fibre of $a = 2$ µm for the bend radii in centimetres shown on the curves and corresponding to $R/a$ values of $1{\cdot}5\times10^4$, $1{\cdot}0\times10^4$ and $0{\cdot}55\times10^4$. The broken curve was obtained for a bend radius of 1·1 cm and shows clearly the shift in effective cut-off frequency caused by additional microbends

the curvature affects only the high-frequency edge. The single-mode ($V = 2{\cdot}4$) cut-off wavelength was found independently[1] to be 0·57 µm and the curves show clearly that the residual microbending was sufficient to produce an apparent cut-off wavelength of 0·5 µm. As a further demonstration of the effect of microbending, additional microbending loss was introduced deliberately by bending the fibre slightly. The same measurement of relative attenuation in the straight and curved fibre was repeated and the broken curve shows that the apparent cut-off wavelength has been moved further to 0·46 µm, again confirming the theoretical predictions of Fig. 1.

*Conclusions:* We have shown theoretically, and confirmed experimentally, that both microbends and curvature must be considered in any study of mode propagation near cut-off. Even in an apparently straight fibre, microbends can produce a high loss above cut-off, thus raising appreciably the effective cut-off frequency. Thus any method of determining *V* from the onset of higher-order modes in the radiation pattern is subject to error.

The same phenomenon affects the measurement of *V* from the relative attenuation of a fibre in straight and curved configurations and can be used to explain the lack of a steep edge to the attenuation curve as expected by Katsuyama[2] *et al.* Indeed, all methods of determining *V* from mode cut-off could be similarly affected. Finally, it will be obvious that, although only microbending has been considered here, other forms of fibre irregularity and inhomogeneity, such as internal stress, birefringence and boundary variations, can have the same effect. All can cause an abnormally high power loss at frequencies just above cut-off.

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