RADIATION FROM CURVED SINGLE-MODE FIBRES

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A study of propagation in curved single-mode fibres shows that the transmission loss increases sharply below a critical bend radius. However the radiation does not leak away uniformly with distance but in a series of discrete well-defined rays.

Introduction: In any application of optical communications, a key element is the transmission medium. Recently, attention has centred on multimode, especially graded-index, glass fibres because they can be used with light-emitting diodes and they present fewer handling problems, such as in launching and jointing, for example, than single-mode fibres. However, the latter can now be fabricated in a relatively simple way[1] by the technique of chemical vapour deposition. Further, it can be deduced from recent theoretical work[2] that the 'microbending' loss can be made small by restricting the core diameter, and, additionally, the potential bandwidth is very large.

We have therefore made a detailed study of propagation in single-mode fibres. In particular, we find that the transmission loss at bends increases rapidly, as expected, as the bend radius is reduced below a critical value. The radius at which this occurs is typically 4 cm for large-core fibres (8 μm diameter, $V = 2$), while our more strongly guiding fibres (4 μm diameter, $V = 2.4$) may be bent to a radius of a few millimetres. As will be reported elsewhere, the agreement between the experimental results and those derived from a theoretical model based on a conformal transformation technique is good. However, in the course of the bend-loss studies, it was observed that the radiation emitted in the transverse direction at a bend was not continuous, but appeared in the form of discrete divergent 'rays'.

Experiment: To observe the 'ray' radiation, several metres of single-mode, or near-single-mode, fibre were coiled around a drum of given diameter, and the HE_{11} mode was excited at the input end by a helium/neon laser operating at 0·633 μm in the TEM_{00} mode. The 'near-single-mode' fibre had a normalised frequency $V = 2{\cdot}78$ and a core diameter of 9·8 μm, but only the HE_{11} mode was present after a few metres because of the launching conditions and any higher-order modes present were only weakly bound, so that they attenuated rapidly. Normally, the radiation emitted from the core at a bend is internally reflected at the outer surface of the cladding and is therefore not easily observed. The 110 μm-diameter fibre was therefore laid on a glass plate and immersed in liquid paraffin, having a refractive index (1·466) slightly higher than that of the cladding. Typical examples of the radiation observed are shown in Fig. 1. The radiated beams have a finite width varying as a function of the radius of curvature. The number of beams per unit length of curved fibre increases as the bend radius increases. The number of rays per complete turn of fibre was measured as the normalised ratio core-diameter/bend-radius was changed from 1·8 to 3×10^{-4} (54 to 33 mm bend radius) and was found to be roughly constant, as predicted by the theory outlined below.

Although the results illustrated in Fig. 1 were obtained with fibres which were capable of propagating more than one mode, the same discrete ray emission was also observed in fibres of $V < 2{\cdot}4$. In addition, smaller-core fibres exhibit the effect at a reduced bend radius.

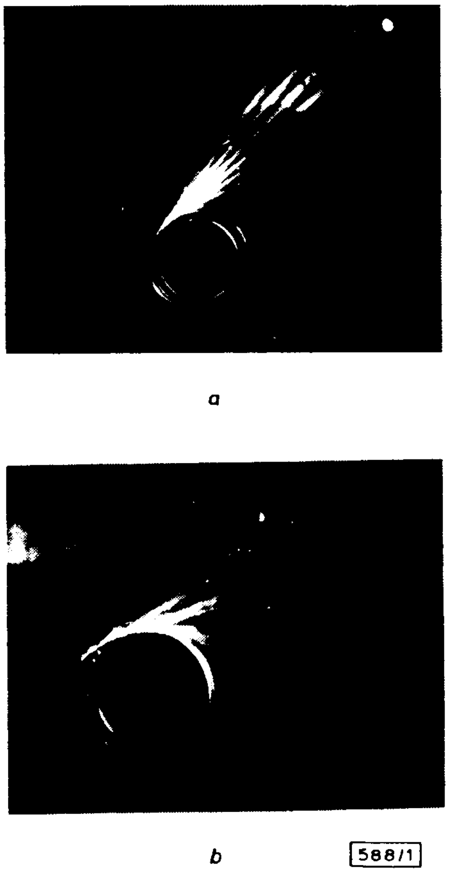

Fig. 1 *Tangential beams emitted by a single-mode fibre*
Bend radii of *a* 12 mm and *b* 16·5 mm

Analysis of leakage from curved single-mode fibre: The analysis is based on a conformal transformation[3] which can be used to map a curved homogeneous cylindrical medium into an inhomogeneous linear one in the transform plane. However, we assume here, for simplicity, a curved slab waveguide of halfwidth d, refractive index n_1 and mean radius of curvature R in the plane of curvature (r, θ). This

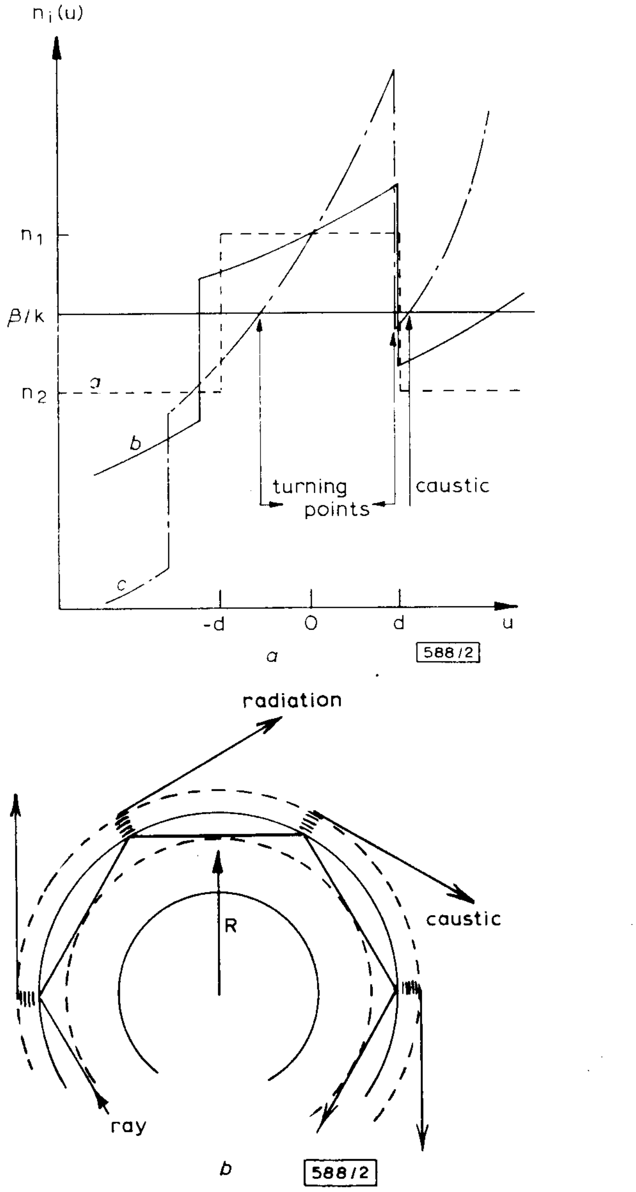

Fig. 2
a Change of refractive index in the transform plane for *a* zero, *b* moderate and *c* large curvatures
b Illustration of the two caustics, 'ray' reflection and tunnelling radiation in a curved fibre

structure is mapped into the (u, v) plane by the transformation

$$\left.\begin{aligned} u &= R \ln (r/R) \\ v &= R\theta \end{aligned}\right\} \qquad (1)$$

in the form of a straight waveguide with the dimensions and refractive-index distribution indicated by curve *b* in Fig. 2*a*. It is clear that the energy distribution in the curved waveguide is shifted towards the outer curved boundary by the increase of refractive index at the right-hand boundary in the transform plane. The effective fibre diameter in the (u, v) plane becomes

$$D = R \ln (R+d)/(R-d) \qquad (2)$$

Both D and the variation of the refractive index in the u direction, $\exp (u/R)$, increase as the curvature increases.

The effect of these changes on the propagation conditions may be deduced from Fig. 2*a*. The broken curve *a* shows the transverse distribution of refractive index in the (u, v) plane for a straight fibre in the (r, θ) plane, i.e. $R = \infty$, so that the width of the guide is the same in both planes, $D = 2d$, and the refractive index is constant. Also shown is the normalised propagation constant of the HE_{11} mode. Strictly speaking, the propagation constant is that of the slab TE_{01} or TM_{01} mode, but calculation shows that the propagation constants of the lowest-order modes in slab and fibre are very similar and the approximation is certainly good enough for our purpose here.

At a moderate radius of curvature, solid curve *b*, the refractive index at $u = d$, increases from n_1 to $n_1(1+d/R)$ in the core and from n_2 to $n_2(1+d/R)$ in the cladding. Similarly, at the inner radius, the refractive index changes from n_1 to $n_1(1-d/R)$ in the core and from n_2 to $n_2(1-d/R)$ in the cladding. As indicated above, there is therefore a narrowing accompanied by a shift in the energy density to the outside of the curve. Further, for $u > d$ the refractive index increases as $n_2 \exp (u/R)$ and exceeds the value β/k at a finite distance from the waveguide. Thus the possibility arises of electromagnetic tunnelling, as discussed by Snyder,[4] which will give rise to bending loss.

When the bend radius is decreased sufficiently, curve *c*, two further effects occur. First, the point can be reached at which $n_2(1+d/R)$ equals or exceeds β/k, so that all guidance ceases and the radiation loss rises to infinity. This, and the tunnelling from a curved fibre, are discussed elsewhere[5] and will not be treated further here. However, the other effect of importance at a small radius of curvature arises when the effective refractive index in the core, $n_1(1-d/R)$, at the inside of the bend falls below β/k. A turning point (caustic) then forms within the core, and, since the inner core boundary no longer has a guiding function, it ceases to have any relevance. The form of the guiding structure therefore changes from that of a single-mode fibre to a single-boundary guide.[6] The HE_{11} mode in the curved fibre may therefore be thought of in terms of the equivalent locus or ray (see, for example, Reference 7) from the straight portion entering an open curved 'whispering gallery' reflecting region of radius R and with an outer caustic at a distance from the core at which the equality

$$\beta/k_2 = n_2 \exp (u/R)$$

is satisfied and through which tunnelling radiation is emitted.

Thus the theory predicts the emission of tangential rays at the points of reflection of the HE_{11} 'ray', as illustrated in Fig. 2*b*. The number of rays per unit length is a function of R, and it can be shown quite simply from the geometric-reflection model that the number of emitted rays per complete turn is almost independent of the normalised inverse bend radius d/R.

As described in the second section, such rays are indeed observed and the number per turn is in approximate agreement with the theoretical predictions.

Conclusions: Experiments indicate the somewhat unexpected result that the radiation emitted by the dominant mode in a curved single-mode fibre is in the form of discrete tangential rays and is not continuous. The tentative analysis we have presented which predicts that the HE_{11} mode becomes leaky at the curved fibre boundary and that electromagnetic tunnelling occurs, is in agreement with the main features of the observations. An analysis by Neumann and Rudolph[8] of radiation from bends in dielectric-rod transmission lines at microwave frequencies assumes a uniform energy loss. However, a careful study of their experimental results indicates that a related phenomenon may have been taking place in this case also.

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