Determination of core diameter and refractive-index difference of single-mode fibres by observation of the far-field pattern

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Abstract: A method is described for determining unambiguously, from the far-field pattern of single-mode fibres, the core diameter and the refractive-index difference between core and cladding. It involves measurements only of the half-power width of the main lobe and the width of the first minimum. Universal curves are presented that can be used with single-mode fibres or any fibre operating in the single-mode regime. Independent determinations of core diameter over the range 4–8 μm by an etching technique are in excellent agreement with those obtained from the far-field pattern.

1 Introduction

Single-mode fibres have several potential advantages when considered for use as transmission lines at optical frequencies. In particular the absence of dispersion due to multimode operation means that the attainable bandwidth can be very large, being ultimately limited by material and mode dispersion when operating with a monochromatic source, and estimates[1] give values in the region of 100 GHz over a 1 km length. In recent years attention has centred on multimode fibres because they can be used with light-emitting diodes and they present fewer handling problems, in launching and jointing for example. However, the fabrication of single-mode fibres has been rendered comparatively simple[2] by the new homogeneous chemical vapour deposition (c.v.d.) technique. Furthermore, it may be deduced from recent theoretical work[3] that the 'microbending' loss can be made small by restricting the core diameter. Together with the longer lifetimes now being reported for semiconductor lasers, these factors indicate that single-mode fibres may become increasingly important in the future.

With any type of fibre, two of the fundamental parameters are the core diameter $2a$ and the difference in refractive index Δn between that of the core n_1 and the cladding n_2 from which the normalised frequency V at the (free-space) wavelength of operation λ may be obtained.

Thus

$$\Delta n = n_1 - n_2 \qquad (1)$$

and

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \simeq (2\pi a/\lambda)(2n_1\,\Delta n)^{1/2} \qquad (2)$$

In multimode fibres, the core diameter, which commonly lies in the range 40–100 μm, can be measured by conventional optical techniques, and Δn can be determined from measurements of the numerical aperture or of the refractive index profile. However, with single-mode fibres, because the core diameter is comparable to a wavelength, the effects of diffraction render such measurements more difficult, particularly at the smaller V values. Sometimes the refractive indices of core and cladding may be accurately known, but in general this is not the case, particularly with fibres produced by the c.v.d. technique. A method of determining a and Δn is therefore required which is simple to apply and which ideally can be used to assess fibres immediately after drawing.

No rapid and satisfactory method exists at present. One possible technique is to detect the onset of higher-mode propagation in the output far-field radiation pattern from the fibre when the exciting wavelength is varied. In this way, the wavelength of the second mode cutoff ($V = 2{\cdot}4$) is found, and, assuming that the dispersive properties of the glasses in the core and cladding are similar, an extrapolation of the V value to other wavelengths can be made. However, the method is insensitive because the wavelength at which a higher mode appears in the output pattern is somewhat indeterminate. A mode becomes extremely lossy as it nears its cutoff point, since the mode volume becomes large and guidance is weak. Thus no clear cutoff wavelength is observed, and the measurement is found to be very sensitive to slight bends and pressure applied to the fibre as these cause premature radiation of the higher mode. We have found, for example, that some fibres operating at a wavelength such that $V = 2{\cdot}8$, i.e. well into the overmoded region, have a radiation pattern indistinguishable from that of a single-mode fibre after a length of about 1 m. Our conclusion from this and other experiments is, therefore, that the mode-cutoff technique is not sufficiently accurate and, moreover, requires a laser source which is tunable over a wide range of wavelength. Furthermore it does not yield the core diameter or refractive-index difference directly.

An alternative method is presented here of determining a and Δn from a simple measurement of the far-field pattern at a single wavelength.

2 Far-field radiation pattern of HE_{11} mode

2.1 Angular width at half-maximum intensity

As with multimode fibres it is clear that the far-field radiation pattern of the HE_{11} mode is a function of both a and Δn, and moreover can be easily observed experimentally.[4] Our first step, therefore, is to calculate this field distribution. It is assumed that $\Delta n \ll n_1$, which is normally the case in practice and greatly simplifies the analysis. The far-field distribution $\Psi(r, \theta, \phi)$ may be ob-

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52

tained from the Fraunhofer diffraction equation[5] expressed in terms of the spherical co-ordinates (r, θ, ϕ) defined in Fig. 1, namely

$$\psi(r, \theta, \phi) \simeq \frac{j}{\lambda r} e^{-jkr} \int_0^{2\pi} \int_0^{\infty} \psi_0 (\rho, \phi_0) \exp$$

$$\{jk\rho \sin\theta \cos(\phi - \phi_0)\} \rho \, d\rho \, d\phi_0 \qquad (3)$$

where $k = 2\pi/\lambda$, and Ψ_0 expresses the near-field at the output end of the fibre where the corresponding co-ordinates are (ρ, ϕ_0).

By using the approximate field equations derived by Snyder[6] for the HE_{11} mode in structures having $\Delta n \ll n_1$, the normalised far-field distribution may be derived as

$$|\psi|^2 = \left[\frac{U^2 W^2}{(U^2 - \alpha^2)(W^2 + \alpha^2)} \left\{J_0(\alpha) - \alpha J_1(\alpha) \frac{J_0(U)}{U J_1(U)}\right\}\right]^2$$

$$\text{for } U \neq \alpha \qquad (4)$$

$$= \left[\frac{U^2 W^2}{2V^2} \frac{1}{U J_1(U)} \{J_0^2(\alpha) + J_1^2(\alpha)\}\right]^2 \quad \text{for } U = \alpha \qquad (5)$$

where $V^2 = U^2 + W^2$, and U and W are the arguments[6] of the Bessel and modified Hankel functions. We have normalised the radiation angle in the form

$$\alpha = ka \sin\theta \qquad (6)$$

and it is, of course, a function of the wavelength.

A convenient parameter to measure experimentally is the output angle θ_h at which the far-field intensity has fallen to one-half that at the central maximum ($\theta = 0$). The normalised half-intensity angle is thus defined as

$$\alpha_h = ka \sin\theta_h \qquad (7)$$

It may be shown from eqn. 4 that α_h is an unambiguous function of V, and the relationship is indicated by the solid line in Fig. 2. Thus for values up to 10 or so, V may be very simply determined if α_h is known, assuming that for $V > 2{\cdot}4$, only the HE_{11} mode is launched and propagated along the fibre. If the core radius a can be found in some other way, measurement of the half-intensity width θ_h enables α_h to be obtained, so that eqn. 4, or in practice the universal curve of Fig. 2, gives V and hence Δn. As described in Section 3, an etching technique can sometimes be used to determine the core diameter, but only with those fibres where the core and the cladding have markedly different etch rates.

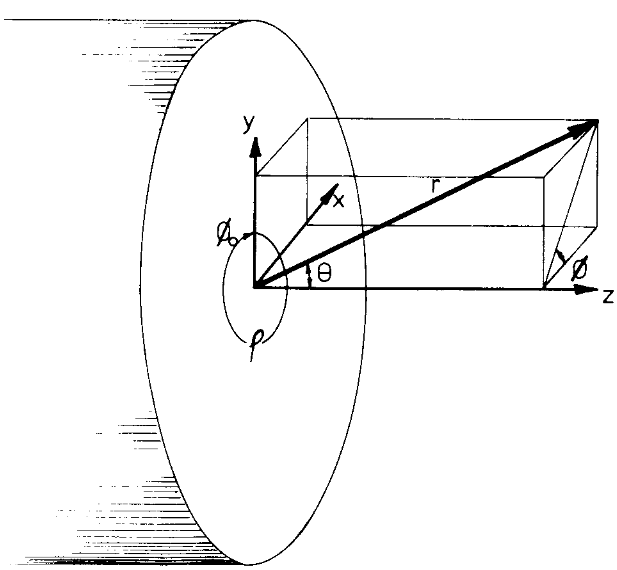


Fig. 1 *Spherical co-ordinate system used in calculations of far-field pattern*

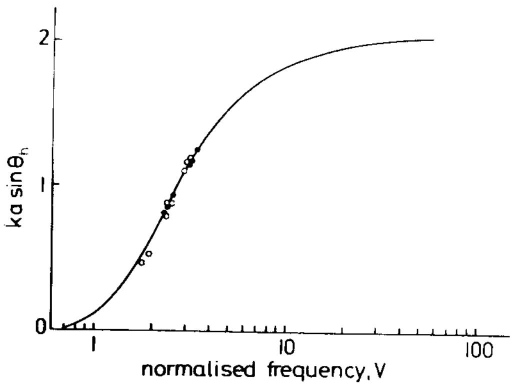


Fig. 2 *Variations of normalised half-maximum angle with V*

——— calculated from eqn. 4
○○○○ measured, core diameter = 6·6μm
●●●● measured, core diameter = 8·1μm
Wavelength range = 0·42–0·9μm for measured points

2.2 *Angular width of first minimum*

The output end of an optical fibre forms a radiating aperture, but it is not generally appreciated that, as with any other form of radiating aperture, such as a microwave aerial for example, the output field pattern contains sidelobes. Thus it can be shown from eqn. 4, and experimentally as in Section 3, that, in addition to the main beam, the far-field pattern exhibits a range of subsidiary peaks, as illustrated in Fig. 3A, at angles and relative intensities that depend on a, n_1, n_2 and λ. Some typical far-field radial intensity distributions are shown in Fig. 3B, in which the radiation angle has again been normalised in

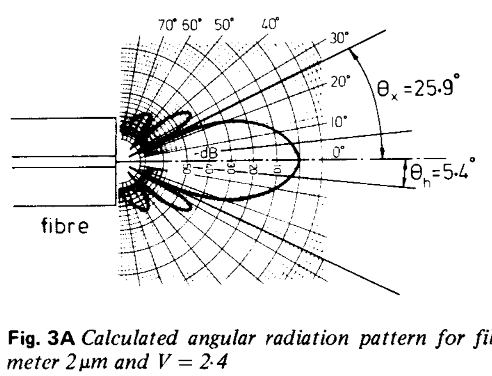


Fig. 3A *Calculated angular radiation pattern for fibre of core diameter 2μm and V = 2·4*

The figure shows the half-intensity angle θ_h and the first minimum angle θ_x

the form $\alpha = ka \sin\theta$, for various values of V. It may be further shown that the angular width θ_x to the first minimum can be used in conjunction with θ_h to obtain V directly without any knowledge of a. Thus, like α_h, the $\sin\theta_x/\sin\theta_h$ ratio is also an unambiguous function of V. The variation of this ratio, together with α_h, is given in Fig. 4 for the range of V values most likely to be encountered in practical single-mode fibres. Thus the interesting and invaluable result is obtained that the simple determination of θ_x and θ_h enables V and a, and hence n, to be obtained without the need for any other measurements. In the example illustrated in Fig. 4, it is assumed that the $\sin\theta_x/\sin\theta_h$ ratio is found experimentally to be 5·25, indicating, using curve (i), that $V = 2{\cdot}14$. From curve (ii) it can be seen that the corresponding value of $ka \sin\theta_h$ is 0·813, and from the measured value of θ_h it is possible to calculate a.

3 Experimental techniques and verification

Experiments have been carried out on a number of single-mode fibres made by the technique[2] of homogeneous chemical vapour deposition. A piece of each fibre of about 1 m length was taken, and laser radiation was launched into the core by a x10 objective lens. In order to avoid the propagation of higher-order modes, which, although

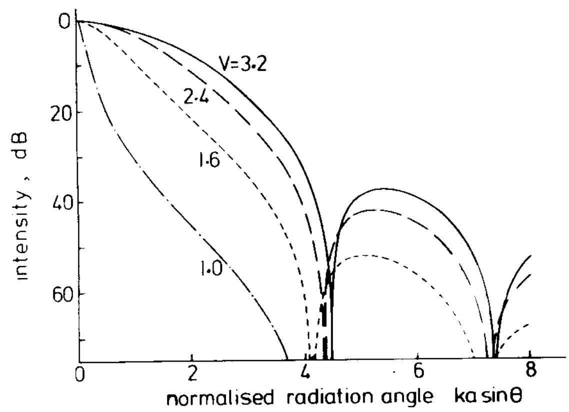


Fig. 3B *Intensity distribution in the far-field as a function of the normalised radiation angle for various V numbers*

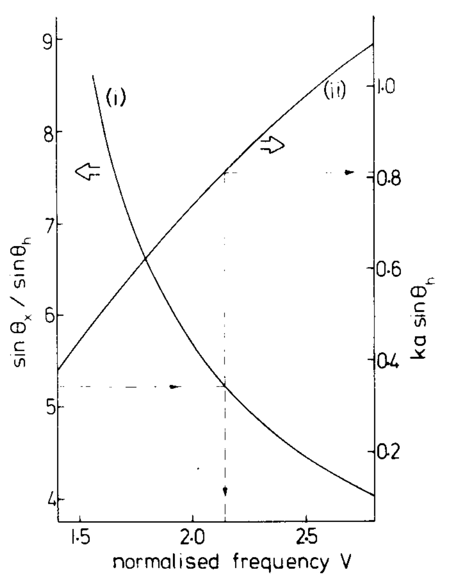


Fig. 4 *Variation of half-intensity angle α_h and $\sin\theta_x/\sin\theta_h$ ratio with V*

beyond cutoff, may be weakly guided over this length, the fibre was slightly curved. Portions near the ends were immersed in a liquid having a refractive index higher than that of the cladding in order to remove any cladding modes.

The angular widths θ_h and θ_x were obtained by monitoring the far-field output pattern with a scanning photodiode array.* Figs. 5*a* and *b* show the outputs from the array displayed on an oscilloscope under conditions of low gain (from which θ_h can be measured) and high gain (showing the positions of the minima θ_x), respectively. In Fig. 5*a*, the response of the photodiode array is linear, and the Gaussian shape of the main beam can be seen. However, in order to show up the first minima in Fig. 5*b* the gain is so high as to cause saturation and distortion at smaller angles. The values of θ_x were confirmed by taking photographs of the far-field pattern as shown in Fig. 5*c*.

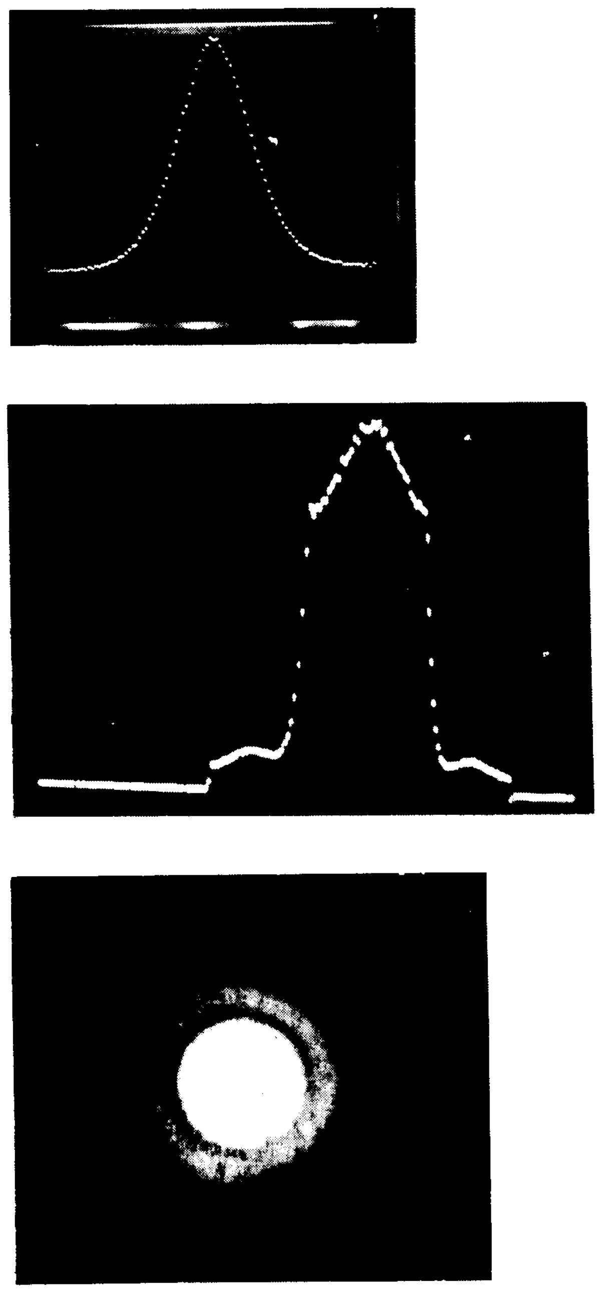


Fig. 5 *Output from scanning photodiode array*

a Under low-gain conditions
b Under high-gain conditions
Note that the central peak is saturated in *b*
c Photograph of the far-field intensity pattern

*Integrated Photomatrix 7000

As a check on the theory given in Section 2, the core diameter can be measured directly in two ways. First, the core at the end of a piece of fibre was etched away with hydrofluoric acid, since phosphosilicate glass dissolves much more rapidly than pure silica. The core diameter was then measured by an optical microscope and could also be determined using a scanning electron microscope. A typical picture of an etched-fibre end obtained with an s.e.m. is shown in Fig. 6 and shows the high degree of resolution that can be obtained at the edge of the core, indicating that the diameter can be measured reasonably accurately. Secondly, the core diameter in the preform was measured optically as well as the outside diameter of the preform. The overall diameter of the resulting fibre was then measured after drawing, so that the fibre core diameter is given by the product of the preform core diameter and the preform/fibre outside-diameter ratio. There are uncertainties in the second method owing to possible diffusion between core and cladding during the drawing process, so that only the values obtained by etching are used here. Nevertheless, the agreement between the two methods was very good (within 2%).

Several single-mode fibres having core diameters ranging from 4 to 8·4 μm have been tested. In the first set of measurements, a tunable laser† was used to vary the V values over a wide range. The angle θ_h was determined at wavelengths between 0·42 and 0·9 μm for two fibres whose core diameters of 6·6 μm and 8·1 μm were obtained by etching. The results are given as the experimental points in Fig. 2 and are in excellent agreement with the theory. The V values for these two fibres were found to be 1·98 and 2·78 at $\lambda = 0{\cdot}63\,\mu$m.

In the second set of measurements, θ_h and θ_x were measured for a number of fibres. For each fibre, the angular measurements were made in ten independent experiments, and the repeatability was within ±2%. The core diameters of two of the samples were again obtained by etching, and the comparison with the far-field measurements is shown in Table 1. The cores were slightly elliptical and the figures in the final column denote the lengths of the major and minor axes. The agreement between the two methods is excellent, particularly since the orientation of the fibre ends for the far-field measurements was not known. For the other three samples, independent diameter measurements were not made, and the results obtained for a and Δn are given in Table 2. These fibres were all drawn from the same preform, which explains the similarities in Δn, but small changes could have occurred because of slight nonuniformities in the deposited layer or because of diffusion during fibre drawing.

It should be noted that the theory presented here assumes a uniform core refractive index, so that Δn is easily specified. However, it is clear from the etched end shown in Fig. 6 that a degree of nonuniformity exists across the core. In particular, the fibre exhibits a dip in refractive index at the core centre caused by depletion of the P_2O_5 from this area during the preform collapse. The fact that the far-field measurement yields accurate values for the fibre core diameter suggests that the index nonuniformity has no great effect on the radiation pattern. Nevertheless care must be exercised in interpreting the deduced value of Δn, as this presumably represents some mean value. It would appear that this mean value, or more accurately the V-number associated with it, has the usual significance in terms of the higher-mode cutoff values.

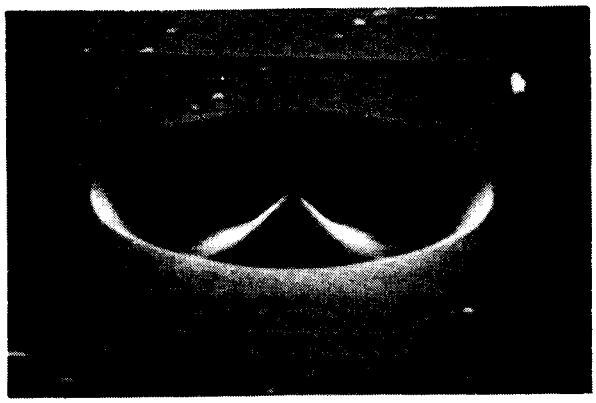


Fig. 6 *Scanning-electron microscope photograph of an etched fibre end*

Core diameter ≃ 7·5 μm

4 Conclusions

It has been shown that the core diameter and refractive-index difference can be obtained unambiguously from the far-field pattern of the HE_{11} mode in fibres of low V value, and the method is now being used in these laboratories for the routine characterisation of single-mode fibres. Independent determinations of core diameter by

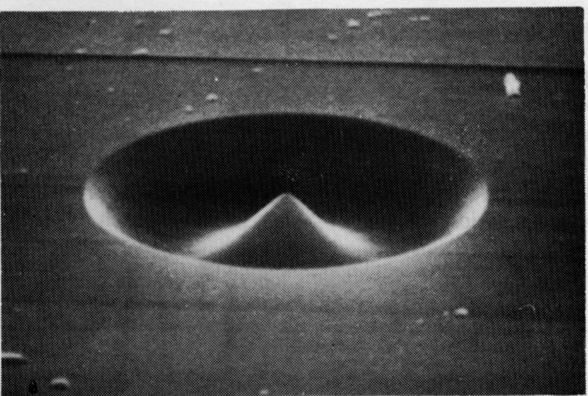
Table 1: Comparison of core diameters obtained from far-field pattern with those measured by etching

Sample	$\sin\theta_h$ (mean)	$\sin\theta_x/\sin\theta_h$ (mean)	V (mean)	Δn	Core diameter: Far-field measurements	Core diameter: Etching measurements
1	0·0211	5·38	2·10	0·00107	7·6	7·36 × 7·82
2	0·0385	5·14	2·18	0·00349	4·3	4·22 × 4·31

Table 2: Values of V, refractive-index difference and core diameter determined from the far-field pattern

Sample	$\sin\theta_h$ (mean)	$\sin\theta_x/\sin\theta_h$ (mean)	V (mean)	Δn	Core diameter
					μm
3	0·0205	6·05	1·91	0·00108	6·9
4	0·0220	4·79	2·32	0·00112	8·2
5	0·0231	4·50	2·48	0·00120	8·4

†Chromatrix CMX4



an etching technique are in good agreement with those found from the far-field pattern and further confirmation has been obtained by preform measurements and the similarity of the index difference for fibres of various dameters pulled from the same preform. The technique is particularly suited to characterisation of single mode fibres produced by the c.v.d. method.

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