LENGTH-DEPENDENT EFFECTS DUE TO LEAKY MODES ON MULTIMODE GRADED-INDEX OPTICAL FIBRES

M.J. ADAMS, D.N. PAYNE and F.M.E. SLADEN
Department of Electronics, University of Southampton, Southampton S09 5NH, UK

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The radiation losses of tunnelling leaky modes in graded-index optical fibres are calculated theoretically, and it is shown that the near-field intensity profile has a length dependence. Consequently measurements of the near-field intensity distribution do not give the refractive index profile directly, and a correction factor must be applied. We have investigated this factor and find that it depends only on a single normalisation parameter involving fibre length, core radius and normalised frequency. A further use of the correction factor is to determine the total power attenuation due to the loss of leaky modes.

1. Introduction

The theory of bound mode propagation in graded-index multimode optical fibres is now well-known, and may be adequately described for many applications by the localised plane wave method [1]. However it has recently been recognised [2–4] that tunnelling leaky modes are usually present in graded-index structures, although the significance of these modes in a practical situation has not been fully clarified. In earlier work in this area [2,5], we have assumed that all leaky modes propagate unattenuated and showed that

i) in contrast to the step-index fibre, tunnelling leaky rays in parabolic index fibres are all contained within the angular limits of the numerical aperture. It is therefore impossible to avoid their excitation when using an apertured lambertian source.

ii) 25% of the power launched by a lambertian source into a parabolic index is carried by the leaky modes.

iii) the near-field intensity distribution is related to the refractive index profile by the factor $1/\sqrt{1-(r/a)^2}$, where r/a is the normalised radius.

It is apparent, therefore, that leaky modes will have a significant effect on at least two fibre measurements which commonly use apertured lambertian sources, namely *a*) evaluation of total attenuation and *b*) determination of the refractive index profile by observation of the near-field intensity distribution [6]. In the former the total output power is measured after transmission through a long length of fibre and then compared with that from a short length. Since the attenuation of the leaky modes is included in the measurement, the result is inclined to be pessimistic. In the latter the refractive index profile is inferred from the intensity distribution across the output face of a metre or so of fibre. In this case the presence of leaky modes can considerably influence the intensity distribution, and therefore introduces a profile error. The object of the present contribution is to calculate the losses inherent in modes of this type, and hence to ascertain the magnitude of the errors introduced in the above two cases.

As a first step, the attenuation coefficient for each leaky mode is found by application of the WKB method, and it is shown that a simple approximation may be applied to give an expression which is not particularly sensitive to the exact form of the index profile. This expression may be used to sum the power remaining in all modes after a given length of fibre, and hence we obtain the near-field distribution at any point along the fibre. A generalised length-dependent correction factor involving only fibre length, core radius and normalised frequency may then be introduced to relate the refractive index profile to the intensity distribution. Finally the theory is applied to calculations of the leaky mode contribution to fibre attenuation measurements, and this gives a useful physical insight into the persistence of tunnelling modes.

2. Leaky mode attenuation coefficient

For a step index fibre the attenuation of leaky modes may be calculated exactly from the known electromagnetic fields, and has been dealt with previously in some detail [7]. For graded-index fibres, on the other hand, some approximate procedure must be adopted, the simplest being the zero'th order WKB approximation. The results so obtained are of comparable accuracy to the more formal first-order approach given by Petermann [3], and in addition more physical understanding is gained in the present treatment. The approximation is valid under the usual WKB restriction, viz. small variation of refractive index distribution over distances of the order of one wavelength. In the case of a graded-index multimode fibre this condition is usually easily satisfied. An assumption which is made throughout is that the fibre core is surrounded by a cladding of infinite extent. Although this is not met in practice, for the purposes of the present calculation the error introduced by the assumption is small, provided that the cladding thickness is at least 50% of the core radius. Most low-loss CVD fibres satisfy this condition.

2.1. General form of the attenuation coefficient

Fig. 1 shows the squared magnitudes of the local plane-wave vector components as functions of radius for a leaky mode in a general graded-index fibre [2]. Here a is the core radius and r_1, r_2, r_3 correspond to the caustics separating regions of oscillatory and evanescent fields. In analogy with the concept of quantum mechanical tunnelling, the probability of a photon from r_2 emerging at the outer caustic r_3 is given by the tunnelling probability T. This is calculated as the inverse ratio of the squares of the field amplitudes $E(r_2), E(r_3)$, at these radii [4]:

$$T = |E(r_3)/E(r_2)|^2 = \exp\left\{-2\int_{r_2}^{r_3} \left(\frac{\nu^2}{r^2} + \beta^2 - k^2 n^2(r)\right)^{1/2} \mathrm{d}r\right\}, \tag{1}$$

where β is the propagation constant, ν = azimuthal mode number, $k = 2\pi/\lambda$, λ = wavelength and $n(r)$ is the refractive index at radius r. The dimensionless attenuation coefficient α (normalised to the core radius) of a given leaky mode (u, ν) can then be calculated from the mode tunnelling coefficient T by the simple relation

$$\alpha(u, \nu) = \frac{2\nu}{a\beta} \frac{T}{1-T}. \tag{2}$$

In order to obtain the net power flow out of the fibre, the component of the Poynting vector normal to the fibre axis must be calculated, and this gives rise to the term $2\nu/a\beta$ in eq. (2) above. The same expression may be deduced directly from Poynting's vector theorem (following Snyder [8]), or alternatively from geometrical optics when the first term arises from the mean distance between points at which a ray meets its caustics (the ray period). Note that implicit in eq. (2) is the assumption that only the least leaky modes are of importance.

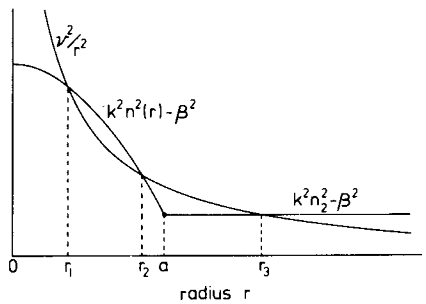

Fig. 1. Squared magnitude of the plane-wave vector components of a tunnelling leaky mode in a graded-index fibre of core radius a. The mode shown has an oscillatory region between caustics r_1 and r_2, indicating bound energy, and an evanescent field between radii r_2 and r_3. Radiation occurs from radius r_3.

2.2. Attenuation coefficient for parabolic index variation

The integral in eq. (1) may be calculated numerically for any refractive index profile $n(r)$. However, an analytic expression may be obtained for the special case of a parabolic index variation:

$$T = (u^4 - 4v^2\nu^2)^{-\frac{u^2}{4v}-\frac{\nu}{2}} \left(\frac{(u^2-v^2)(2\nu x + 2\nu^2 - u^2)}{(\nu + x)^2}\right)^{\nu} (2vx + 2v^2 - u^2)^{\frac{u^2}{2v}} \exp(x), \tag{3}$$

where $x = [\nu^2 - (u^2 - v^2)]^{1/2}$, and the conventional notation $u^2 = a^2[k^2n^2(0) - \beta^2]$, $v^2 = a^2k^2[n^2(0) - n_2^2]$ has been used; $n(0)$ is the refractive index at core centre and n_2 that of the cladding.

2.3. Approximate form of the attenuation coefficient

Although eq. (1) can be evaluated for any index profile $n(r)$, a more useful general result can be achieved by a simple approximation. From fig. 1 it is intuitively seen that for most forms of the index profile $n(r)$, the central caustic r_2 is not too far from the core–cladding boundary at radius a. Using this fact as a basis for approximation and replacing $E(r_2)$ by $E(a)$, the tunnelling coefficient T becomes

$$T \simeq |E(r_3)/E(a)|^2 = [(u^2 - v^2)/(\nu + x)^2]^{\nu} \exp(2x). \tag{4}$$

It may be seen that T, and hence the attenuation coefficient α, is no longer a function of the specific form of the index profile $n(r)$. This is an important result as it implies that, at least within the limits of the approximation used here, the loss of a leaky mode having a given designation (u, ν) is independent of the profile of the structure within which it is propagating. Thus we might expect that the influence of leaky modes on, for example, the near-field intensity, would be relatively insensitive to the core index profile, and this is indeed the case, as will be shown later.

In the following sections eq. (4) will be used in place of eq. (1), as this allows a considerable reduction in numerical computation whilst yielding results of comparable accuracy. Furthermore, we note that provided only the least leaky modes are considered ($x \simeq \nu$), eqs. (4) and (2) reduce to give the leaky mode attenuation coefficient derived by Gloge [9] for the *step* index fibre.

3. The length-dependent near-field intensity distribution

Reverting temporarily to geometrical optics, the near-field intensity $I(r)$ at radius r on the output face of a fibre excited by a lambertian source may be found by summing a cosine source function over angles of incidence θ and projected angles ϕ [2]. The attenuation of the leaky modes is included by way of the ray equivalent of the attenuation coefficient α derived above:

$$\frac{I(r)}{I(0)} = \frac{4}{\pi[n^2(0) - n_2^2]} \int_0^{\pi/2} \mathrm{d}\phi \int_0^{\pi/2} \sin\theta \cos\theta \exp[-\alpha(\theta,\phi) z/a]\, \mathrm{d}\theta, \tag{5}$$

where $\alpha(\theta,\phi)$ is the attenuation of mode (u, ν) associated with rays launched at angles (θ,ϕ), and z is the fibre length. The integral over angles of incidence θ in eq. (5) may be split into three regions corresponding to bound, leaky and refracted rays [2]. The attenuation coefficient $\alpha(\theta,\phi)$ of a bound ray is taken to be zero, that of a refracted ray infinite, and that of a leaky ray is given by the mode attenuation $\alpha(u, \nu)$ defined in eq. (2). If we consider only bound rays to be propagating, the upper limit on θ in eq. (5) becomes $\sin^{-1}[\sqrt{n^2(r) - n_2^2}]$ and the expression reduces to that given by Gloge and Marcatili [1].

Converting eq. (5) back into the mode notation [2] (u, ν), and assuming an external medium of $n = 1$, we deduce that the intensity distribution $I(r)$ is

$$\frac{I(r)}{I(0)} = \frac{n^2(r) - n_2^2}{n^2(0) - n_2^2}\, C(r,z) \simeq \frac{n(r) - n_2}{n(0) - n_2}\, C(r,z), \tag{6}$$

where

$$C(r,z) = 1 + \frac{4}{\pi[n^2(r) - n_2^2]} \int_0^{\nu_{max}} \frac{d\nu}{a^2 r k^3} \int_{v}^{\sqrt{v^2+\nu^2}} \frac{u\, du \exp[-\alpha(u,\nu) z/a]}{[n^2(r) - n_2^2 + (u^2/a^2k^2) - (\nu^2/r^2k^2)]^{1/2}}, \tag{7}$$

The upper limit ν_{max} is given by the appropriate zero of the square-root term in the integral*.

Thus $C(r, z)$ may be considered as a "correction factor" relating the near-field distribution $I(r)$ to the refractive index profile $n(r)$. As expected this factor reduces to unity after an infinite length of fibre when no leaky modes remain; a close resemblance will then exist between the intensity and index profiles, a result first reported in ref. [1].

The intensity distribution as a function of fibre length for any given index profile may now be evaluated numerically from eqs. (6) and (7) using the attenuation coefficients given by (2) and (1), or the approximate form (4). A specific example of this is given in fig. 2 where the near field is calculated using the more accurate eq. (3) for a typical parabolic index fibre of 1 metre in length. The index profile is also shown, and it can be seen that the presence of leaky modes causes a marked difference between the two curves. In order to illustrate the process of converting near-field intensity to refractive index, and also to test the accuracy of the approximate expression (4), as compared to the more precise eq. (3), an index profile calculated from the near-field using eq. (4) is plotted. It is clear that this result is very close to the true profile, thereby illustrating the level of accuracy obtainable by use of the approximate tunnelling coefficient.

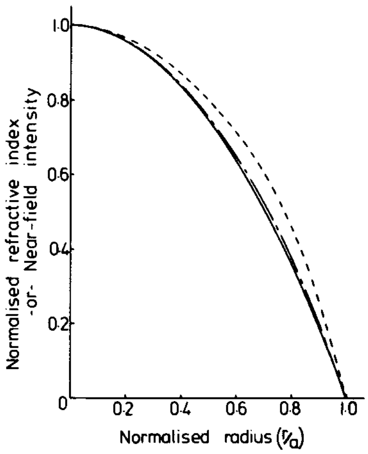

Fig. 2. Calculated near-field intensity distribution (upper dashed curve) for a parabolic index fibre compared with the refractive index profile (lower solid curve). Also shown is the index profile (chain-dotted curve) inferred from the intensity distribution by means of the approximate correction factor developed in the text. Fibre parameters are: length 1 m, core radius 50 μm, numerical aperture 0.2, wavelength 0.93 μm, giving $X = (1/\nu) \ln(z/a) = 0.15$.

4. Generalised near-field correction factors

Although eq. (7) may be used as it stands for computing near-field correction factors, it has proved possible to further simplify the results by use of a general normalisation parameter. It can be shown that an analytic approximation for eq. (7) may be realised by considering only the least leaky modes, since these contribute most to the integrals. The result of this approximation is dependent principally on the normalisation parameter $X = (1/\nu) \ln(z/a)$, and this suggests that this parameter may be a general normalisation even when eq. (7) is evaluated so as to include all modes. Fortunately this hypothesis is well founded provided $z/a > 10^3$; the validity of the normalisation has been verified to within 2% for a wide range of index profiles by numerical integration of eq. (7) using the approximation (4) introduced earlier.

As a consequence both of the approximation (4) and the above normalisation, a single set of curves $C(r, z)$ is all that is now required to completely specify the near-field intensity distribution for a length of a fully-excited fibre having an arbitrary circularly sym-

* Note that in fibres having index exponent > 2 and when launching from a lower index medium, not all leaky modes are necessarily excited by a lambertian source. Care must therefore be exercised in the evaluation of eq. (7) to ensure that the upper limits in the integrals do not allow the local numerical aperture [sin θ in eq. (5)] to exceed unity. By suitable choice of limits, eq. (7) may also be applied to lambertian sources truncated in angle, which similarly do not excite all leaky modes.

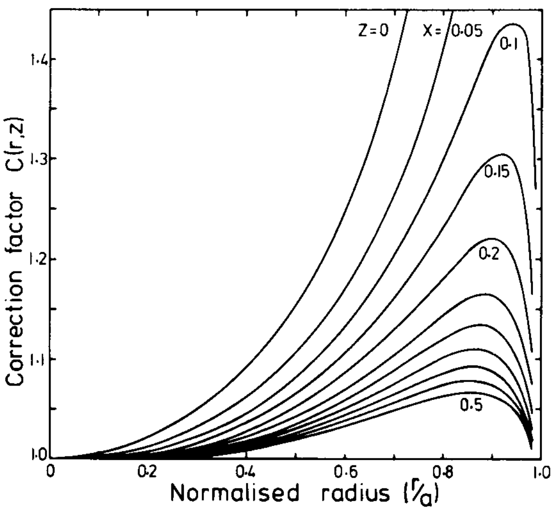

Fig. 3. Near-field correction factors $C(r, z)$ given as a function of normalised fibre radius for X values from 0.05 to 0.5 in increments of 0.05. The normalisation parameter X describes the fibre core radius, length and numerical aperture, and is given by $X = (1/v) \ln (z/a)$. The curves may be used for fibres having any graded-index profile. Also shown is the result for $z = 0$, when all leaky modes are present.

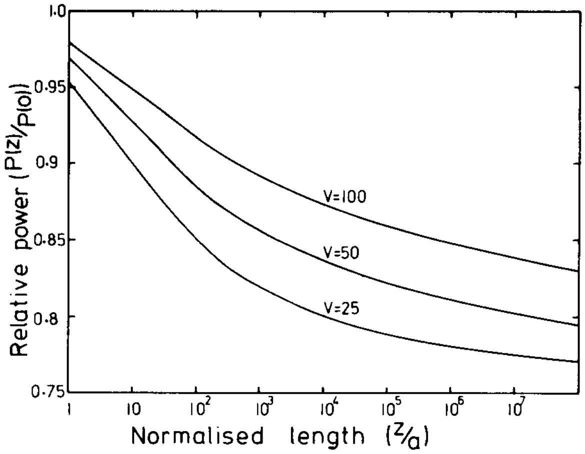

Fig. 4. Plot of output power $P(z)$ relative to input power $P(0)$ as a function of length for parabolic index fibres having the v-values shown. Provided $z/a > 10^3$ the curves are applicable to fibres having any combination of numerical aperture, core radius and wavelength. For shorter lengths the curve is drawn for the specific example of a numerical aperture of 0.2 and $\lambda = 0.93\ \mu$m. The decrease of power with length is caused by the radiation losses experienced by tunnelling modes. The curves are asymptotic to $P(z)/P(0) = 0.75$.

metric index profile. Conversely, and more importantly, the same set of curves may be used to correct a measured intensity distribution to give the refractive index profile. This fact forms the basis of a near-field scanning technique which has been developed to determine refractive index profiles [6].

The curves $C(r, z)$ are shown in fig. 3 plotted against normalised radius for several values of the parameter X. Also shown is the curve for $z = 0$ when all leaky modes are present and equally excited, viz. $C(r,0) = 1/\sqrt{1-(r/a)^2}$. The figure represents a good approximation not only over all X values normally encountered, but also for a wide range of near-parabolic and power-law [1] variations; it is somewhat less accurate for a step index distribution. Taking a worst case example of a profile having an index exponent of 3 and an X value such that a large correction factor of 1.4 is required, we estimate that an index error of 1.5% of the centre value occurs at $r/a = 0.8$. Greater accuracy may be expected when using smaller correction factors, and for profiles closer to parabolic.

As an example of the use of the curves let us take a 1 metre length of graded-index fibre having a core diameter of 80 μm and a numerical aperture of 0.18. The exciting wavelength is 0.9 μm. We calculate the X value as 0.2, and the figure indicates that the near field is 8% greater than the index profile at a normalised radius of 0.6, rising to 20% at 0.85.

5. Power attenuation

The near-field distributions given by eqs. (6) and (7) may be integrated over radius r to give the total power remaining in all modes as a function of fibre length z. In thise case the normalisation parameter X may be used for values of $z/a > 10^3$, although the result is now sensitive to details of the refractive index profile as a consequence of the differing number of leaky modes supported by various structures. Results for a parabolic index fibre are shown in fig. 4, where the normalised power $P(z)/P(0)$ after length z is plotted as a function of normalised length z/a for a number of v-values. The curves are computed for a fixed numerical aperture (0.2) and wavelength (0.93 μm) and the v-values are determined by different core radii. For other numerical aperture/wavelength/radius combinations the curves will be similar to those of fig. 4 for $z/a > 10^3$, but somewhat different for lower z/a values as a result of the invalidity of the normalisation parameter X.

As a numerical example, consider a fibre of numerical aperture 0.3 and core radius 25 μm at the wavelength of GaAs emission (0.93 μm). This yields a v-value of 50 so that, from the graph, after 50 cm ($z/a = 2 \times 10^4$) the power in the leaky modes has decayed from an initial 25% of the total to 9.7%; after 1 km ($z/a = 4 \times 10^7$) the proportion is further reduced to 6.1%. Thi' illustrates the persistence of some proportion of the leaky modes for considerable distances.

As noted earlier the radiation losses suffered by leaky modes will contribute an error to the total attenuation measurement of a fully excited fibre. Taking the initial fibre length to be 1 km and the shortened length 50 cm, as in the example given above, the fibre loss would be pessimistic by 0.17 dB/km. If, however, the length available was only 100 metres ($z/a = 4 \times 10^6$) the extrapolated loss would be in error by the more significant figure of 1.3 dB/km.

6. Conclusion

It has been shown that the near-field intensity distribution in graded-index fibres excited by lambertian sources has a length-dependence caused by the radiation losses of leaky modes. Although many of the leaky modes are lost within distances of less than 1 cm, other can persist for a kilometre or more, giving a near-field that departs considerably from that predicted by a bound mode analysis. Length-dependent correction factors have been computed which enable near-field intensitites to be calculated, given the refractive index profile. The inverse of this process yields a technique for the experimental measurement of refractive index profiles [6].

The correction factors may also be used to calculate total power attenuation as a function of length. This would indicate that a small error is incurred under normal attenuation measurement conditions, but that care should be taken in extrapolating results obtained on short fibre lengths. The normalisation parameter deduced here, $X = (1/v) \ln (z/a)$, provides a convenient characterisation of a given length of fibre.

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