LEAKY RAYS ON OPTICAL FIBRES OF ARBITRARY (CIRCULARLY SYMMETRIC) INDEX PROFILES

Indexing terms: Fibre optics, Geometrical optics, Optical waveguides

The local plane-wave decomposition approach used to analyse optical fibres of arbitrary refractive-index profiles has been extended to include the case of so called 'leaky' rays. The result thus obtained for acceptance angle represents a generalisation of results derived previously for simple forms of the profile by geometrical-optics methods.

Introduction: Recent publications[1,2] derive an expression for the acceptance angle of a graded-index fibre by a geometrical-optics technique. The results so obtained indicate that the acceptance angle at a point on the fibre input face varies not only with position, but also with the projected angle of incidence of the ray (ϕ in Fig. 1). This is contrary to the approach developed by Gloge and Marcatili,[3,4] which predicts that the local numerical aperture is a function of radius only. The purpose of this letter is to extend the plane-wave decomposition method to include weakly leaky or tunnelling rays[5,6] and to show that these rays account for this discrepancy. In addition, we show that tunnelling rays exist in the general class of circularly symmetric guiding index profiles, and that their presence strongly influences the observed near-field power distribution in short fibres excited by incoherent sources.

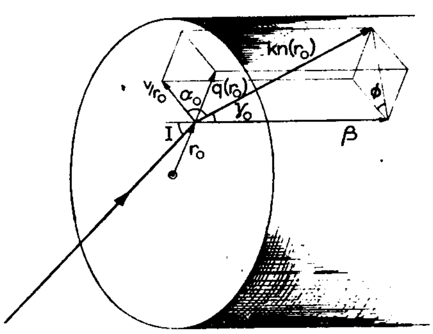

Fig. 1 *Local wave-vector diagram for ray incident on fibre face at radius r_0 and with angle I*

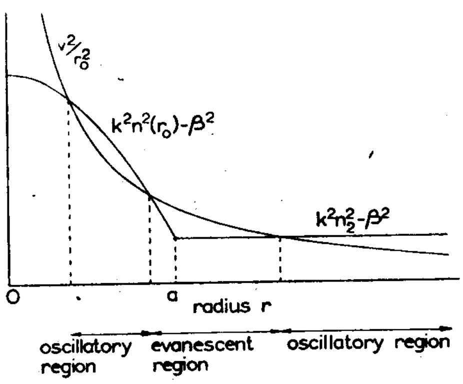

Fig. 2 *Squared magnitude of components of eqn. 2*
Showing a mode just below cutoff ($\beta < kn_2$); a is the core radius

Theory: The local plane-wave decomposition[3,4] is shown in Fig. 1, where the angles α_0, γ_0, ϕ are those appropriate for a ray entering the fibre at radius r_0. The relationships between the launching conditions (given by r_0, α_0, γ_0) and the wave-optical decomposition components are[3]

$$\cos\gamma_0 = \frac{\beta}{kn(r_0)} \text{ and } \cos\alpha_0 = \frac{\nu}{r_0\, kn(r_0)} \qquad (1)$$

where k is the wavenumber, ν is the azimuthal wavenumber, $n(r_0)$ is the refractive index and β is the propagation constant. The radial component of the wave vector at r_0 is given by

$$q(r_0) = \left[k^2 n^2(r_0) - \beta^2 - \frac{\nu^2}{r_0^2}\right]^{\frac{1}{2}} \qquad (2)$$

In Fig. 2, the squared magnitude of the various components of eqn. 2 are shown as a function of radius r_0. The Figure is drawn for a fibre core of arbitrary index profile and radius a, surrounded by a cladding of constant index n_2, which may be air. The case shown is that of a mode just below cutoff, i.e. $\beta < kn_2$. It can be seen that this mode has

(*a*) a region of radial periodicity within the core, representing bound power

(*b*) a region of evanescent field within the cladding

(*c*) a further region having an oscillatory field solution within the cladding, representing radiated power.

The mode may therefore be identified as a leaky tunnelling mode. The limiting values for the propagation constant between which these modes may exist are given from Fig. 2 by

$$k^2 n_2^2 - \frac{\nu^2}{a^2} \leqslant \beta^2 < k^2 n_2^2 \qquad (3)$$

Inserting eqn. 1 into eqn. 3 yields

$$n_2^2 - (r_0/a)^2\, n^2(r_0)\cos^2\alpha_0 \leqslant n^2(r_0)\cos^2\gamma_0 < n_2^2 \qquad (4)$$

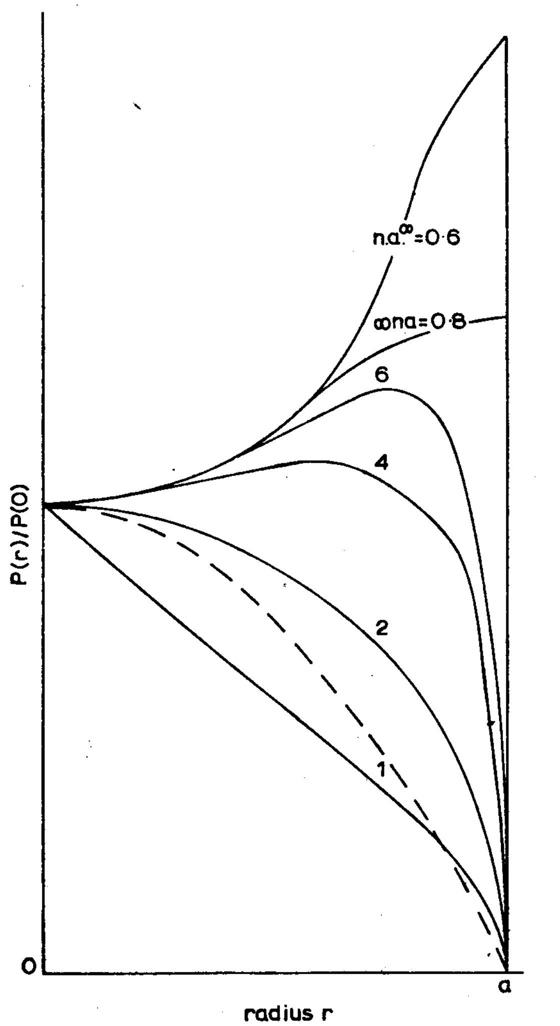

Fig. 3 *Near-field intensity plots*
Calculated from eqn. 12 for class of profiles given by $n^2(r) = n^2(0)[1-2\Delta(r/a)^\alpha]$ with $\alpha = 1, 2, 4, 6, \infty$. Also shown (broken line) is the index-profile plot for $\alpha = 2$

The physical significance of this result may be seen more readily by expressing it in terms of the angle of incidence I and the projected angle ϕ of a ray incident on the fibre face (Fig. 1). We see that, for an external medium of unity index,

$$\sin I = n(r_0)\sin\gamma_0 \qquad (5)$$

$$\cos\phi = \cos\alpha_0/\sin\gamma_0 \qquad (6)$$

Inserting eqns. 5 and 6 into eqn. 4 gives the required expression:

$$\frac{n^2(r_0) - n_2^2}{1-(r_0/a)^2\cos^2\phi} \geqslant \sin^2 I > n^2(r_0) - n_2^2 \qquad (7)$$

Eqn. 7 defines the angular region in which we find leaky tunnelling rays at a given radius on the endface of a fibre with an arbitrary circularly symmetric index profile. It may be seen that one requirement is that the angle of incidence I is greater than the local acceptance angle defined in Reference 3. In addition, I must be less than some value dependent on both the radius r_0 and the projected angle ϕ. As expected, no leaky-ray region exists for $\phi = \pi/2$, as this defines a meridional ray.

Solutions for simple profiles: By using only the l.h.s. of eqn. 7, we define a local acceptance angle that includes both the bound rays, given by $\sin^2 I \leqslant n^2(r_0) - n_2^2$, and the tunnelling rays. It is then possible to verify the result by geometrical optics. Solutions to the following simple cases are already available:

(*a*) *Step-index fibre:* Eqn. 7 is equivalent to Snyder's conditions[6] for leaky rays, if expressed in terms of the angles θ_z, θ_N, θ_c (see Reference 6 for definitions):

$$\pi/2 \geqslant \theta_z > \theta_c \qquad \pi/2 \geqslant \theta_N > \pi/2 - \theta_c$$

In addition, eqn. 7 gives the maximum acceptance angle at radius r_0 as

$$\sin^2 I_{max} = (n_1^2 - n_2^2)\left[1 + \left(\frac{(r_0/a)\sin\delta}{1 + (r_0/a)\cos\delta}\right)^2\right]^{\frac{1}{2}} \tag{8}$$

where n_1 is the core refractive index and δ is the angle defined by Matsumura[1] as

$$\delta = \cos^{-1}\left(\frac{-r_0}{a}\cos\phi\right) + \phi \quad . \quad . \quad . \quad . \quad . \quad . \tag{9}$$

This is identical to the result obtained by a geometric optics analysis of skew-ray propagation,[1,7] confirming that, in a step-index fibre, leaky tunnelling rays are those rays travelling at an angle greater than the meridionally defined numerical aperture, although predicted by geometrical optics to be trapped.

(*b*) *Parabolic-index fibre:*

$$n^2(r_0) = \begin{cases} n^2(0)[1 - 2\Delta(r_0/a)^2] & r_0 \leqslant a \\ n^2(0)[1 - 2\Delta] & r_0 \geqslant a \end{cases} \quad . \quad . \tag{10}$$

where $n(0)$ is the refractive index at the core centre and Δ is the maximum refractive-index difference. Eqn. 7 yields

$$\sin^2 I \leqslant \frac{2\Delta n^2(0)[1 - (r_0/a)^2]}{1 - (r_0/a)^2\cos^2\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \tag{11}$$

This relationship was recently derived by Matsumura,[1] using a geometrical technique.

(*c*) *Fourth-order index profile:* Ikeda's skew and meridional acceptance angle (eqns. 39 and 40 of Reference 2) may be similarly derived by setting $\phi = 0$ and $\phi = \pi/2$, respectively.

Near-field intensity distribution: To determine the effect of leaky rays on fibre propagation, the near-field intensity distribution $P(r)$ may be calculated by the method of Reference 3, suitably corrected for the increased local acceptance angle. Assuming an incoherent source (all modes equally excited), it can be shown that

$$P(r)/P(0) = \frac{n^2(r) - n_2^2}{n^2(0) - n_2^2}\,\frac{1}{\sqrt{\{1 - (r/a)^2\}}} \quad . \quad . \quad . \tag{12}$$

where $P(0)$ is the intensity at the fibre centre.

Fig. 3 shows the near-field intensity plots for a range of index profiles and clearly indicates a substantial departure from the plots of Reference 3. The most obvious difference occurs for the step-index fibre, plotted here for two different meridional numerical apertures. The dependence on numerical aperture is a result of truncation of eqn. 7 at some value of r_0 to ensure that the local acceptance angle does not exceed $\pi/2$.

Conclusions: By extending the plane-wave decomposition technique, a simple generalised expression has been derived defining in angular terms the region in which tunnelling leaky rays are found. This region varies with both index profile and position on the fibre face, and accounts for the somewhat larger local acceptance angles found in References 1 and 2.

The inclusion of the additional angle defined by the leaky-ray region causes a large deviation from the expected near-field intensity distribution. The close resemblance between the index profile and the near-field intensity is no longer found. Although not shown here, a similar departure occurs in the far field. However, these results assume all rays propagate unattenuated, and, whereas experiments show this to be substantially true for 1 m lengths of graded-index fibre, account must be taken of the slow radiation loss of tunnelling rays in longer lengths.

The full significance of leaky tunnelling rays on graded-index fibres has yet to be determined. In step-index fibres, it is normally possible to avoid the excitation of tunnelling rays by simply arranging the launching conditions to just fill the numerical aperture of the fibre. This is not so for the parabolic-index fibre, as all leaky tunnelling rays are contained within the angular limits of the meridionally defined numerical aperture. Thus an incoherent source excites all tunnelling modes, even if arranged to only just fill this aperture.

At least for the step-index fibre, it has been shown[6] that some of the tunnelling modes may persist for several kilometres if excited by the source. It seems likely that this will also be so for the graded-index fibre. Since it is difficult to avoid their excitation, modes of this type may therefore have a significant effect on the measured loss and pulse dispersion in graded-index fibres.

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