OPTICAL FIBRES AND THE GOOS-HÄNCHEN SHIFT

Indexing term: Fibre optics

It is shown, and confirmed experimentally, that, despite the existence of the Goos-Hänchen shift, the propagation delay of a ray in a multimode cladded fibre is given to a very good approximation by simple ray theory.

In the study of the transmission of light along multimode cladded fibres, the ray propagation model has proved useful in predicting the propagation delay and dispersion[1,2] in satisfactory agreement with experiment. Naturally, a ray theory cannot give exact results and must be used with care, but, for most practical purposes, the approximation is a good one. In principle, and certainly at small values of normalised frequency, a modal approach is necessary, but normally, in multimode fibres, the number of propagating modes is inconveniently large. Recently, it has been noted[3] that a more complete analysis should take into account the well known Goos-Hänchen shift,[4] and this is certainly true when the ray theory is applied to certain calculations, such as the coupling between closely spaced waveguides.[5] However, we disagree with the claim[3] that the simple ray model is inadequate to deal with propagation delay and dispersion, and show theoretically and experimentally that the resulting errors are negligible.

In the ray theory, light totally reflected at the interface between two dielectric media is assumed to make a geometric reflection, as indicated by the ray normals to the wavefront, OA and AB, in Fig. 1*a*. However, the more accurate wave analysis indicates that, for a beam of finite cross-section, this is not strictly true, and the reflected beam is displaced in the axial direction by an amount Δ, thus following the path A′B′, where AA′ = Δ. This displacement has become

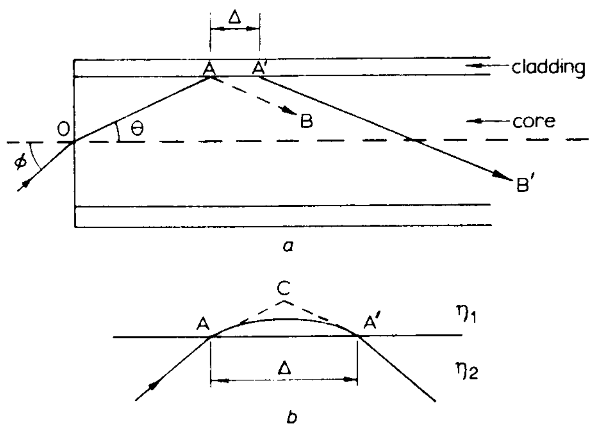

Fig. 1 *Paths of ray normals at core–cladding interface*

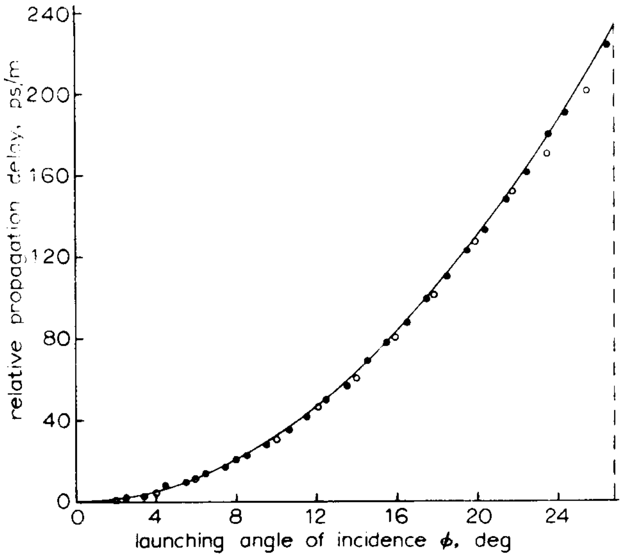

Fig. 2 *Variation of relative propagation delay of input beam of small angular width with input angle of incidence in air*

The line shows theoretical values obtained from eqn. 6
○ measured at wavelength of 0·633 μm
● measured at wavelength of 0·9 μm

known[4] as the Goos-Hänchen shift. Thus, although in the simple ray theory the path length in the core of a fibre travelled by a ray making an angle θ to the axis is given by

$$P_2 = n_2 L \sec\theta \qquad (1)$$

the more correct result, taking into account the Goos-Hänchen shift, is

$$P_2' = \frac{n_2 L}{\cos\theta + (\Delta/d)\sin\theta} \qquad (2)$$

where n_2 and n_1 are the refractive indices of the core and the cladding, respectively, L is the length of the fibre and d is the fibre diameter. Sodha *et al.*[3] deduce an expression equivalent to eqn. 2, but then assume that P_2' is the total pathlength of the beam in the fibre, and accordingly obtain the following expression for the time-delay difference Δt_s between the axial ray and a ray at an angle θ to the axis:

$$\Delta t_s = (n_2 L/c)[\{\cos\theta + (\Delta/d)\sin\theta\}^{-1} - 1] \qquad (3)$$

However, this result is incorrect, because it ignores the transit time between the points A and A′, and a revised form of eqn. 3 must be obtained, in which the propagation delay over the displacement path is taken into account.

A complete wave analysis is complex, but it is possible to

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obtain approximate results by some crude assumptions about the effective 'ray' path in the cladding. Thus it is evident that the delay time will be intermediate between that of a wave travelling the direct path AA′, i.e. a distance Δ, and the extreme path ACA′, comprising a distance $(n_1/n_2)\,\Delta \sec\theta$, in the cladding, as in Fig. 1*b*. If it is assumed that the delay time at each reflection corresponds to the path ACA′, the total equivalent optical path in the cladding for all reflections is

$$P_1 = \frac{(n_1^2/n_2)(\Delta/d)\,L\,\tan\theta}{\{\cos\theta + (\Delta/d)\sin\theta\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The total path length in the fibre through the core and the cladding is thus given by

$$\begin{aligned} P &= P_1 + P_2' \\ &= n_2 L\,\frac{1 + (n_1/n_2)^2\,(\Delta/d)\tan\theta}{\cos\theta + (\Delta/d)\sin\theta} \\ &= n_2 L \sec\theta\left[1 - \frac{\{1-(n_1/n_2)^2\}(\Delta/d)\sin\theta}{\cos\theta + (\Delta/d)\sin\theta}\right] \qquad (5a) \end{aligned}$$

On the other hand, if the direct path AA′ in the cladding is assumed, one obtains

$$P = n_2 L \sec\theta\left[1 - \frac{\{1-(n_1/n_2)\cos\theta\}(\Delta/d)\sin\theta}{\cos\theta + (\Delta/d)\sin\theta}\right] \qquad (5b)$$

However, the second term in the brackets in each of these equations is negligibly small compared with the first (unity), for all practical purposes. For example, in our fibres $n_1 = 1{\cdot}48$, $n_2 = 1{\cdot}55$ and, even if we assume Δ to be so large that $(\Delta/d) = 0{\cdot}1$ and that $(\pi/2) - \theta$ is near the critical angle, the second term is only 3×10^{-3} in both cases. In fact, it has been shown[4] that, as the angle of incidence on the core–cladding interface approaches the critical angle, the shift Δ reaches a maximum and constant value, which is always considerably smaller than the effective beamwidth. The latter cannot exceed the core diameter, and hence $\Delta/d \ll 1$. At incident angles that are not close to the critical angle, Δ is even small compared with a wavelength, and thus negligible error is introduced by using eqn. 1.

To check this result experimentally, the propagation delay relative to that of the axial ray has been measured for beams of narrow angular width, over a wide range of input angles of incidence, at wavelengths of 0·633 and 0·9 μm. In one case, the source was a mode-locked He–Ne laser producing 0·6 ns pulses at a rate of 80 MHz, and measurements were made with a beam semiangular width of 0·3° on a fibre of core diameter of 50 μm and length 110 m. In the second experiment, the source was a semiconductor laser producing pulses of 200 ps duration and 0·4° width, and the fibre was of core diameter 100 μm and length 111 m. For both, the core consisted of hexachlorobuta-1, 3-diene in a cladding of glass.* For the propagation path length of eqn. 1, the propagation delay of a ray at an angle ϕ to the axis in air is given by

$$\Delta t = (n_2 L/c)[\sec\{\sin^{-1}(\sin\phi/n_2)\} - 1] \quad . \quad . \quad . \quad (6)$$

where

$$\theta = \sin^{-1}(n_2^{-1}\sin\phi)$$

The normalised propagation delay per unit length is plotted in Fig. 2 as a function of the angle of incidence ϕ. The solid line shows the theoretical curve derived from eqn. 6, and is in excellent agreement with the experimental results for the full range of angles from zero to that corresponding to the critical angle; i.e. over the full numerical aperture of the fibre.

In carrying out these measurements, it is necessary to avoid mode conversion in the fibre, which will reduce the observed propagation delay. For the experiments reported, the fibres were carefully examined and found to exhibit single-mode propagation under the appropriate launching conditions.[6]

We may conclude, therefore, on both theoretical and experimental grounds, that, in the absence of mode conversion, and despite the Goos–Hänchen shift, the equivalent optical path length of a ray in a multimode cladded fibre is given to a good approximation by the simple ray theory, as in eqn. 1.

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