THEORY OF SCATTERING FROM THE CORE OF A MULTIMODE FIBRE WAVEGUIDE

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A theory suitable for the interpretation of measurements of light scattering from cladded multimode optical fibres is presented. The model used is based on geometrical optics and enables the effects of depolarization, the angular distribution of the propagating beam, as well as refraction and reflection at the core–cladding interface to be taken into account. Good agreement is obtained between the theory and experiment.

1. Introduction

In an earlier publication [1] experimental measurements of scattering, as a function of both angle and wavelength, from bulk glass and from multimode cladded-glass fibres were presented. In order to compare the measured results for the fibre with those predicted by theory, corrections are necessary to take account of a number of factors, including the presence of the cladding surrounding the core. Details of the theory and the corrections, together with the associated scattering functions, are now given here for the first time. In a recent paper [2] a simplified analysis is used to calculate the Rayleigh scattering loss of liquids from measurements of the total integrated energy scattered from the fibre. The present theory is more accurate, enabling depolarization in the core, the angular distribution of the propagating beam, as well as reflection and refraction at the interface, to be taken into account. Measurements of scattering from single-mode fibres have been presented by Rawson [3] but details of the analysis leading to the required corrections were not given. However, these could not easily be applied to multimode fibres.

As outlined previously [1], the fibres are initially assumed to be immersed in an index-matching liquid of refractive index equal to that of the cladding. The far-field radiation patterns outside the core from a given 1 cm length of fibre are then calculated for a given scattering distribution in the core. The theory is applicable to cladded multimode fibres of core diameter large compared with the wavelength and is based on a ray analysis which has been successful in explaining many of the propagation properties of fibres [4].

2. Generalized theory

In calculating the corrections to be made to the externally-measured angular distribution of scattering from the core, which arise from the presence of the cladding, the following assumptions are made:

(a) The fibre is cylindrical, straight and of dimensions large compared with the radiation wavelength.

(b) The refractive index of the matching liquid is equal to that of the cladding (in practice the difference can be made less than 0.001).

(c) The propagating radiation has a uniform transverse illumination across the core and at each point has an angular intensity distribution which is symmetrical about the z (i.e. fibre axis) direction. (The latter assumption is unnecessary for low-angle propagation in the fibre.) As will be shown later non-uniform transverse distributions can also be taken into account.

(d) The scattered radiation intensity from each point is symmetrical about the z-direction. This is consistent with the launching of unpolarised light and assumptions (a) and (c).

(e) All rays striking the core–cladding interface at an incident angle less than the critical angle event-

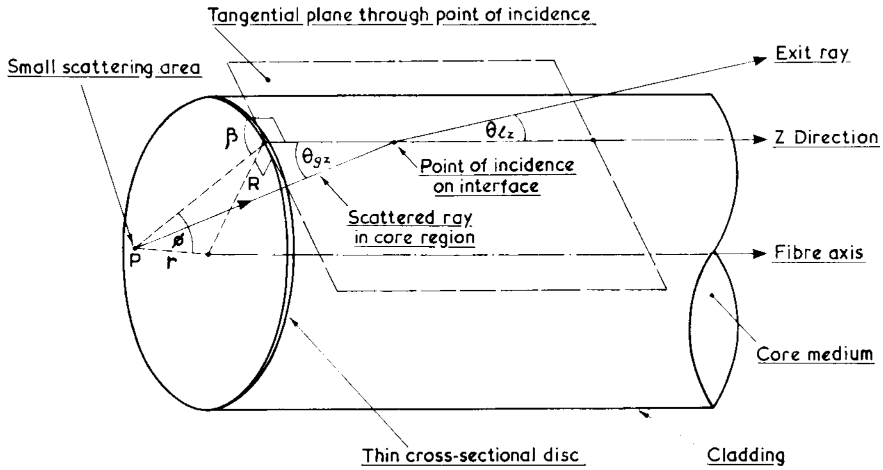

Fig. 1. Path of a ray scattered from a point P at a distance r from the fibre axis in a thin cross-sectional disc.

ually leave the core, albeit after several partial reflections.

The main steps in the analysis are as follows. Firstly, the angle (or range of angles) outside the core, corresponding to a given scattering angle in the core must be found. Secondly, a correction must be applied for the fraction of the radiation scattered in the core that is trapped by total internal reflection at the interface. Finally it is necessary to calculate the "lossless" changes in radial intensity produced by refraction effects at the interface.

From assumption (b) the system reduces to a cylindrical core surrounded by an infinite cladding medium of lower refractive index (fig. 1). The scattering from a cross-sectional disc in the core, of small thickness δz, is then considered. From assumption (c) the incident illumination at any point on the disc may be expressed as:

$I_{\mathrm{i}}(\theta'_{\mathrm{g}z})$ Power/unit area/steradian,

where $\theta'_{\mathrm{g}z}$ is the angle an incident ray makes with the z direction. The scattering function is the same for all points in the disc and may be expressed as $F_1(\theta_{\mathrm{g}z})$, where $\theta_{\mathrm{g}z}$ is the angle a scattered ray makes with the z direction. This gives the far-field scattered intensity distribution if the core region were infinite and the incident radiation were confined to the point. However depending on the position of the scattering point only a certain fraction of this radiation actually leaves the fibre.

Fig. 1 shows a single scattered ray, from a generalized point P, making an angle $\theta_{\mathrm{g}z}$ with the z direction. The angle inside the cladding–liquid after refraction (if total internal reflection does not take place) is given by $\theta_{\ell z}$, again measured between the ray and the z direction. Using Snell's law and simple solid geometry it may be shown that:

$$\cos\theta_{\mathrm{g}z} = (\mu_\ell/\mu_\mathrm{g})\cos\theta_{\ell z}\,, \tag{1}$$

where μ_ℓ, μ_g are the refractive indices of the liquid (or cladding) and the core respectively.

This relation is independent of the position of the point in the disc and any measured scattering angle $\theta_{\ell z}$ outside the core has associated with it a unique angle of scattering within the core.

It is now necessary to evaluate the fraction of the total scattered power from the disc at an angle $\theta_{\mathrm{g}z}$ to the axis that leaves the core. In the limiting condition just before total internal reflection occurs, the exit ray lies in a tangential plane to the interface and the angle $\theta_{\ell z}$ also lies in this plane. For this case, and for fixed $\theta_{\mathrm{g}z}$, let us define the value of the angle β between the projection on the cross-sectional disc of the scattered ray in the core and the tangential plane as β_c where, by straightforward geometrical considerations,

$$\cos\beta_\mathrm{c} = +\sqrt{\frac{\mu_\ell^2(1-\cos^2\theta_{\ell z})}{\mu_\mathrm{g}^2 - \mu_\ell^2\cos^2\theta_{\ell z}}}\,. \tag{2}$$

If $\beta < \beta_c$ the ray is trapped by total internal reflection. As the scattering function at each point is symmetrical about the z direction, the fraction, A, of this radiation leaving the core only depends on the scalar distance, r, from the axis and

$$A(r) = \frac{\text{Range of } \phi \text{ for which } \beta > \beta_c}{2\pi}$$

where ϕ is defined in fig. 1. From the sine rule

$$\sin^2\phi = \frac{R^2 \cos^2\beta}{r^2} \quad ,$$

we find

$$\begin{aligned} A(r) &= 1, && \text{when } r \leqslant R\cos\beta_c, \\ &= \frac{2}{\pi}\sin^{-1}\left(\frac{R}{r}\cos\beta_c\right), && \text{when } r > R\cos\beta_c, \end{aligned} \qquad (3)$$

assuming the principal value of $\sin^{-1}(Rr^{-1}\cos\beta_c)$ is taken. Recalling assumption (c) the proportion of scattered radiation which leaves the core, integrated over the entire disc, is given by:

$$T_1(\beta_c) = \frac{\int_0^R A(r)\, 2\pi r\, dr}{\int_0^R 2\pi r\, dr}. \qquad (4)$$

Substituting (3) into (4) and integrating gives

$$T_1(\beta_c) = 1 - \frac{2}{\pi}\beta_c + \frac{2}{\pi}\cos\beta_c \sin\beta_c. \qquad (5)$$

The function $T_2(\theta_{\ell z}) = T_1(\beta_c)$ is obtained by substitution of the principal value of β_c from (2) into eq. (5) and gives the fraction of transmitted radiation in terms of $\theta_{\ell z}$. It now remains to calculate the lossless radial intensity change due to refraction at the interface. In the core, rays scattered between angles θ_{gz} and $\theta_{gz} + \theta_{gz}$ are contained in a solid angle $\Delta\omega_g$ where

$$\Delta\omega_g = 2\pi \sin\theta_{gz} \Delta\theta_{gz} \text{ for small } \Delta\theta_{gz}. \qquad (6)$$

In the cladding these rays (less any trapped by total internal reflection) leave between the angles $\theta_{\ell z}$ and $\theta_{\ell z} + \Delta\theta_{\ell z}$ in a solid angle $\Delta\omega_\ell$ where

$$\Delta\omega_\ell = 2\pi \sin\theta_{\ell z} \Delta\theta_{\ell z} \text{ for small } \Delta\theta_{\ell z}. \qquad (7)$$

Thus transmission from the core into the cladding produces a reduction in the radial intensity by the factor:

$$\frac{d\omega_g}{d\omega_\ell} = \frac{\sin\theta_{gz}\, d\theta_{gz}}{\sin\theta_{\ell z}\, d\theta_{\ell z}}. \qquad (8)$$

Eq.(8) is obtained by dividing eq. (6) by eq. (7) and letting $\Delta\theta_{\ell z}$ and $\Delta\theta_{gz}$ tend to zero. By differentiating eq. (1) the factor $d\omega_g/d\omega_\ell$ is found to be constant and equal to μ_ℓ/μ_g.

Now using eqs. (1), (2), (5) and (7) it is possible to correct the bulk scattering function $F_1(\theta_{gz})$ in the core to obtain the corresponding scattering function $F_2(\theta_{\ell z})$ outside the fibre by using the relation:

$$F_2(\theta_{\ell z}) = F_1\left[\cos^{-1}\left(\frac{\mu_\ell}{\mu_g}\cos\theta_{\ell z}\right)\right] T_2(\theta_{\ell z}) \frac{\mu_\ell}{\mu_g}. \qquad (9)$$

Thus, subject to the assumptions specified above, eq. (9) can be used for any type of multimode fibre and core scattering function.

3. Fibres exhibiting anisotropic Rayleigh scattering

Taking the particular example of Rayleigh scattering together with parallel unpolarised incident light, and allowing for anisotropic scattering centres [5], the scattering function $F_0(\theta_{gz})$ is given by

$$F_0(\theta_{gz}) = F_0(90^\circ)\left(1 + \frac{1-\rho_u}{1+\rho_u}\cos^2\theta_{gz}\right), \qquad (10)$$

where ρ_u is the depolarisation ratio of the core material. For the simple case of non-parallel incident light where the radial intensity distribution of the illumination is uniform over a cone of half angle θ_0:

$$\begin{aligned} I_i(\theta_{gz}) &= \text{Constant}, && \theta_{gz} \leqslant \theta_0, \\ &= 0, && \theta_{gz} > \theta_0. \end{aligned}$$

By integration of $F_0(\theta_{gz})$ it can then be shown that

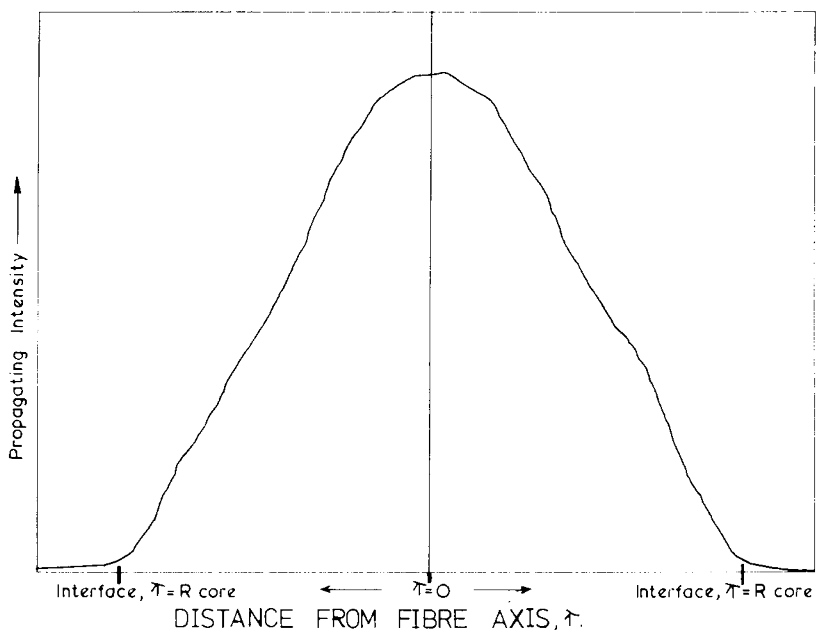

Fig. 2. Transverse intensity distribution of light propagating in a glass fibre of 57 μm core diameter at 0.5 m from the launching end. The white light input beam was focussed to a 12 μm spot at the centre of the core region by a lens of numerical aperture 0.25. The measurement was taken by scanning a projected image of the fibre end with a small-area silicon photodiode. A similar distribution was obtained at 4 m, 8 m and 23 m from the launching end and little change was evident when the white light was filtered.

$$F_1(\theta_{gz}) = F_1(90) \times \frac{1+\frac{1-\rho_u}{1+\rho_u}\left(\frac{\cos^2\theta_{gz}(\cos\theta_0+\cos^2\theta_0)}{2}+\frac{2-\cos\theta_0-\cos^2\theta_0}{6}\right)}{1+\frac{1-\rho_u}{1+\rho_u}\,\frac{2-\cos\theta_0-\cos^2\theta_0}{6}} \qquad (11)$$

The function $F_2(\theta_{\mathrm{gz}})$ outside the fibre may then be found from eqs. (11) and (9).

Rawson [3] has presented curves for Rayleigh scattering from single-mode fibres which are of similar shape to those for the particular multimode fibres treated here, see fig. 3, despite the obvious theoretical differences between the two cases.

4. Experiment

The analysis has been applied to the case of a glass-core fibre made in these laboratories, consisting of a Schott F7 core of 57 μm diameter, refractive index 1.63 and a Chance–Pilkington ME1 cladding 16 μm thick, refractive index 1.49. A value for θ_0 was obtained by measuring the divergence of the output beam when the fibre was cut at the point where scattering measurements had previously been made and the end inserted in an index-matching mount [6]. In practice the intensity distribution across the fibre core may not be uniform and recent measurements have shown that when a spot of 12 μm diameter is focussed onto the fibre end at the centre of the core with a lens of N.A. 0.25 the propagation distribution is more nearly triangular, as shown in fig. 2. (These in fact were the launching conditions for the scattering measurement.) Such cases can be treated by inserting the appropriate function as a factor in the integrals in eq. (4). A comparison of the calculated angular scattering distributions, for both uniform and triangular transverse intensity distributions, is given in fig. 3. It can be seen that there is good agreement with experiment for most angles, particularly for the triangular (i.e. measured) distribution. This agreement also confirms that the scattering in the core is of Rayleigh form. The reason for the small amount of scattering represented by the peaks at large and small angles is still being investigated. The wavelength dependence of the 90° scattered light is given in the earlier publication [1].

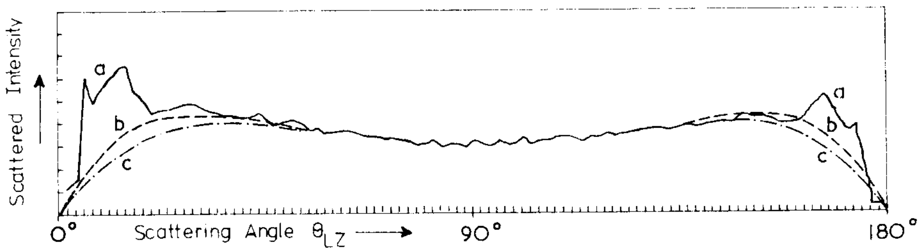

Fig. 3. (a) Measured scattering distribution at 430 nm (launching conditions as in fig. 1) at 1.5 m from the launching end. (b) Calculated Rayleigh curve assuming triangular transverse intensity distribution. (c) Calculated Rayleigh curve assuming uniform transverse intensity distribution. (For cases (b) and (c) the measured depolarisation ratio ρ_u = 0.1 and the measured angular spread of the propagating light θ_0 = 9° were taken into account. The curves are normalised to the experimental curve in the region close to 90°.)

5. Conclusions

An analysis has been made of the corrections to be used when the scattering distribution in the core of a cladded multimode fibre is to be related to the angular intensity distribution outside the cladding. Factors taken into account include the trapping of scattered radiation inside the core and refraction at the core–cladding interface. Results are given for anisotropic Rayleigh scattering but other forms of scattering may be handled in a similar manner. Using the measured power distribution in the core the calculated angular distribution of scattering outside the cladding is in good agreement with experiment.

For a cladded fibre in air (rather than immersed in an index-matching liquid) having a thick cladding, hte power P_t trapped by the cladding–air interface is given by

$$P_t = 2\int_0^{2\pi} d\phi \int_0^{\cos^{-1}(\mu_\ell^{-1})} F_2(\theta_{\ell z}) \sin\theta_{\ell z} d\theta_{\ell z}.$$

This equation differs from that of Stone [2] who uses the isotropic Rayleigh scattering function, $(1 + \cos^2\theta)$, whereas the scattering function $F_2(\theta_{\ell z})$ includes the effect of depolarization, transverse intensity distribution in the core, the finite angular width of the propagating beam, as well as refraction at the core–cladding interface. For an accurate determination of scattering loss it is necessary to obtain measurements for the first three factors. For the more general case where the cladding thickness is comparable with the core diameter the analysis is more complex but can be solved using eqs. (1), (2) and (3) at the cladding–air boundary.

Finally Stone's claim to have reported the first measurement of the Rayleigh (wavelength)$^{-4}$ dependence for liquids over a wide spectral range must be refuted. Such a measurement for benzene (as well as for several glasses) was reported in 1970 by Laybourn, Dakin and Gambling [7] for the wavelength region 0.5 to 1.0 μm.

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