PULSE RESPONSE OF A GRADED-INDEX OPTICAL FIBRE

Indexing terms: Fibre optics, Dispersion (wave)

An analysis has been made of pulse dispersion in an optical fibre having a continuous radial variation of refractive index. Solutions are presented for Selfoc fibre showing the effects of mode and material dispersion and of group delay. The predicted dispersions range from very low values up to about 1 ns/km, depending on the launching conditions.

Introduction: Single-mode cladded fibres are predicted[1, 2] to have bandwidths of tens of gigahertz per kilometre, while, with multimode cladded fibres, bandwidths approaching 1 GHz/km have been measured[3] with a mode-locked helium–neon laser. Another type of fibre is that having a continuous variation in refractive index, of which Selfoc[4] is the best known variety, for which some experimental measurements have been made. We present here the results of the corresponding theoretical analysis.

Consider a continuous cylindrical medium (r, θ, z) in which the relative permittivity varies only in the transverse (radial) direction r as

$$\varepsilon(r) = \varepsilon(0)\{1-(gr)^2+b(gr)^4\} \qquad (1)$$

where $\varepsilon(0)$ is the relative permittivity on the axis, and b and and g are constants. Using a scalar-wave approximation, the transverse electric field E is obtained by substituting eqn. 1 into the appropriate wave equation, which, if bg^4 is small, can be solved using the stationary perturbation theory.

Initially, we make $bg^4 = 0$, and obtain an expression for the transverse electric field of the unperturbed modes. It is then assumed that the perturbation (i.e. making $bg^4 \neq 0$) changes only the phase constant, so that the transverse electric field $E_n(r,z)$ of mode n can be expressed in terms of the normal mode of the perturbed medium. Thus, in terms of the normal modes, the total field amplitude is

$$E(r,z) = \sum_{n=0}^{\infty} A_n E_n(r,z) \qquad (2)$$

where A_n is the mode amplitude, and

$$E_n(r,z) = \pi^{-\frac{1}{2}} w_0^{-1}\{L_n(r^2/w_0^2)\} \exp(-r^2/2w_0^2) \exp(-j\beta_n z) \qquad (3)$$

where

$$\left.\begin{aligned} \beta_n &= k(0)\{1-2(2n+1)g/k(0)+2(3n^2+3n+1)bg^2/k^2(0)\}^{\frac{1}{2}} \\ k(0) &= w_c\{\mu\,\varepsilon(0)\}^{\frac{1}{2}} \\ &\text{and} \\ w_0 &= \{gk(0)\}^{-\frac{1}{2}} \end{aligned}\right\} \qquad (4)$$

is the characteristic spot size of the fibre and $L_n(\ )$ is the Laguerre polynomial of degree n.

Assuming that a Gaussian input beam of spot size w_i is launched along the axis of the fibre,

$$E(r,0) = (\pi^{\frac{1}{2}} w_i)^{-1} \exp(-r^2/2w_i^2) \qquad (5)$$

and, from eqns. 2 and 5,

$$A_n = \frac{2\pi^{\frac{1}{2}}}{w_0}\int_0^{\infty} E(r,0)\, L_n\left(\frac{r^2}{w_0^2}\right) \exp\left(-\frac{r^2}{2w_0^2}\right) r\,dr$$

$$= \frac{2W}{1+W^2}\left(\frac{W^2-1}{W^2+1}\right)^n \qquad (6)$$

where $W = w_0/w_i$.

If the input beam is correctly matched to the fibre, $w_0 = w_i$, so that $W = 1$, and only the $n = 0$ mode in the fibre is excited.

To find the pulse broadening of a given mode n, an input pulse with a Gaussian temporal distribution of carrier frequency ω_c and halfwidth a is assumed. Using Fourier-transform techniques, it can be shown[5] that the output-pulse shape as a function of time is

$$|q_n(t)|^2 = \frac{2}{a^2} \frac{1}{\sqrt{\left\{1+\left(\frac{2z\alpha_{2n}}{a^2}\right)^2\right\}}} \exp\left[\frac{-2(t-z\alpha_{1n})^2}{a^2\left\{1+\left(\frac{2z\alpha_{2n}}{a^2}\right)^2\right\}}\right] \qquad (7)$$

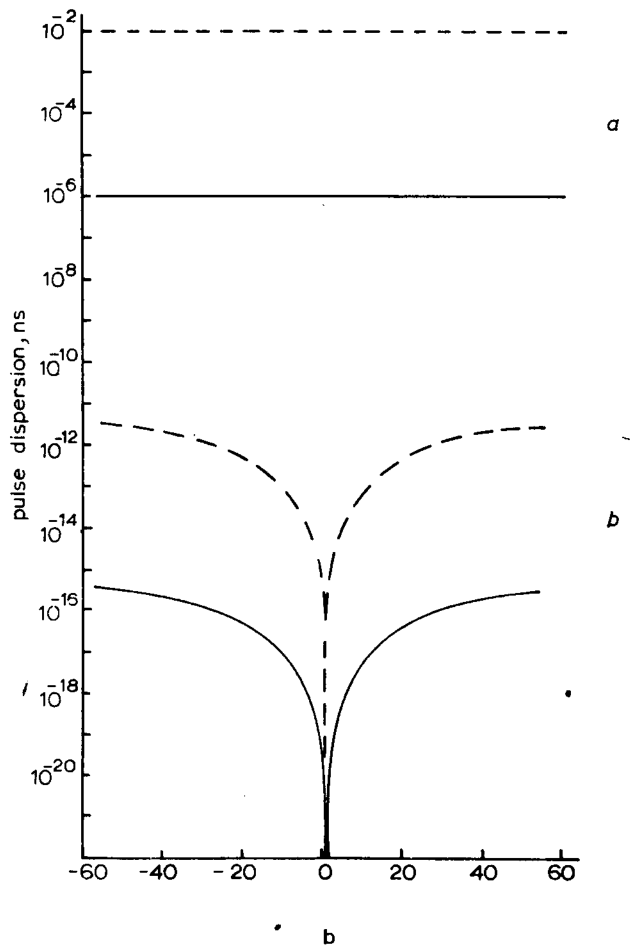

Fig. 1 *Variation of pulse dispersion with b*
a Finite bulk dispersion, $\partial n/\partial \omega \neq 0$
b Zero bulk dispersion, $\partial n/\partial \omega = 0$
—— length of 1 km
– – – length of 100 km

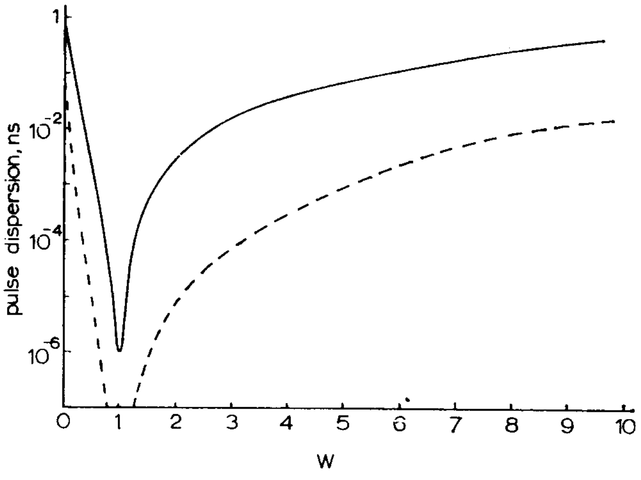

Fig. 2 *Pulse dispersion as a function of W for b = 1 and a length of 1 km*
— — — assumes zero material dispersion

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where

$$\alpha_{m,n} = \frac{d^m}{d\omega^m}\left[k(0) - (2n+1)g - \frac{g^2}{k(0)}\left\{2(1-3b/2)(n^2+n) + (\tfrac{1}{2}-b)\right\}\right]_{\omega=\omega_c} \quad (8)$$

Eqn. 7 shows that, during transmission, the pulse is reduced in amplitude by the factor $1/\surd\{1+(2z\alpha_{2n}/a^2)^2\}$, and the delay time $\Delta\tau$ relative to a plane wave is

$$\Delta\tau = z\alpha_{1n} - n_0 z/c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where n_0 is the refractive index at ω_c, and c is the velocity of light in a vacuum.

Using eqn. 7, the pulse dispersion of a particular mode can be calculated. However, when light is launched into a fibre, the energy is, in general, distributed between many different modes. For a laser (monochromatic) source, it is necessary to take into account, even for a single mode, both mode and material dispersion. Then, when more than one mode is present, it is necessary to determine the dispersion due to group-delay differences between the modes, so that the individual mode amplitudes A_n must be found from eqns. 5 and 6. Then the total pulse temporal distribution is given by

$$|q(t)|^2 = \sum_{n=0}^{\infty} |A_n|^2 \, |q_n(t)|^2$$

$$= \frac{4W^2}{(1+W^2)^2} \sum_{n=0}^{\infty} \left[\frac{W^2-1}{W^2+1}\right]^{2n} \frac{2}{a^2}$$

$$\times \frac{1}{\sqrt{\left\{1+\left(\frac{2z\alpha_{2n}}{a^2}\right)^2\right\}}} \exp\left[\frac{2(t-z\alpha_{1n})^2}{a^2\left\{1+\left(\frac{2z\alpha_{2n}}{a^2}\right)^2\right\}}\right] \quad . \quad . \quad . \quad (10)$$

Results: Dispersions have been computed for mode-locked TEM_{00} helium–neon laser pulses of halfwidth $2a = 0{\cdot}65$ ns and for the parameters of Selfoc glass.[5] When the input spot size w_i is matched to the fibre so that $W = 1$, only the fundamental mode is excited, and Fig. 1 shows the dispersion of this mode as a function of the radial parameter b for two values of material dispersion, namely zero and that of Selfoc. The amount of pulse broadening in the former case depends strongly on b, and is zero when $b = \frac{1}{2}$. However, this value of b is not the ideal one, since it gives rise to a certain amount of mode conversion, and, from this rather more important point of view, the optimum value is $b = 0{\cdot}67$. Nevertheless, the argument is perhaps an academic one, in that, for the wide range of values of b shown, the pulse dispersion is, in any case, negligible, and is completely dominated, in practice, by the effects of material dispersion.

For different beamwidths, $W \neq 1$, and higher-order modes are excited. The pulse dispersion is now determined by the group delay between modes, and Fig. 2 shows, for a fibre length of 1 km, the rapid rise in dispersion with the departure of W from unity. It is clear therefore that, for maximum bandwidth, an input beam having a Gaussian spatial distribution, matched to the fibre and launched axially, must be used.

On the other hand, for large departures from $W = 1$, as would be caused by nonoptimum sources such as multimode (including the GaAs) lasers, the pulse dispersion becomes asymptotic to a value between 0·5 and 1 ns over 1 km. It is interesting to note that experimental values of pulse dispersion in Selfoc fibre[6,7] corresponding to 0·6 and 1·4 ns/km, and therefore in encouraging agreement with Fig. 2, have been reported.

Conclusion: It may be concluded that, for maximum bandwidth in Selfoc fibre, the input beam must have a Gaussian distribution and be matched to the fibre. However, if the input beam is not ideally matched, or if mode conversion occurs, the pulse dispersion can exceed 1 ns/km.

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