BANDWIDTHS OF SINGLE-MODE AND MULTIMODE OPTICAL FIBRE

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Received 22 March 1973
Revised manuscript received 24 April 1973

An analysis is given of the effect of carrier spectral width on bandwidth of optical fibre transmission lines and a simple generalized expression is deduced relating carrier bandspread and maximum permissible pulse rate. It is shown that for the case of a semiconductor laser carrier source the bandwidth of a multimode fibre can be made to approach that expected from single-mode and graded-index fibres.

1. Introduction

Theoretical estimations [1, 2] of the bandwidth, and hence the maximum pulse rate, attainable in single-mode optical fibre waveguides have predicted values in the region of tens of gigahertz over tens of kilometres but have assumed that a single-frequency source is used. However it has been pointed out [3] that in practice the frequency spread in the carrier will have an important influence. The magnitude of the effect has been estimated [4–6] using various approximations but we present here a generalized analysis which results in a simple expression relating maximum pulse rate and carrier bandspread.

In order to illustrate the magnitude of the effect we calculate the pulse rate limitations due to mode dispersion in a single-mode fibre and also due to dispersion in the core material. As shown in previous work the latter predominates. However when a gallium arsenide injection laser in used as the carrier source the effect of carrier bandspread is to reduce the pulse rate from the above values to less than 1GHz over 1 km. Thus the difference in the bandwidths which can be achieved with single-mode, graded-index and multimode fibres [7] in a practical system is appreciably less than has been claimed.

2. Analysis

Consider first the case of a pure, single-frequency carrier. If the fibre dispersion is $\Delta (= \mathrm{d}v_g/\mathrm{d}f)$ $\mathrm{ms^{-1}Hz^{-1}}$ then it is possible to estimate [2] the maximum pulse rate R Hz which it is possible to transmit through a guide of length L m as

$$R = \tfrac{1}{2} v_g (2L|\Delta|)^{-1/2} \quad (1)$$

The pulses are assumed to have unity mark-space ratio, and a relative time delay δT equal to half the pulse width is allowed between frequency components at the extremes of the modulated carrier.

On the other hand, in the more practical case the carrier will have a finite bandspread B_c and the maximum pulse rate R_1 will be less than that given by eq. (1). The frequency range of modulation is thus $2R_1 = B_m$. Let us assume that both carrier and modulation have gaussian amplitude distributions in the frequency domain, of the form (taking no account of phase):

$$F_c(\omega) = \exp\left[-\tfrac{1}{2}(\omega-\omega_c)^2\sigma_c^2\right] \qquad \text{for the carrier;}$$

$$F_m(\omega) = \exp\left[-\tfrac{1}{2}\omega^2\sigma_m^2\right] \qquad \text{for the modulation.}$$

With a single-frequency carrier at ω_c, the amplitude spectrum of the modulated carrier is of the form:

$$|F(\omega)| \propto |F_m(\omega+\omega_c) + F_m(\omega-\omega_c)| \cdot |F_c(\omega_c)|,$$

where $|F_c(\omega_c)|$ is the amplitude of the carrier component at ω_c. The power density spectrum may thus be written

$$P(\omega) \propto |F_m(\omega+\omega_c) + F_m(\omega-\omega_c)|^2 \cdot |F_c(\omega_c)|^2$$

$$\simeq \{|F_m(\omega+\omega_c)|^2 + |F_m(\omega-\omega_c)|^2\} \cdot |F_c(\omega_c)|^2.$$

Because of the symmetry of the modulation spectrum

$$|F_m(\omega+\omega_c)| = |F_m(\omega-\omega_c)|,$$

and

$$P(\omega) \propto |F_m(\omega-\omega_c)|^2 \cdot |F_c(\omega_c)|^2.$$

The power density spectrum for the case of the finite-width carrier is now obtained by integrating over all values of ω_c. Thus

$$P_T(\omega) \propto \int_{-\infty}^{\infty} |F_m(\omega-\omega_c)|^2 \cdot |F_c(\omega_c)|^2 d\omega_c \qquad (2)$$

which is the convolution of the two gaussian functions $\exp[-\omega^2\sigma_m^2]$ and $\exp[-(\omega-\omega_c)^2\sigma_c^2]$. The form of $P_T(\omega)$ is found to be also a gaussian:

$$P_T(\omega) \propto \left(\frac{\pi}{\sigma_m^2+\sigma_c^2}\right)^{1/2} \exp\left(\frac{-\sigma_m^2\sigma_c^2\omega^2}{\sigma_m^2+\sigma_c^2}\right). \qquad (3)$$

Defining bandspread in terms of the e^{-1} points on the amplitude spectrum (e^{-2} points on the power-density spectrum):

$$B_m = 2\sqrt{2}/\sigma_m \text{ and } B_c = 2\sqrt{2}/\sigma_c.$$

The bandspread B_T of the power-density spectrum of the modulated wide-band carrier $P_T(\omega)$ is thus

$$B_T = 2\sqrt{2}(\sigma_m^2+\sigma_c^2)^{1/2}/\sigma_m\sigma_c = (B_c^2+B_m^2)^{1/2}. \qquad (4)$$

Since B_T must now represent the maximum spread of frequencies transmitted through the fibre, the total dispersion of velocities δv_{gmax} may be written:

$$\delta v_{gmax} = B_T|\Delta|.$$

Now

$$\delta v_g = -(L/T^2)\delta T,$$

where T is the average transmission time and δT is the relative time delay between frequency components. If as before we take δT_{max} as half the pulse width then

$$|\delta T_{max}| = 1/4R_1 = 1/2B_m.$$

Hence $|\delta v_{gmax}| = L/2T^2 B_m = v_g^2/2LB_m = B_T |\Delta|$.

Thus

$$B_m B_T = v_g^2/2L|\Delta| = 4R^2, \tag{5}$$

where R is the maximum pulse rate for a single-frequency carrier, from eq. (1). Substituting now for B_T from eq. (4) gives

$$2R_1(B_c^2 + 4R_1^2)^{1/2} = 4R^2,$$

whence

$$R_1^2 = (B_c^4/64 + R^4)^{1/2} - B_c^2/8, \tag{6}$$

or in normalised form:

$$(R_1/B_c)^2 = [\tfrac{1}{64} + (R/B_c)^4]^{1/2} - \tfrac{1}{8}. \tag{7}$$

The variation of R_1/B_c with R/B_c is plotted in fig. 1. As indicated in the figure this expression has two asymptotes; namely for $B_c \gg R$ the permitted normalized pulse rate is $R_1/B_c \simeq 2(R/B_c)^2$, while for $B_c \ll R$ then $R_1 \simeq R$ as expected. In fact no great error (< 20%) is incurred by using the former approximation for $B_c > 2R$ and the latter when $B_c < 2R$, which is a very convenient simplification.

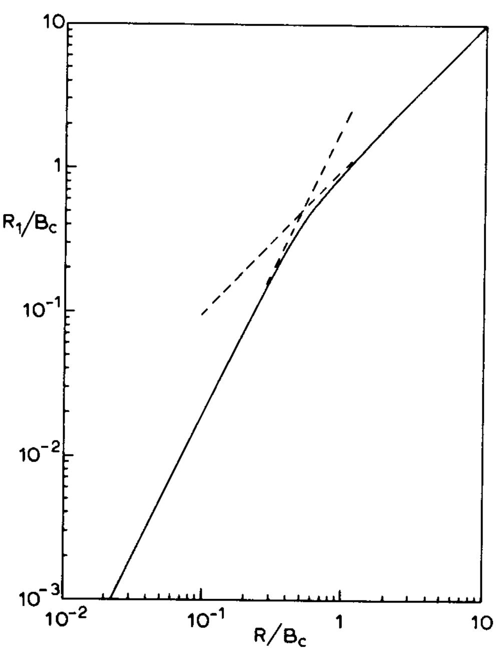

Fig. 1. Normalized maximum permissible pulse rate, R_1/B_c, as a function of the permissible pulse rate, R/B_c, with a single-frequency carrier.

3. Bandwidth of single-mode fibre with monochromatic source

In order to determine the effect of frequency spread in the carrier on the information capacity of an optical fibre communications system it is first necessary to calculate the bandwidth for a monochromatic carrier source. For a single-mode fibre the effect of dispersion of the lowest-order, HE_{11}, mode may be obtained from the solution of Maxwell's equations. For a small difference between the refractive indices of core and cladding, which will be the case in practice, a generalized solution [8] and set of curves may be obtained. Using this result and taking as an example the case of a fibre with a core diameter of 2 μm and refractive index 1.6, together with a cladding having a refractive index 1% lower, then the maximum permissible pulse rate due to mode dispersion is shown by the dashed curve in fig. 2. Thus at the semiconductor wavelength of 0.9 μm the pulse rate R = 28.7 GHz for 1 km.

The second fundamental limitation is that due to the natural dispersion of the material of the fibre. For a bulk refractive index n and free-space wavelength λ the group velocity may be written

$$v_g = \mathrm{d}\omega/\mathrm{d}\beta = c[n - \lambda_0(\mathrm{d}n/\mathrm{d}\lambda)] . \tag{8}$$

Thus

$$\Delta = \frac{-\lambda_0 \mathrm{d}^2 n/\mathrm{d}\lambda_0^2}{n - \lambda_0(\mathrm{d}n/\mathrm{d}\lambda_0)^2} . \tag{9}$$

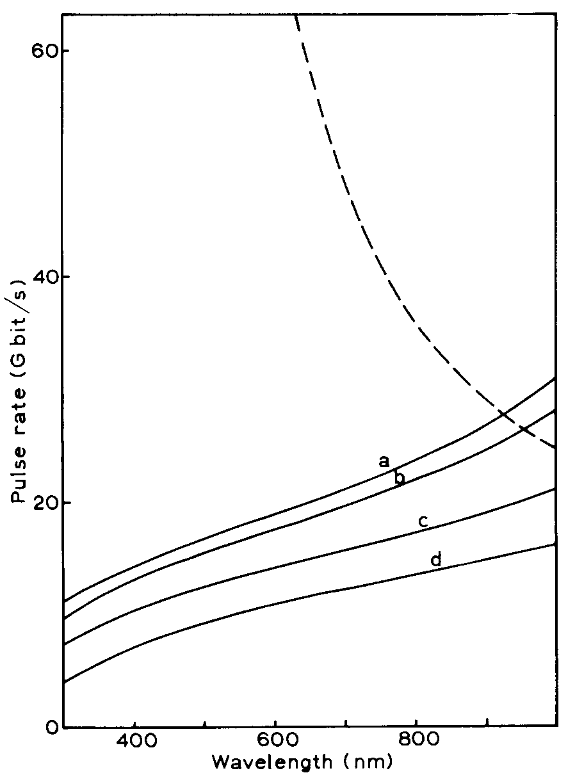

Fig. 2. Maximum permissible pulse rate, as limited only by material dispersion, for silica and various glasses as a function of wavelength. The materials are as follows. (a) Spectrosil; (b) BK5; (c) SSK4; (d) F7. The dashed curve shows the effect of mode dispersion alone in a single-mode fibre of core diamter 2 μm, refractive index 1.6 and core/cladding indices differing by 1%.

The refractive index may be expressed in terms of a polynomial in λ_0, the constants of which are published for many glasses [9] and may be computed for others where the variation of n with wavelength has been measured:

$$n^2 = A_0 + A_1\lambda_0^2 + A_2\lambda^{-2} + A_3\lambda_0^{-4} + A_4\lambda_0^{-6} + A_5\lambda_0^{-8}.$$

Obtaining $dn/d\lambda_0$ and $d^2n/d\lambda_0^2$ from this expression we calculate v_g and Δ from eqs. (8) and (9) and hence the limiting pulse rate R may be found from eq. (1). R is plotted as a function of λ_0 in fig. 2 for silica and several optical glasses.

The limiting pulse rates over 1 km vary from 26.5 GHz for silica to 14.7 GHz for F7 glass at a wavelength of 0.9 μm. It is clear therefore that bulk dispersion is a more serious factor than mode dispersion, although only marginally so far silica, but (in principle) the combined effect of both should be considered. The limitation due to bulk dispersion applies also to graded-index fibres of course. Furthermore, the doping of glass or silica with heavy metal ions to produce a small change of refractive index increases bulk dispersion appreciably.

4. Effect of finite carrier spectral width

Now consider the effect of using a gallium arsenide laser as the source of the carrier. With a driving current only just above threshold and operating in only a single longitudinal mode the linewidth may be 0.04 nm or less [10] which is equivalent to a linewidth of 14.8GHz. If the drive current is increased to three or four times the threshold value in order to achieve a reasonable output power then the linewidth may increase [11] to 2 nm (740 GHz). With operation near threshold the effect of carrier frequency spread is found from eq. (7) to reduce the pulse rate from 14.7 GHz to 13.8 GHz over 1 km; at the higher drive level the pulse rate falls to 0.58 GHz. The realisable bandwidth of a single-mode fibre is therefore completely determined by the linewidth of the semiconductor laser source and is likely to be ~ 1GHz over 1 km.

5. Comparison with multimode fibres

Initially it was thought that the bandwidth of multimode fibres was limited to the equivalent of a few megahertz over 1 km because of the effect of mode conversion due to scattering and inhomogeneities in the fibre. However recent work has shown [7] that in well-made multimode fibres the above effects can be made negligible and the bandwidth is determined mainly by the launching conditions and by mode conversion due to bending of the fibre. Furthermore the inherent dispersion can be reduced, and the bandwidth increased by suitable mode filtering techniques. For example with a bend radius of ~ 1 m and an input gaussian beam of semi-angular width 0.3° the dispersion can be as low as 1.6 psec/m corresponding roughly to a bandwidth ~ 0.5 GHz. This is comparable with that expected of a single-mode fibre and a semiconductor laser source.

However a more valid comparison must be made for the same sources and launching efficiencies. Thus a single-mode fibre will only accept, from a semiconductor (or any other) laser, that radiation which will set up the HE_{11} mode. To a good approximation, and for a core diameter of 2 μm, this corresponds [12] to a gaussian beam of semi-angle about 5°. Under these conditions the bandwidth, as shown in section 4, may be as low as 0.58 GHz over 1 km depending on the drive level. The same launching conditions and efficiency, can be obtained in multimode fibre by using an aperture, conceptually at least, to reduce the beam width to the same value of 5° and the dispersion at a continuous bend radius of 1 m can be [7] as low as 5 ns/km corresponding to a bandwidth of 0.1 GHz, again over 1 km. Compared with this value the effect of bulk material dispersion can be neglected. In practice a fibre will not all be coiled on a drum but will be laid in relatively straight lines with some bends. Moreover the angular width of the input beam could be reduced using a simple (miniature) lens with no loss of launching efficiency. Alternatively the launching efficiency could be increased (without the need of a lens) at the expense of the bandwidth. In practice, therefore, the bandwidth of suitable multimode fibres can be made to approach that of single-mode fibres, for the case of a semiconductor laser source. The former are, of course, easier and cheaper to make and result in fewer handling, launching and jointing problems.

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