LIMITING BANDWIDTH OF A GLASS-FIBRE TRANSMISSION LINE

Indexing terms: High-frequency transmission lines, Optical waveguides, Glass

The effect of natural dispersion on the bandwidth of a glass-fibre transmission line is analysed. It is shown that, if typical optical glasses are used, the limiting pulse rate, in both cladded fibres and those having a parabolic radial variation of refractive index, is not likely to exceed 10 Gbit/s over a distance of 10 km. The effect of the optical breakdown strength of the glass is also considered.

Interest in the possibility of a glass-fibre communication system has led to several studies of the bandwidth of various configurations. Experiments have indicated[1] that pulse rates of tens of megahertz over a distance of 1 km are possible with cladded fibres having a core of 50 μm diameter, while calculations[2, 3] of mode dispersion indicate that the corresponding figure for single-mode fibre is $\sim$1 GHz. More recently, it has been claimed[4, 5] that a single fibre having a parabolic variation of refractive index is capable of an enormously greater bandwidth, since the optical path lengths of the axial ray and all paraxial rays are identical. This conclusion is optimistic on three counts. First, the analysis on which the preceding statement is based neglects the effect of skew rays. Secondly, any imperfections in the fibre are likely to have a significant effect on the dispersion.[6] Thirdly, as has been pointed out previously,[7] the factor which ultimately limits the bandwidth of any fibre transmission line is the natural dispersion of the constituent materials. Obviously the lower the mode dispersion, the larger the relative effect of natural dispersion. It is of interest therefore to make an estimate of this limiting bandwidth.

Since, as the analysis will show, the bandwidth B is small compared with the (optical) carrier frequency f_c, the phase velocity in the range of interest may be assumed to be a linear function of the frequency. If the dispersion is $\Delta\,\mathrm{ms^{-1}\,Hz^{-1}}$ and the allowable bandwidth is B Hz, the spread in signal velocities is $\delta v = B\Delta\,\mathrm{ms^{-1}}$.

If the signal velocity is $v(f) = L/T(f)$, where L is the length of the line and $T(f)$ the transmission time,

$$\frac{\partial v}{\partial T} = -\frac{L}{T^2} \qquad (1)$$

For a unity mark/space ratio, the limiting relative delay δT between frequency components at the extremes of the bandwidth may be taken as equal to one-half of the pulsewidth; thus

$$|\delta T_{max}| = \frac{T^2\,\delta v}{L} = \frac{1}{2B} \qquad (2)$$

from which

$$\delta v_{max} = \frac{L}{2T^2 B} = \frac{v^2}{2LB} = B\Delta$$

or

$$B = \left(\frac{v^2}{2L\Delta}\right)^{\frac{1}{2}} \qquad (3)$$

The refractive index n of most glasses can be expressed[8] in the form

$$n^2 = A_0 + A_1\lambda^2 + A_2\lambda^{-2} + A_3\lambda^{-4} + A_4\lambda^{-6} + A_5\lambda^{-8} \qquad (4)$$

(The constants $A_0, \ldots, A_5$ give in the Schott catalogue are calculated for values of λ quoted in microns and must be corrected when, as in this analysis, the more standard unit of the metre is used.)

Now $\lambda = c/f$, and we may write

$$v = \frac{c}{n} = \frac{\lambda f}{n}$$

from which

$$\Delta = \frac{dv}{df} = \left(\frac{\lambda}{n}\right)^2 \frac{dn}{d\lambda} \qquad (5)$$

Eqn. 4 gives n, and differentiation results in

$$\frac{dn}{d\lambda} = \tfrac{1}{2}(A_0 + A_1\lambda^2 \ldots)^{-\frac{1}{2}}$$
$$(2A_1\lambda - 2A_2\lambda^{-3} - 4A_3\lambda^{-5} - 6A_4\lambda^{-7} - 8A_5\lambda^{-9})$$
$$= n^{-1}(A_1\lambda - A_2\lambda^{-3} - 2A_3\lambda^{-5} - 3A_4\lambda^{-7} - 4A_5\lambda^{-9})$$
$$= n^{-1}F(A)$$

Hence

$$\Delta = \frac{\lambda^2}{n^3}F(A) \qquad (6)$$

and the maximum pulse rate R is given by

$$R = \tfrac{1}{2}B = \left\{\frac{nf_c^2}{8LF(A)}\right\}^{\frac{1}{2}} \qquad (7)$$

Values of R have been calculated from eqn. 7 for several typical glasses over a range of wavelengths (carrier frequencies) for a length L = 10 km, and the results are shown in Fig. 1.

The pulse rate is a maximum in the wavelength region of approximately 800–1000 nm, corresponding to the output wavelength of injection lasers, and where loss due to scattering is small. The maximum rate over 10 km is between 7 and 10 Gbit/s, depending on the glass, and these values are comparable with those allowed by mode dispersion in a single-mode cladded glass fibre. These results show that there is a finite limit on the bandwidth of a fibre having a parabolic variation of refractive index. For comparison, the limiting pulse rate due to mode dispersion in a 2 μm-core-diameter cladded glass fibre (for a refractive-index difference between core and cladding of 1%) in the region where only the HE_{11} mode can propagate has a minimum value of 6 Gbit/s. Fig. 2 shows the maximum pulse rate computed from the dispersion characteristics[3] of the HE_{11} mode.

Bearing in mind the greater influence of inhomogeneities on the graded index fibre, it does not appear to have any great advantage over the cladded fibre for communications purposes, unless it can be made from glasses having appreciably smaller dispersion than those listed in Fig. 1. On the other hand, the ion-diffusion technique could make the manufacture of low-loss fibres easier, since it obviates the necessity for an interface where impurities, inclusions and inhomogeneities can have an appreciable effect on the attenuation.

It is also interesting to consider the effect of the dielectric strength of the glass on the system bandwidth. The surface breakdown power is much smaller than that of bulk glass, and a typical value[9] is 500 kW/mm². The maximum core diameter for a single-mode cladded fibre at 0·9 μm is roughly 3 μm (again assuming a refractive-index difference between core and cladding of 1%). If, as a first approximation, we assume that 50% of the power is contained in the core, calculation shows that the maximum permissible power in a single pulse (neglecting the effect of heating) is 7 W. If we now assume that the minimum level to which a signal is allowed to fall before being amplified is a factor of ten greater than the limiting photon noise, i.e. $10hfB$ where h is Planck's constant, then, for a bandwidth of 10 GHz (corresponding to a pulse rate of 5 Gbit/s), the maximum permissible attenuation is 85 dB. For a repeater spacing of 10 km, this would require a fibre attenuation of 8·5 dB/km, or, conversely, for a fibre with an attenuation of 20 dB/km, the repeater spacing would have to be roughly 4 km. If smaller pulse powers are used, the pulse rate will be correspondingly reduced. In practice, the limiting factor is likely to be the peak power available from suitable optical sources, and this will apply to cladded and graded index fibres equally.

It may be concluded that the choice for an optical transmission line between cladded and graded index fibres is not a simple one and will have to be made on factors other than calculated mode dispersions.

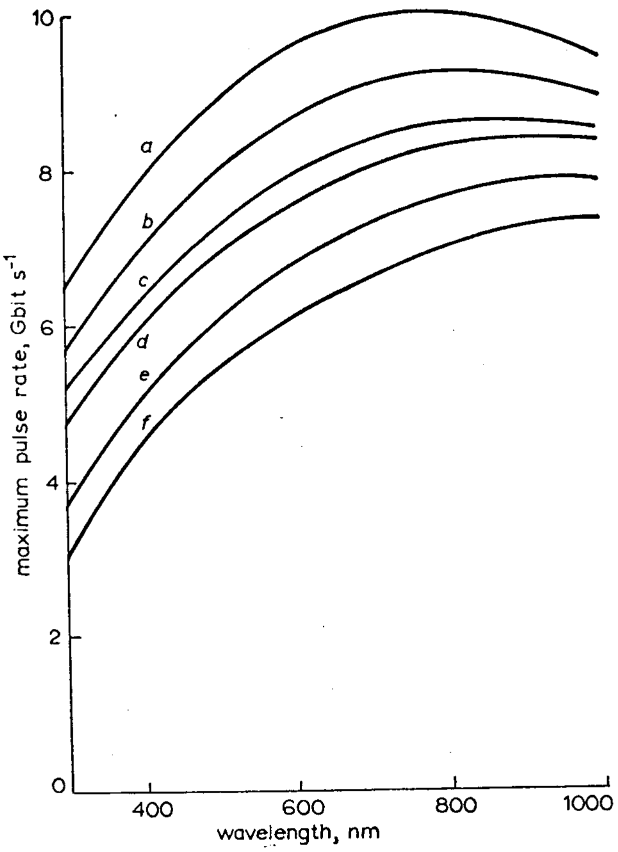

Fig. 1 *Limiting pulse rate due to natural dispersion of several glasses as a function of carrier wavelength*
a FK5 *b* BK5 *c* SK7
d SSK4 *e* LF5 *f* F7

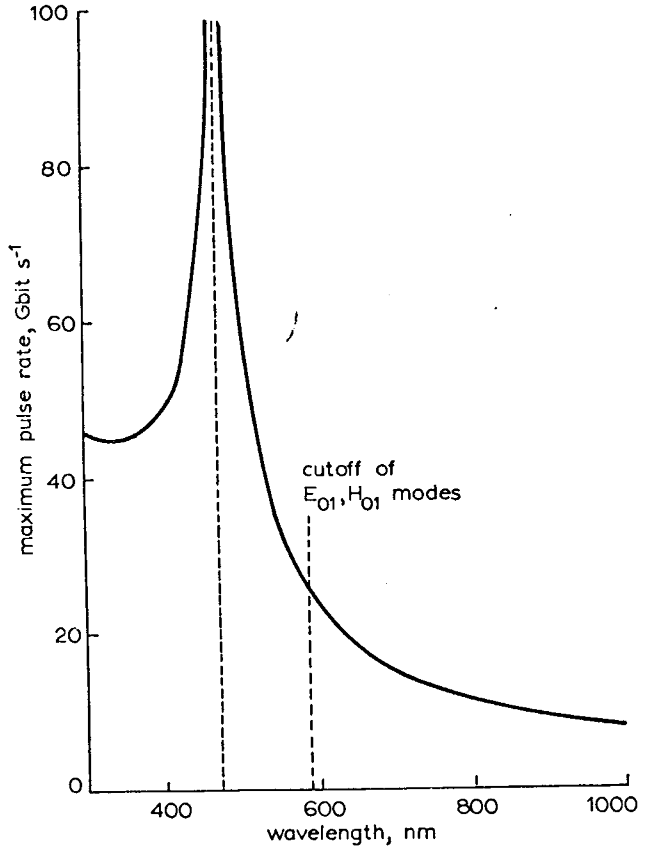

Fig. 2 *Limiting pulse rate due to mode dispersion for HE_{11} mode transmission in a 2 μm-core-diameter fibre, 10 km long*
Core refractive index = 1·6; $\Delta n = 10^{-2}$

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References

1 GAMBLING, W. A., LAYBOURN, P. J. R., and LEE, M. D.: 'Pulse propagation along glass fibres', *Electron. Lett.*, 1970, **6**, pp. 364–365
2 KAO, K. C., and HOCKHAM, G. A.: 'Dielectric-fibre surface waveguides for optical frequencies', *Proc. IEE*, 1966, **113**, pp. 1151–1158
3 LAYBOURN, P. J. R.: 'Group velocity of dielectric waveguide modes', *Electron. Lett.*, 1968, **4**, pp. 507–509
4 UCHIDA, T., FURUKAWA, M., KITANO, I., KOIZUMI, and MATSUMURA, H.: 'Optical characteristics of a light-focusing fiber guide and its applications'. Presented at the conference on laser engineering and applications, Washington, DC, 1969
5 KAWAKAMI, S., and NISHIZAWA, J.: 'An optical waveguide with the optimum distribution of the refractive index with reference to waveform distortion', *IEEE Trans.*, 1968, **MTT-16**, pp. 814–818
6 MARCATILI, E. A. J.: 'What kind of optical fiber for long-distance transmission?' Presented at the SPIE conference on fiber optics, 1970
7 GAMBLING, W. A., and LAYBOURN, P. J. R.: 'Communicating with light', *Sci. J.*, 1969, **5** A, pp. 40–46
8 Optical Glass Catalogue, Jenaer Glaswerk Schott and Gen., Mainz, 1970
9 DAVIT, J.: 'Mechanism for laser surface damage of glasses', *J. Appl. Phys.*, 1968, **39**, pp. 6052–6056

LEAKAGE FROM BALANCED FEEDERS

Indexing terms: Magnetic leakage, High-frequency transmission lines

The findings of Martin on field reduction due to twisting of feeders are compared with theoretical and experimental results obtained by Baruah for fields at distances much greater than the pitch of the helix.

In connection with radiocommunication in mines and tunnels, Martin mentioned[1] that his results on the twisting of balanced feeders contradicted the findings of some American workers. It may be of interest to compare some theoretical and experimental work on the low-frequency field from twisted pairs which was carried out in this department by Baruah.[2] His analysis showed that, for pitch of twist s and radius a of the helix described by one of the conductors (so that a is approximately half the distance between a pair before twisting if $s \gg a$), the maximum magnetic field from unit current at a distance ρ large compared with s should be

$$B_{max} = \frac{\pi\mu a}{\sqrt{(\rho s^3)}}\left(2 + \frac{3\pi^2 a^2}{s^2}\right) e^{-2\pi\rho/s} \quad . \quad . \quad . \quad . \quad (1)$$

For $\rho \gg s$, the exponential term is dominant, and the field decays most rapidly for small s, i.e. for tight twisting. There are, however, two caveats in addition to $\rho \gg s$.

(*a*) Twisting the pair introduces components of field in directions in which the field was previously zero, e.g. parallel with the direction of run of the feeder.