Now consider the response, under closed-loop feedback control, of a system satisfying both these conditions to a set of step changes in the reference inputs. Thus let the reference input transform vector be

$$r(s) = \frac{1}{s}\bar{r}$$

where $\bar{r}$ is a vector whose elements are real constants. The conditions (eqns. I and J) on the behaviour of $R(jw)$ at low and high values of frequency, together with a use of the final-value and initial-value theorems of Laplace transform theory, then enable us to infer the initial and final values of system controlled outputs as

$$y(0+) = Q(\infty)\bar{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (K)$$

$$y(\infty) = H^{-1}(0)\bar{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (L)$$

Eqn. K shows that any multivariable feedback system for which $Q(s)$ has significant diagonal terms at high frequencies will be prone to interactive crosstalk immediately following a reference input transient. The example system[A] discussed is particularly prone to this, as $G(s)$ has one entry which tends to a constant at high frequencies. Thus the high-frequency short-duration breakthrough term is enormous as it has the full loop gain, so to speak, behind it. In mitigation of this it should be pointed out that any realistic model for a practical system would have $G(\infty) = 0$, and thus a very much more attenuated interaction term would result from a design study on a practical system. Furthermore this particular transient term will tend to die away relatively quickly. Nevertheless the computational disadvantages of spectral analysis together with the points discussed above combine to make the direct application of a commutative technique impracticable.

Allwright[C] has pointed out that most of the computational difficulties involved in the spectral analysis aspects of commutative control may be avoided by using other diagonal forms such as the Smith–MacMillan form. This is essentially a return to techniques previously explored by Rosenbrock.[D] The above discussion shows that the phenomenon described by Layton would still remain as a disadvantage although, if $Q(\infty) = 0$, the interaction terms may not be too severe. It would be useful and interesting to examine this in detail, since the ability to monitor directly closed-loop stability remains as an advance on the original Rosenbrock[D] methods.

Since, as discussed by Rosenbrock,[D] the direct diagonalisation of $Q(s)$ is known to suffer disadvantages as a design method, we are led to conclude that, if noninteraction is an important feature of the design specification for a feedback system, eigenvalue-locator design techniques[B] such as the inverse Nyquist array[E] method seem currently to offer the most practicable route to a workable design procedure.

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PULSE PROPAGATION ALONG GLASS FIBRES

Indexing terms: Optical waveguides, Optical propagation effects

Mode-locked helium–neon lasers have been used to propagate pulses of ~1 ns duration along multimode cladded glass fibres. Any pulse spreading due to dispersion in a 33 m length of fibre is less than 0·5 ns which is the limit of resolution of the measuring equipment. This result indicates that a pulse transmission rate of at least 33 MHz may be possible over a distance of 1 km.

Since the advent of the laser, the possibility of obtaining a communication system of large capacity by using an optical carrier has been of great interest. Assuming that suitable sources, modulators and detectors can be developed, there remains the problem of the propagation medium. In outer space, propagation over considerable distances is possible since the only limitations are those of diffraction and alignment of the transmitting and receiving systems. However, for terrestrial communication the optical carrier must be protected from temperature gradients and turbulence in the atmosphere. One possibility[1,2] is to provide periodic refocusing of the beam using a tubular guiding system which must either be evacuated or maintained at a constant uniform temperature. In this way, a system of considerable bandwidth and low transmission loss may result but the cost of development, installation and maintenance will be high. Such a beam-guiding system might be suitable, in the future, for transcontinental routes where a high rate of information transfer is required between points many hundreds of miles apart. On the other hand, in the United Kingdom the requirement is more likely to be for a cheaper system having a bandwidth which is more modest but nevertheless much greater than any presently available. Thus there is great interest in propagation along cladded glass fibres.[3] If the attenuation loss can be reduced to a reasonable value, a cheap small versatile glass cable may result having a bandwidth of ~ 1 GHz over distances ~1 km.

Suitable single-mode fibre is not yet available and there is interest in the bandwidth which might be possible with a multimode fibre. Calculation of the dispersion is not easy because the effects on mode initiation and mode conversion of inhomogeneities, bends, scattering etc. in a practical fibre are not known, and it is necessary to rely on experiment. Initial measurements were made with pulses of 50 ns duration generated by a gallium-arsenide laser operated at 77 K with an output wavelength of 0·847 μm. The authors[4] observed a pulse broadening of 3 ns after propagation through 30 m of fibre with a core of 80 μm diameter and a cladding thickness of 5 μm, although they note that the effects are close to the limits of experimental observation. We have made measurements on a similar fibre but using the output of mode-locked helium–neon lasers producing pulses of less than 2 ns width, thus giving a much higher time resolution.

The fibre was 33 m long with a 50 μm core* ($n = 1{\cdot}611$) and a cladding of thickness 15 μm. The ends of the fibre were sealed into fine-bore capillary tube and polished flat. The laser output was focused onto the fibre with a 50 mm lens with a positional accuracy of 5 μm and an angular accuracy of 0·1° by means of a micromanipulator.

Two types of mode-locked helium–neon laser were used. The first laser† (nominally 75 mW) had passive mode locking. The axial modes self-locked preferentially at the rate $c/L = 167$ MHz, where $c = 3\times10^8$ m/s and L is the cavity length. Under these conditions, the pulse power was 30 mW and the measured pulse half-width of less than 2 ns was limited by the detector response. Because stable mode locking is not always achieved by passive means, the results were checked by using the second laser which was actively mode-locked using an electroacoustic delay line as a loss modulator. The resonator had a plane mirror and a concave mirror of radius 3 m separated by 2·5 m. The delay line, consisting of a block of fused silica with a quartz transducer mounted at one end, was inserted in the laser cavity near the plane mirror and driven by a 30 MHz stable oscillator. Owing to acoustic standing waves in the silica, the resonator loss was modulated by diffraction, by the acoustic grating so formed, at the rate of 60 MHz. Precise tuning to the intermode spacing of the laser was effected by movement of the plane mirror. The resulting pulse power was 7·5 mW at the rate of 60 MHz.

The detector was a 56 CVP photomultiplier, and the shape

* Schott type F7 glass
† Spectra Physics model 125

of the output pulses from the lasers were checked with a faster p–i–n silicon photodiode. Unfortunately the latter was not sufficiently sensitive to detect the pulses after transmission through the fibre. The output from the detector was fed either to a sampling oscilloscope* or to a spectrum analyser.†

The displays of the pulse shapes indicate that, if any pulse broadening is produced by transmission through the 33 m of fibre, the amount is not more than 0·5 ns and is certainly much less than the 3 ns reported earlier. These results indicate that, if the same relative dispersion occurs in a 1 km length of fibre, the pulse broadening would be not greater than 15 ns at 0·633 nm. Thus, if a fibre of sufficiently low attenuation were available, a pulse transmission rate over 1 km of more than 33 MHz would be possible.

Components of the beat spectrum of the pulses at frequencies up to 855 MHz were also displayed. Only the components above 770 MHz were missing after transmission through the fibre, which is in agreement with the pulse-broadening measurements. In subsequent work we hope to measure the relative phase change between the Fourier components as a further check on dispersion in the fibre.

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PROBABILITY-DENSITY FUNCTION OF A SPECULARLY FADING SIGNAL-PLUS-NOISE MINUS NOISE

Indexing terms: Noise, Fading, Error statistics

A general expression for the probability-density function of a specularly fading signal-plus-noise minus noise has been obtained. Such an expression is frequently required for the analysis of different types of communication systems such as f.s.k. systems or pulse radar systems.

Introduction: For the analysis of different types of communication and radar systems, knowledge of the probability-density function (p.d.f.) of signal-plus-noise minus noise is necessary. Marcum[1] obtained such an expression for nonfading signals. However, signals after propagation through a fading transmission medium will in general consist of a specular (or nonfading) component together with a fading component. For such a case, Marcum's results are not applicable. Such cases are generally treated in a roundabout way depending on the application. In this letter, we shall determine the probability-density function of a specularly fading signal-plus-noise minus noise using a new and direct method. Such an expression has direct application in the analysis of frequency-shift keying systems or pulse radar systems.

Determination of probability-density function: In general, signal-plus-noise may be represented as

$$s(t) = \underbrace{A\cos(\omega_0 t+\psi_0)+Q(t)\cos\{\omega_0 t+\psi(t)\}}_{\text{signal}} + \underbrace{r(t)\cos\{\omega_0 t+\theta(t)\}}_{\text{noise}} \quad (1)$$

where the first term represents the nonfading specular component and the second term represents a fading component assumed to have Gaussian distribution. Out of the total signal power σ_s^2, let $m\sigma_s^2 = A^2/2$ = power in the specular component and $n\sigma_s^2$ = power in the fading component where $n = 1-m$ and $m \leqslant 1$. The third term in eqn. 1 represents the narrowband Gaussian noise so that its amplitude has a Rayleigh distribution and its power $= \sigma_n^2$. The fading component of the signal can be considered together with noise as noise for the sake of analysis. This approach was first used by Murarka[2] for treating the case of purely fading signals. Using the same approach for the case of specularly fading signal (Reference 3), $s(t)$ may be considered to consist of a specular component of power $m\sigma_s^2$ and a noise component of total power $N = (\sigma_n^2+n\sigma_s^2)$.

The p.d.f. of the amplitude x_1 of $s(t)$ is given by the well known Rician function

$$p(x_1) = \frac{x_1}{N}\exp\left(-\frac{x_1^2+A^2}{2N}\right)I_0\left(\frac{x_1 A}{N}\right) \quad \text{for} \quad x_1 \geqslant 0 \quad (2)$$

The p.d.f. of the signal amplitude after square-law detector is

$$p(y_1) = \frac{1}{2aN}\exp\left(-\frac{y_1+aA^2}{2aN}\right)I_0\left\{\frac{A}{N}\sqrt{\left(\frac{y_1}{a}\right)}\right\} \quad \text{for} \quad y_1 \geqslant 0 \quad (3)$$

where $y = ax^2$ is the square-law detector characteristics.

The amplitude x_2 of noise alone has Rayleigh p.d.f. which changes to the following exponential function after square-law detection:

$$p(y_2) = \frac{1}{2a\sigma_n^2}\exp\left(-\frac{y_2}{2a\sigma_n^2}\right) \quad \text{for} \quad y_2 \geqslant 0 \quad (4)$$

For determining the p.d.f. for signal-plus-noise minus noise, a new variable z given by $z = y_1 - y_2$ is to be considered. The subtraction of the variable y_2 from y_1 is equivalent to the addition of the negative variable $-y_2$ to y_1. Thus, $z = y_1+(-y_2)$. The p.d.f. of z is obtained by convoluting the p.d.f.s of y_1 and $-y_2$. Assuming that these variables are uncorrelated, $p(z)$ is given by

$$p(z^*) = \frac{\exp[-\{xq/(1+q)\}]\exp(-|z^*|)}{1+q} \quad \text{for} \quad z^* \leqslant 0$$

$$= \frac{\exp[-\{xq/(1+q)\}]\exp(+z^*)}{1+q} \times Q\left\{\sqrt{\left(\frac{2x}{1+q}\right)},\ 2\sqrt{\left(\frac{z^*(1+q)}{2q}\right)}\right\} \quad \text{for} \quad z^* \geqslant 0 \quad (5)$$

where

$$z^* = \frac{z}{2a\sigma_n^2} \qquad x = \frac{mR}{1+nR} \qquad q = 1+nR \quad . \quad (6)$$

and $R = \sigma_s^2/\sigma_n^2$ = total-signal-power/total-noise-power ratio, and

$$Q(\alpha, \beta) = \int_\beta^\infty v\exp\{(v^2+\alpha^2)/2\}I_0(\alpha v)\,dv \quad . \quad . \quad (7)$$