UNIVERSITY OF SOUTHAMPTON

FACULTY OF ENGINEERING, SCIENCE AND MATHEMATICS

School of Civil Engineering and the Environment

Two-dimensional Cut Plan Optimization for Cutter Suction Dredgers

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ABSTRACT

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by M.J.M de Ruyter

Optimal cut plans for cutter suction dredgers aim to maximize operational efficiency. Maximizing operational efficiency involves minimization of stoppage time resulting from non-productive dredger movements. To automate a systematic search for optimal two-dimensional cut plans for cutter suction dredgers two models with an adaptive simulated annealing-based solution approach were developed.

The first model, the dredge cut nesting model, optimizes irregular stock cutting problems where stencils represent dredge cuts and sheets represent dredging areas. Stencils are collections of unit dredge cuts with dimensions related to an effective cutting width which can be achieved with the cutter suction dredger considered. The objectives of the dredge cut nesting model are to maximize sheet coverage and to minimize stencil overlap. Centroids of unit dredge cuts of final nest layouts are extracted and used as grid nodes in the second model.

The second model, the dredger routing model, optimizes asymmetric travelling salesperson problems with turning costs. The objectives of the dredger routing model are to minimize total route length and sum of turning angles, and to maximize average link length. A link consists of two or more route edges which are aligned with each other to within specified limits.

A significant result of this research is that an engineering application of both models showed that two-dimensional cut plans for cutter suction dredgers can be systematically optimized and that dredger routes with minimum turning costs can be found. However, results also showed that the dredger routing model is not yet sophisticated enough to find cut plans for cutter suction dredgers for which overall project execution time is minimal.

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Transportation Problems (pp15-16)

x_{ij}	Number of units transported from one location to another, decision variable.
Z	Total transportation cost.
c_{ij}	Unit transportation cost, cost to transport a unit from one location to another.
a_i	Supply of transportation units available at a source.
b_j	Demand of transportation units required at a destination.

Earthwork Transportation Problems (p16)

x_{ij}	Volume of earth transported from one location to another, decision variable.
Z	Total haul.
c_{ij}	Transport distance.
a_i	Cut volume available at a source.
b_j	Fill volume required at a destination.

Assignment Problems (p22)

x_{ij}	Assignment of an agent to a task, decision variable.
c_{ij}	Cost of assigning an agent to a task.

Dredged Material Disposal Modelling (p23)

V	Total volume of dredged material in a disposal area.
F	Reduction factor applied to total volume of dredged material in a disposal area to allow for shrinkage, swelling or bulking.

Nesting Problems (pp35-36)

A_{escape}	Total escape area, the total area of stencils situated outside sheet profile(s), decision variable.
$A_{overlap}$	Total overlap area, the total area of overlap between stencils, decision variable.
$A_{non\text{-}placement}$	Total non-placement area, the total area of sheet profile(s) not covered by placed stencils, decision variable.
α_{escape}	Escape penalty factor, to influence the weight of total escape area in the objective function.
$\alpha_{overlap}$	Overlap penalty factor, to influence the weight of total overlap area in the objective function.
$\alpha_{non\text{-}placement}$	Non-placement penalty factor, to influence the weight of total non-placement area in the objective function.
$s_{escape\ i}$	Area of a stencil situated outside the sheet profile(s).

L_{area}	Total area of sheet profile(s) in which stencils are to be placed.
s_i	Area of a stencil.
$s_{overlap\ ij}$	Area of overlap area between two stencils.

Travelling Salesperson Problems (pp38-39)

x_{ij}	Arrival at a city from another, problem decision variable.
d_{ij}	Distance between two cities.

Simulated Annealing Theory (pp50-51)

$P(\Delta E)$	Probability of a thermodynamic system in equilibrium assuming a higher energy state.
ΔE	Increase in energy for a thermodynamic system in equilibrium to assume a higher energy state.
k	Boltzmann's constant, used for coping with different materials.
T	Temperature or cost annealing temperature.
P	Acceptance probability for a modelled system assuming a worse state.
ΔL	Change in objective function value.
r	Random number between 0 and 1.

Simulated Annealing Algorithm Pseudo-code (p53)

ω	Initial or current solution.
ω'	Modified solution.
Ω	Solution space comprising all possible solutions.
p	Temperature change counter.
$T(p)$	Temperature cooling schedule as a function of temperature change counter.
$T(0)$	Initial (cost) temperature.
$N(p)$	Repetition schedule as a function of temperature change counter.
q	Repetition counter.
$g(T)$	Probability generation function used for modifying a solution.
$\eta(T)$	Neighbourhood function used for modifying a solution.
δ	Difference in objective function cost of initial/current and modified solution.
$f(\omega)$	Total objective function cost of initial or current solution.
$f(\omega ')$	Total objective function cost of modified solution.

Adaptive Simulated Annealing Theory (pp54-58)

k	Cost annealing time.
α^i_n	Value of a problem parameter in a dimension at current cost annealing time.
A_i	Lower limit of the finite range of values a problem parameter can assume.
B_i	Upper limit of the finite range of values a problem parameter can assume.
y_i	Random variable between -1 and 1.
u_i	Random variable between 0 and 1.
t_{0i}	Initial problem parameter temperature.
t_{in}	Current problem parameter annealing temperature.
n	Parameter annealing time.
$\mathrm{sgn}(x)$	Sign function: Equal to -1 for all x < 0; 0 for x = 0; and 1 for all x > 0.
c_i	Parameter tuning factor, to influence annealing of the problem parameter temperature.
m_i	Control coefficient, to influence the value of the parameter tuning factor.
n_i	Control coefficient, to influence the value of the parameter tuning factor.
D	Total number of dimensions of problem parameter space or problem parameters.
$\partial\alpha_i$	Small change in problem parameter value.
s_i	Problem parameter sensitivity.
s_{max}	Largest problem parameter sensitivity found during sensitivity analysis.
∂L	Change in objective function value resulting from $\partial\alpha_i$.
n'_i	Rescaled problem parameter annealing-time.

Stencil Motion (p66)

x_i	Initial or current x-coordinate of stencil reference point.
x_{i+1}	Modified x-coordinate of stencil reference point.
y_i	Initial or current y-coordinate of stencil reference point.
y_{i+1}	Modified y-coordinate of stencil reference point.
δ_x	Translation of stencil reference point in x-direction.
δ_y	Translation of stencil reference point in y-direction.
δ_r	Rotation of stencil around stencil reference point.

Dredge Cut Nesting Problems (p67)

A_{escape} Total escape area, the total area of dredge cuts situated outside sheet profile(s), decision variable.

$A_{overlap}$ Total overlap area, the total area of overlap between dredge cuts, decision variable.

$A_{non\text{-}placement}$ Total non-placement area, the total area of sheet profile(s) not covered by placed dredge cuts, decision variable.

s_i Area of a dredge cut, represented by a stencil.

e_j Area of an escape region where dredge cuts can be placed outside the dredging area(s).

L_k Total area of dredging area(s), represented by sheet profile(s).

α_{esc} Escape penalty factor, to influence the weight of total escape area in the objective function.

α_{ovl} Overlap penalty factor, to influence the weight of total overlap area in the objective function.

α_{npl} Non-placement penalty factor, to influence weight of total non-placement area in the objective function.

β_{esc} Escape penalty exponent, to influence the weight of total escape area in the objective function.

β_{ovl} Overlap penalty exponent, to influence the weight of total overlap area in the objective function.

β_{npl} Non-placement penalty exponent, to influence weight of total non-placement area in the objective function.

Dredger Routing Problems (pp69-70)

x_{ij} Arrival of dredger at a route node from another, decision variable.

y_{ijk} Arrival and departure of dredger at a route node from/to other nodes, decision variable.

L_{TOUR} Total length of a dredger route.

d_{ij} Distance between two nodes visited in a dredger route.

F_j Reduction factor applied to the route edges which make up a dredger route.

A_{TOUR} Sum of turning angles in a dredger route, measured between consecutive route edges.

$\Delta\gamma_j$ Difference in plane angles of incoming and outgoing route edge at a node visited in a dredger route.

α_{length} Length penalty factor, to influence the weight of total route length in the objective function.

α_{angle} Angle penalty factor, to influence the weight of the sum of turning angles in the objective function.

β_{length}	Length penalty exponent, to influence the weight of total route length in the objective function.
β_{angle}	Angle penalty exponent, to influence the weight of the sum of turning angles in the objective function.

<u>Optimal Dredger Routes on Regular Square Grids (p72)</u>

L_{MIN}	Minimum dredger route length.
A_{MIN}	Minimum sum of turning angles.
d	Square grid spacing.
n_L	Number of nodes along the length of a continuous rectangular grid.
n_W	Number of nodes along the width of a continuous rectangular grid.
M_{MAX}	Maximum link length, the maximum number of route edges in a straight line.
N_{MAX}	Maximum number of maximum length links.

<u>Dredger Route Edge Length Reduction Factor (p74)</u>

F_j	Reduction factor applied to the edges of a dredger route.
w	Reduction constant, user defined.
n	Number of aligned dredger route edges before node visited in a dredger route.
n_{max}	Maximum expected number of aligned edges in a dredger route.

Author's Declaration

I, Marcus J. M. de Ruyter, declare that the dissertation titled:

Two Dimensional Cut Plan Optimization For Cutter Suction Dredgers

and the work presented therein are both my own, and have been generated by me as the result of my own original research and confirm the following:

- This work was done wholly while in candidature for research degree at this University.
- Where any part of this dissertation has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this dissertation is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.
- None of this work has been published before submission.

Signed: ______________________________.

Date: ________________________________.

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1 Introduction

The work in this dissertation addresses two-dimensional cut plan optimization for cutter suction dredgers. The dredging industry is a specialized and capital-intensive sector of the construction industry (Dolmans, 2001). In 2007 the world market for dredging, including maritime construction, was worth and estimated 12.8 billion euros with strong growth predicted for 2008 (Tewes, 2007). Dredging itself is described by Williams (2003) as a complex task that is carried out using varying types of equipment to accomplish diverse goals.

Bray *et al*. (1997) defines the act of dredging as the excavation or movement of soil or rock with vessels or floating plant known as dredgers. Dredging is done to deepen waterways, to create or protect land, to substitute or win material for construction, to win minerals and to improve the environment (*ibid*.). Over time dredging projects, environmental concerns and competitive pressures have become more complex. As such there is need for the dredging industry to continually investigate updated management procedures and tools that can improve the economy of dredging activities (Mayer *et al*., 2002).

Cutter Suction Dredgers are often used for dredging navigable waterways (Randall *et al*., 2000). The total installed power on cutter suction dredgers varies from 200 to well in excess of 20,000 kilowatts (Bray *et al*., 1997, Vercruijsse, 2007). Large cutter suction dredgers can excavate to depths in excess of 35 metres below water level and achieve single cut widths in excess of 100 metres (Katoh *et al*., 1985). Figure 1.1 depicts a medium size cutter suction dredger working offshore in the United Arab Emirates.

Figure 1.1 Medium size cutter suction dredger (Courtesy Gulf Cobla L.L.C.).

Between 2005 and 2008 the Belgian dredging contractor Dredging, Environmental & Marine Engineering reportedly invested 460 million euros in the building of 7 new dredgers (Bertrand *et al*., 2008). The same contractor planned to invest another 500 million euros to build 10 more new dredgers between 2008 and 2011 (*ibid*.), amounting to an average cost of around 55 million euros per dredger over both periods. The high costs of building, operating and maintaining dredgers (Dolmans, 2001, Wang *et al*., 2006) have led to continuous attempts to increase operational efficiencies and productivities of such plant (Van Oostrum, 1979). Reducing stoppage time for dredgers is considered important because increasing the operational efficiency of a dredger equates to increasing a dredger's productivity (Brouwer, 1986). When dredging work is paid for by a fixed amount per unit volume excavated, stoppages not only equate to a loss of productivity but also to a loss of income (Miertschin *et al*. 1998). Less stoppage time means less time spent on a dredging project which in turn means less wear on machinery and fewer overheads, such as fuel and wages (*ibid*.).

On many projects done with cutter suction dredgers, the first part of preparing a dredge plan consists of dividing a larger dredging area into smaller adjoining dredge cuts of varying length and widths equal to or less than the cut width which can be achieved by the cutter suction dredger employed (Tang *et al*., 2008). After that, a sequence in which the smaller dredge cuts are to be excavated needs to be determined. When multiple dredge cuts have to be dredged, then at some point the cutter suction dredger employed is likely to be relocated and/or change its working direction from one cut to another. When a cutter suction dredger is relocated or changes its working direction it is considered idle. The time spent on moving a cutter suction dredger is unproductive as the actual dredging process itself is interrupted (Swart, 1995, Dirks *et al*., 1999, Blasquez *et al*., 2001). Therefore preparing a dredge plan which minimizes the total expected stoppage time resulting from unproductive dredger movements can increase the productivity of cutter suction dredgers. The aim of the work presented here is to investigate how optimal dredge plans for cutter suction dredgers can be determined systematically.

The next section of this thesis presents a background of cutter suction dredger operation and the research problem. The background section is followed by a review of literature on earthwork and dredging optimization to see if the research problem has been studied before. Next a review of literature on so-called nesting and routing problems is presented, special forms of which bear relevance to the research problem. The literature review ends with a review of a number of optimization methods. After the literature review the models of the research problem are presented and series of experiments are described to see if optimal solutions to these models can be found. The results of the experiments are then presented and discussed. Finally conclusions are drawn and further work is suggested.

2 Background

This section addresses cutter suction dredger operation and the research problem.

2.1 Cutter Suction Dredger Operation

Cutter suction dredgers are versatile dredgers (Herbich, 2000) with hulls consisting of pontoons and usually they do not have their own means of propulsion (Bray *et al.*, 1997). To move a cutter suction dredger when it is not dredging use of support vessels, such as workboats, is made (Bray *et al.*, 1997, Tang *et al.*, 2008). The rotating cutter apparatus of a cutter suction dredger, the cutterhead, can be designed to cope with a variety of materials: Peat, clay, silt, sand, gravel and boulders to sedimentary rock such as limestone, dolomite and carbonaceous rocks (Herbich, 2000). Larger more powerful cutter suction dredgers can dredge rock-like formations such as coral and softer types of basalt without pre-treatment by blasting and/or drilling (*ibid.*). In 2000 a single cutter suction dredger dredged around 7,000,000 cubic metres of limestone, glacial tills and sand and gravel deposits to excavate a 3.5 kilometre long tunnel trench between Denmark and Sweden (Maddrell *et al.*, 1998, Dirks *et al.*, 1999). Figure 2.1 depicts the main features of a typical medium size cutter suction dredger.

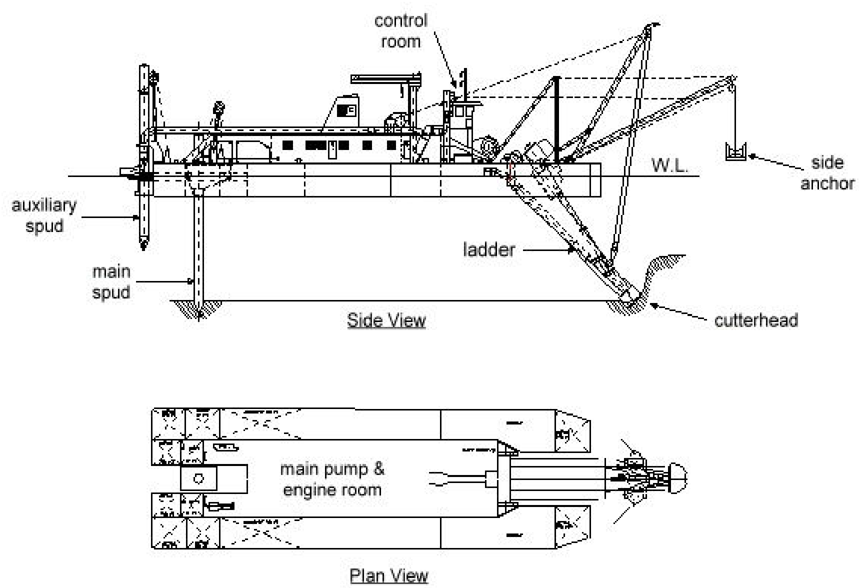


Figure 2.1 Overview medium size cutter suction dredger (Courtesy Gulf Cobla L.L.C.).

When dredging a cutter suction dredger is kept in position with side anchors at its front and often a spud system at its back (Bray *et al.*, 1997, Herbich, 2000, Tang *et al.*, 2008). Once in position, the pivoting ladder – a fabricated steel structure at the end of which the cutterhead is mounted – is lowered below the water level to start dredging. Upon immersion of the cutterhead, one or more on-board dredge pumps are engaged in order to create a desired flow rate in the dredger's suction and discharge pipes (Bray *et al.*, 1997, Wang *et al*., 2006). The cutterhead is set in motion before contact with the seabed is made (Tang *et al*., 2008). Figure 2.2 depicts a cutterhead designed for dredging sands and clays.

Figure 2.2 Routine inspection of a sand/clay cutterhead.

Once in contact with the seabed, the cutterhead loosens the bed material and a mixture of soil and water is drawn into the suction intake as a result of the vacuum created by the first dredge pump behind the cutterhead (Randall *et al*., 2000, Herbich, 2000, Tang *et al*., 2008). Suction intakes are usually located inside the lower half of the cutterhead. To ensure continued excavation of bed material the dredger is made to swing from side to side using winches, wires of which are connected to the dredger's side anchors (Bray *et al*., 1997, Tang *et al*., 2008). Side anchors are positioned outside the cut being dredged, on either side of the front of the dredger. Figure 2.3 depicts a typical swing track made by the cutterhead of a dredger fitted with a central spud carriage whilst dredging a single layer in plan view.

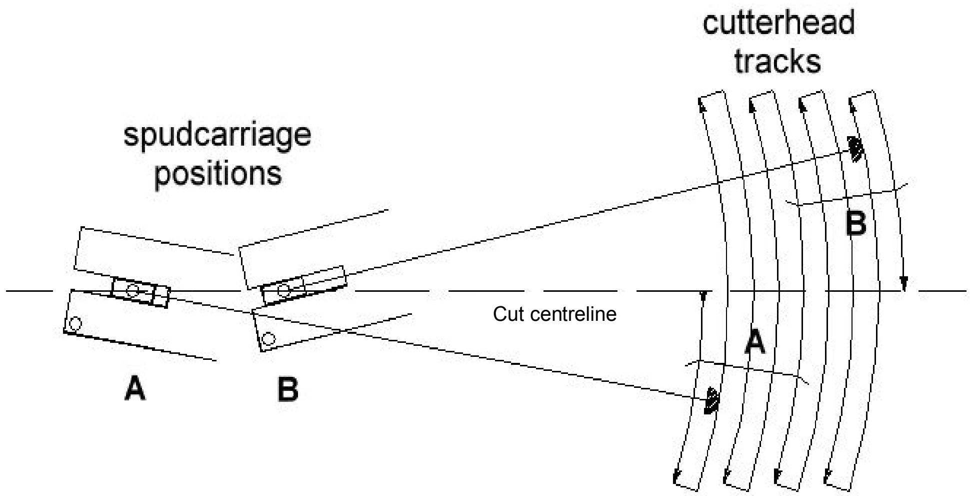


Figure 2.3 Typical cutterhead swing track (Corrected after Bray *et al*., 1997).

Figure 2.3 shows that the track at the excavation face across which the cutterhead is traversed is semi-circular (Bray *et al*., 1997). The radius of the arc across which the cutterhead is traversed has its centre point at the position of the main spud of the dredger. The main spud's position is normally on the centreline of the cut being dredged. Once an arc has been dredged over the full width of the cut, the swinging motion of the dredger is temporarily slowed down near to, or is stopped altogether on the edge of the cut to advance further into the excavation face. The dredger and cutterhead are advanced by pushing back the spud carriage which holds the main spud (Tang *et al*., 2008).

Newer cutter suction dredgers usually have a hydraulic ram cylinder with which the spud carriage holding the main spud is pushed out or pulled back against the hull (Herbich, 2000). When a spud carriage cylinder has been fully extended, for example over distance A in Figure 2.3, dredging is stopped in the centreline of the cut, or on a line parallel to it, to change spuds (Bray *et al*., 1997). To change spuds the auxiliary spud is first lowered after which the main spud is raised. Then the spud carriage cylinder is retracted while the grounded auxiliary spud keeps the dredger's aft in position. When the spud carriage cylinder is fully retracted the main spud is lowered and grounded after which the auxiliary spud is raised (Bray *et al*., 1997). After changing spuds the cutter suction dredger is ready to continue dredging a new length of cut, for example over distance B in Figure 2.3.

The core function of a cutter suction dredger's main spud system is to resist the backward thrust generated by the cutterhead as it is traversed across and advanced into a cut face (Steinbusch *et al*., 2001). As a cutter suction dredger advances, side anchors are periodically moved ahead (Randall *et al*., 2000) to keep the angle of pull on the side wires within the allowable limits. Usually angles are kept within 40 degrees to the cut centreline (Bray *et al*., 1997). The anchors used can vary but their main purpose is to resist the pulling forces of the dredger's side winches and additional sideways forces generated by the rotating cutterhead (Degenkamp *et al*., 1992).

Dredged soil-water mixtures are discharged from cutter suction dredgers into a series of floating pipeline sections connected to the dredger or into hopper barges for further transport (Bray *et al*., 1997, Randall *et al*., 2000, Tang *et al*., 2008). To deposit dredged soils into a containment area directly from the dredger use can be made of floating, submerged and/or land-based pipeline sections. Such pipeline networks can incorporate auxiliary parts such as ball joints and/or rubber hoses to increase flexibility, y-pieces with valves for branching off secondary pipelines and bends (Bray *et al*., 1997).

Within the limitations of vacuum, pressure, critical velocity and available power of a particular cutter suction dredging system, field experience indicates that for a given soil the dredging productivity of a cutter suction dredger is a function of depth of cut and lateral and rotational speeds of the cutterhead (Herbich, 2000). The soil-water mixture dredged by a cutter suction dredger can contain as much as 20% solids by volume (Randall *et al*., 2000). Cutter suction dredgers are usually rated either according to the internal diameter of their discharge pipe or by the power driving their cutterhead (Bray *et al*., 1997). Internal discharge pipe diameters range from under 150 millimetres to over 1,100 millimetres and cutterhead power can vary from 15 kilowatts to over 4,500 kilowatts (*ibid*.). Generally, small cutter suction dredgers are powered by diesel-hydraulic systems and larger ones by diesel-electric systems (*ibid*.).

A well-designed cutter suction dredger with an internal discharge pipe diameter of 750 millimetres, 1,500 kilowatts powering its cutterhead and 3,500 to 6,000 kilowatts driving its dredge pump will discharge between 1,500 to 3,500 cubic metres per hour in soft material and 150 to 1,500 cubic metres per hour in soft to medium hard rock through pipeline lengths of up to 4,500 metres (Herbich, 2000). To assess the performance of cutter suction dredgers their operational and dredging productivities can be calculated: Operational productivities of a cutter suction dredger are calculated by dividing the total volume removed divided by the sum of operational time (Herbich, 2000), where operational time is defined as time when a dredger is fully manned (Bray *et al*., 1997). Dredging productivities are calculated by dividing the total volume removed by the sum of operational time less stoppage time, where stoppage time is defined as time when the dredger is fully manned but not dredging (*ibid*.). On most dredging projects average operational and dredging productivities of cutter suction dredgers are calculated and recorded on a daily basis.

For a well-managed cutter suction dredger, under average site conditions, the total stoppage time incurred can be in the range of 20 to 30 per cent of the total operational time (Bray *et al*., 1997). However, specific site conditions can result in higher losses (*ibid*.). Stoppage times can be grouped depending on whether they are considered avoidable or unavoidable. Stoppages causing loss of productivity which are unavoidable are those inherent to the cutter suction dredging process itself. Examples of unavoidable stoppages are those incurred when changing the position of anchors and spuds, without which progress cannot be made. Stoppages resulting from sub-standard maintenance, operation and/or management of the dredger can be considered as avoidable. Unnecessary dredger movements resulting from poor operation and/or management of the dredger are avoidable.

An analogy can be made with the *Lawn Mowing Problem* (Arkin *et al*., 2005), where the lawnmower represents a cutter suction dredger and the lawn equates to a dredging area. For a single covering exercise and constant mowing rate, an objective can be to achieve the highest possible operational productivity with the lawnmower. To achieve this the lawn has to be cut such that stopping the lawnmower to change its working direction or to teleport it to another part of the lawn is minimized since stoppages add to the total time the manned lawnmower is operational: The total area to be cut is fixed so any increase in operational time reduces the operational productivity of the lawn mower. Using a lawn mower is less complex than operating a cutter suction dredger, but the lawn mowing problem, as described here, can serve as a model for the work done with such a dredger. The following section describes the research problem of two-dimensional cut planning for cutter suction dredgers in more detail.

2.2 Research Problem

Capital dredging normally denotes projects which involve dredging as a one-off operation (Bray *et al*., 1997). Maintenance dredging is used to describe dredging which is of a recurrent nature (*ibid*.). Usually the scope of capital dredging works is described in detail by contractual documents specifying horizontal and vertical excavation limits, including tolerances if applicable (*ibid*.). Figure 2.4 depicts a cutter suction dredger dredging a single straight cut as part of a capital dredging project to widen an existing channel.

Figure 2.4 Single cut dredging project (Courtesy Gulf Cobla L.L.C.).

Usually, a single cut dredged by a cutter suction dredger is of constant width and of a length greater than its width. Unless purposely dredged otherwise, the start and end of a dredge cut resemble near-identical arcs in plan view. In Figure 2.4 the semi-circular end of the single cut can be seen on the left where channel widening is in progress and the dredger advances. Preparing a cut plan for a single straight cut requires the selection of one cut centreline. The project can be completed as the dredger progresses naturally along that single centreline. Only if the required depths or widths of cut are not achieved will the dredger have to be moved back or turned around in the opposite direction. However, such events are not taken into account in this work. The research problem of two-dimensional cut planning for cutter suction dredgers presents itself when a dredging area cannot be excavated in a single dredge cut.

Preparing a plan to excavate a continuous dredging area which cannot be covered with a single cut of a cutter suction dredger consists of two stages. In the first stage the larger dredging area is divided into smaller adjoining dredge cuts in plan view. The first planning stage will be referred to here as the *Dredge Cut Nesting Problem*. The second planning stage consists of determining a sequence in which to excavate adjoining dredge cuts and will be referred to here as the *Dredger Routing Problem*. In order to model dredge cuts they can be approximated by rectangles with lengths and widths equal to exact multiples or fractions, or a combination of both, of effective cut widths which can be achieved with cutter suction dredgers. In such an approximation the semi-circular shape of the starts and ends of real cuts dredged by cutter suction dredgers is neglected.

In practice, the dredging of two adjoining cuts is made to overlap to some degree to avoid leaving behind undredged ridges. The selected amount of overlap usually depends on the type of material being dredged. Equally so, additional dredging is usually carried out on the edges of dredging areas to ensure the required slope profiles are realized. Effective cut widths of cutter suction dredgers are defined here as excluding the extra widths dredged to achieve overlap or side slope profiles. The maximum effective cut width of a cutter suction dredger is therefore less than the maximum cut width which can be achieved with the same dredger.

To illustrate the two planning stages which make up the research problem a hypothetical dredging project is considered. The dredging area of this project is a rectangle of 300 metres wide and 400 metres long. A cutter suction dredger selected to excavate the area is capable of achieving an effective cut width of 100 metres. It is assumed the dredger can float anywhere in and around the dredging area. There no constraints on the length or total number of dredge cuts which can be selected or on the order in which dredge cuts, once selected, can be dredged. In addition, it is given that one pass will suffice to achieve required depths and that site conditions are homogeneous throughout so that all dredging directions are equally productive. Figure 2.5 depicts two arrangements of 100 metre wide cuts in the hypothetical dredging area.

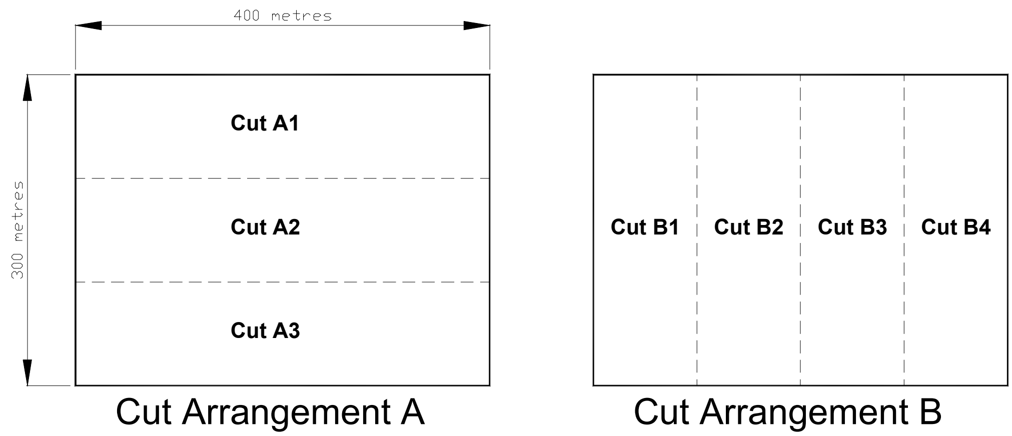


Figure 2.5 Hypothetical dredging area divided into cuts.

Figure 2.6 depicts two hypothetical dredging sequences, one for each cut arrangement depicted in Figure 2.5. Dredging sequences in Figure 2.6 are shown with arrows indicating the working direction of the dredger and dredge cut outlines are adjusted for clarity.

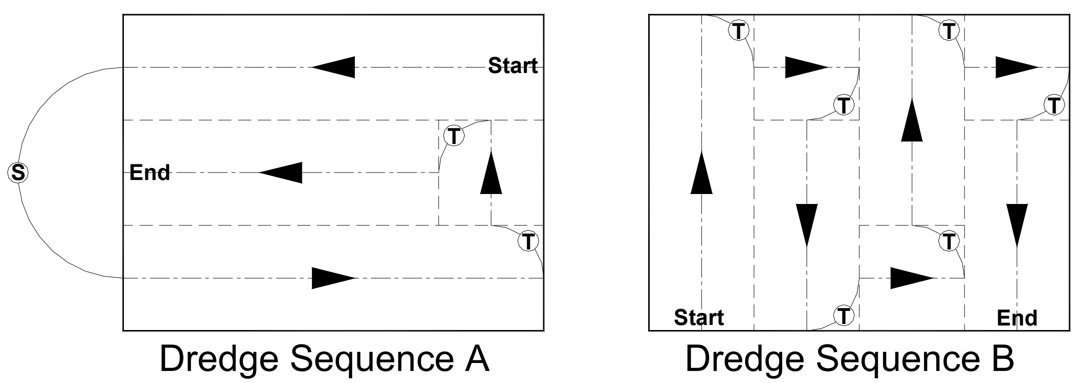


Figure 2.6 Hypothetical dredging sequences.

Cut plan A is made up of cut arrangement A of Figure 2.5 and dredging sequence A of Figure 2.6, and cut plan B is made up of cut arrangement B of Figure 2.5 and dredging sequence B of Figure 2.6. Each cut plan can now be evaluated by summing the associated hypothetical dredger movements. Minor dredger movements are those when the dredger turns into the next cut more or less on the spot, which are indicated by an encircled ‘T’ in Figure 2.6.

Major dredger movements are those when the dredger has to be relocated over some distance to the next cut, which are indicated by an encircled ‘S’ in Figure 2.6. The total number and types of hypothetical dredger movements for each cut plan are: One major movement and two minor movements for cut plan A and six minor movements for cut plan B.

If it is said that a major dredger movement equals two hours of stoppage time and a minor dredger movement equals one hour of stoppage time, then the total stoppage time resulting from dredger movements is: Four hours for cut plan A and six hours for cut plan B. All else being the same, executing cut plan B would result in two hours more stoppage time than if cut plan A were executed. Executing cut plan A, therefore, will result in the cutter suction dredger achieving a higher overall operational efficiency and productivity since less operational time is lost.

The hypothetical example and its outcome reflect the essence of the research problem. However, in the hypothetical example only two possible solutions out of many were considered. To find the best solution all possible combinations of dredge cuts and cutting sequences need to be evaluated and compared. The hypothetical cut planning problem is a combinatorial problem with a search space depending on problem size. The dredger routing problem alone is a combinatorial problem depending on the total number of locations through which the dredger can pass. For a dredger routing problem with n locations, where the dredger has to return to where it started from, there are $(n - 1)!/2$ possible tours: If n is 21, there are more than 10^{18} possible tours (Helsgaun, 2000). Finding optimal solutions to combinatorial problems with very large search spaces can be made easier by modelling the problem and using computer-programmed optimization methods (Winston, 1994). The main objectives of the research presented here are to develop a model of two-dimensional cut planning for cutter suction dredgers and use and evaluate a computer-based solution approach which systematically optimizes the developed model.

Next a literature review consisting of five sections is presented. In the first two sections literature on optimizing earthworks and dredging is reviewed to see if the research problem presented here has been identified, modelled and/or optimized before. In the third and fourth section literature on nesting and routing problems is reviewed to see how they can be used to model the *Dredge Cut Nesting Problem* and the *Dredger Routing Problem*. In the last section literature on optimization methods is reviewed to see how the developed model of the research problem can be optimized.

3 Literature Review

This section reviews literature which is of direct relevance or provides context to the work presented here.

3.1 Earthwork Optimization

Literature on earthwork optimization is considered relevant to the work presented here since dredging, as defined by Bray *et al*. (1997), concerns the movement of soil and rock, albeit done under water. Construction projects, which often involve significant amounts of earthwork, possess unique characteristics which make the individual planning of each project essential (Askew *et al*., 2002). Earthworks are costly and reducing the total distance travelled by earthmoving vehicles leads to cost savings in fuel consumption, time and equipment maintenance (Henderson *et al*., 2003). Planning and estimating earthmoving operations involves three steps: Formulation, representation and the evaluation of a plan (Kannan *et al*., 1997). The application of automation technologies to planning earthmoving operations is desirable because they are machine-oriented, repetitive, tedious, and consist of physically demanding tasks (Kim *et al*., 2003a). In addition, automated approaches to earthwork planning benefit worker safety, skilled worker requirements and productivities (*ibid*.).

A common tool used for finding the most economical distribution of earthwork on road-, rail- and runway projects is the mass-haul diagram (Easa, 1988a). A mass-haul diagram graphically represents the cumulative volume of earth along a project (Oglesby *et al*., 1982), where haul is defined as the movement of a unit volume over a unit of distance (Mayer *et al*. 1981). Appendix A gives a numerical example of a conventional mass-haul diagram. Stark *et al*. (1972) suggests using a *Linear Programming* model to optimize earthwork allocation problems instead of mass-haul diagrams. Linear programming is a classical *Operations Research* optimization method (Lirov, 1992).

Operations Research originated in England when it was used for making decisions how to best use war material during World War II (Taha, 2003). After the war the ideas on military operations were adapted to improve efficiency and productivity in the civilian sector (*ibid*.). Taha (1982) describes Operations Research as a problem-solving approach, involving the use of mathematical techniques to model decision problems, which seeks the determination of the best (optimum) course of action for decision problems under the restriction of limited resources.

Models are abstractions of assumed real systems: They simplify the complexity of a real system by concentrating primarily on identifying the dominant variables, parameters and constraints which control the behaviour of the real system (Taha, 1982). Figure 3.1 depicts the modelling process used in Operations Research schematically.

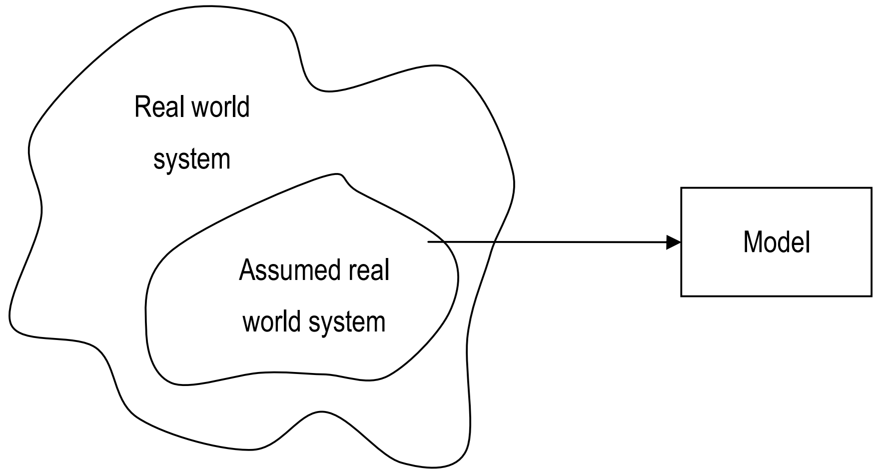


Figure 3.1 Operations Research modelling overview (Taha, 1982).

In Operations Research a model of an assumed real system is stated as an objective subject to constraints: The objective reflects the desired end result and the constraints identify important relationships of the modelled system (Taha, 1982). For example, the common objective in money-making endeavours is to maximize profit or minimize cost (*ibid*.). Constraints of a money-making endeavour can be, for example, the limited amount of components available for making finished products. When the objective and constraints of a decision problem are known, the optimum course of action can be decided upon by identifying values of variables which best meet the objective (*ibid*.). The quality of solutions arrived at depend on how accurate a model represents a real problem (Taha, 2003).

Three main types of Operations Research models exist: Exact (or mathematical) models; simulation models; and heuristic (or approximation) models (Taha, 1982, Winston, 1994). Exact models assume that all the relevant variables, parameters, and constraints as well as the objective are quantifiable, and are generally successful at optimizing the problems they model (*ibid.*). Simulation models imitate the behaviour of a system over time so that information about the performance of the system can be collected when pre-defined events occur (*ibid.*). For example, a business may decide to simulate different inventory systems rather than experiment with the real-world system (Winston, 1994). The information indicating the performance of the simulated system is accumulated and stored as a set of statistical observations (Taha, 1982). Because simulation models do not need explicit mathematical functions to relate variables, it is usually possible to simulate complex systems that cannot be modelled or solved exactly (*ibid*.). The main drawback of simulation is that the analysis of a system is equivalent to conducting experiments and therefore is subject to experimental error (*ibid*.). In addition, although optimization with simulation is possible, simulation is not an optimization method: Simulation models are mostly used to analyze ‘what if’ questions (Winston, 1994). Optimization of simulation models is generally a slow process and can be costly (*ibid*.).

When for an exact formulation of a problem the determination of an optimal solution is problematic, heuristic models can be used to determine optimal or near-optimal solutions (Taha, 1982). Heuristic models make use of local search techniques that intelligently move from one solution point to another with the aim of improving the value of the model’s objective function (*ibid*.). When no further improvements can be achieved, the best attained solution is an optimal or near-optimal solution to the model (*ibid.*).

Linear programs are exact models of problems which have a linear objective function subject to linear equality and inequality constraints (Taha, 2003). Solving linear programming models gives values of previously unknown variables which result in a minimum or maximum value of the objective function (*ibid*.). Mayer *et al*. (1981) uses a linear programming model to optimize an earthwork allocation problem in which three categories of cost are defined: The first for excavation and loading, the second for haul, and the third for placement and compaction. The costs of excavation and placement on a construction project are typically considered to be proportional to the earthwork quantities involved (*ibid*.). The cost of haul, however, is proportional to both earthwork quantity and haul distance. When for a given quantity of earth to be moved, the costs of excavation and placement are fixed, Mayer *et al*. (1981) states that the most economical distribution of cut and fill is that which minimizes haul. Mayer *et al*. (1981) optimizes an earthwork allocation problem by modelling it as a *Transportation Problem*.

The transportation problem is representative of many linear programs which model the movement of specific amounts of identical items from a discrete number of sources to a discrete number of destinations (Kannan *et al*., 1997). Figure 3.2 depicts a model of a transportation problem as a network with *m* sources and *n* destinations. Each source and destination is represented by a square, also referred to as a node.

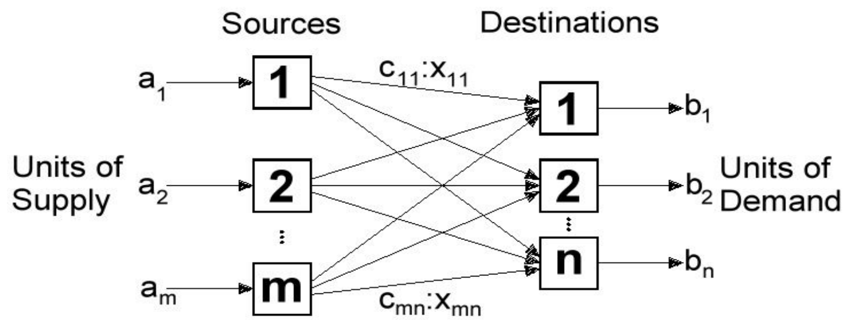


Figure 3.2 Single-period Transportation Problem as a network (Schrage, 1999).

The network in Figure 3.2 only has two node levels indicating activities are planned over a single period. More complex multi-period transportation problems can be modelled by adding levels of nodes (Schrage, 1999). The routes along which products can be transported are represented by lines with arrows joining the nodes. The total cost of transport along a given route is the product of the unit transportation cost for that route and the number of units transported along that route. How a unit of transportation is defined depends on the item transported. A unit of transportation can be a single item or a multiple thereof, for instance a consignment required for the assembly of a larger item. The units of supply and demand must correspond to the definition of the transported unit.

To model a transportation problem, the amount of supply available at each source, the amount of demand present at each destination, and the unit transportation cost from each source to each destination must be known. The unknown variables of a transportation problem are the amounts of items transported from each source to each destination, expressed as the problem decision variable, x_{ij}. Since all items are identical, a destination can receive its demand from more than one source. Taha (2003) states that the linear program of the transportation problem depicted in Figure 3.2 is generally formulated as follows:

$$minimize\ Z = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij} x_{ij} \tag{3.1}$$

where: Z = total cost; c_{ij} = unit transportation cost from i to j; x_{ij} = units transported from i to j; M = total number of sources; and N = total number of destinations.

Subject to the following constraints:

$$\sum_{j=1}^{N} x_{ij} \le a_i, \quad i = 1, 2, ..., m \tag{3.2}$$

$$\sum_{i=1}^{M} x_{ij} \ge b_j, \quad j = 1, 2, ..., n \tag{3.3}$$

$$x_{ij} \ge 0, \quad \text{for all } i \text{ and } j \tag{3.4}$$

$$\sum_{i=1}^{N} a_i \ge \sum_{j=1}^{M} b_j \tag{3.5}$$

where: a_i = the supply available at source *i*; and b_j = the demand present at destination *j*.

Equation 3.1 states the objective of the problem is to minimize the total cost of the transport process. Equation 3.2 specifies that the sum of units transported from a source cannot exceed supply; Equation 3.3 stipulates that the sum of units transported to a destination must satisfy its demand; and Equation 3.4 ensures units transported are not negative. Lastly, Equation 3.5 specifies that total supply must be equal or greater than total demand, which is considered an optional constraint. If for a transportation problem total supply equals total demand the problem is said to be a balanced transportation problem (Winston, 1994).

Table 3.1 redefines variables of Equations 3.1 to 3.5 so that the total amount of haul can be minimized for earthwork allocation problems for which transportation cost is linearly proportional to transport distance. Mayer *et al*. (1981) defines haul as the movement of one unit volume over one unit of distance.

Table 3.1 Redefinition Transportation Problem variables – Earthwork optimization

Variable	Transportation Problem Definition	Earthwork Problem Definition
Z	Total cost	Total haul
a_i	Source supply	Cut volume
b_i	Destination demand	Fill volume
c_{ij}	Unit transportation cost	Transport distance
x_{ij}	Transported items	Transported volume

Solving an earthwork allocation problem modelled as a transportation problem using linear programming identifies volumes transported between cut and fill locations for which the total haul is minimal. Mass-haul diagrams are best for optimizing relatively narrow works such as road-, rail- and runway projects but can not cope with haul costs and soil properties which vary along the roadway (Easa, 1988a). Using linear programs of transportation problems, linear earthwork allocation problems covering wide areas can be modelled and optimized, as Mayer *et al*. (1984) demonstrates with a hypothetical example of a dredged-material allocation problem.

Linear programs can also take into account project set-up costs associated with the use of borrow pits and landfills. The model proposed in Mayer *et al*. (1981) uses constant unit costs of haul for borrow pits and landfills. Constant unit costs for haul make haul costs linearly proportional to haul distance. Easa (1987) proposes a mixed-integer linear program for optimizing an earthwork allocation problem which uses non-constant unit costs for haul for borrow pits and landfills. Mixed-integer linear programs describe problems with a mix of linear and integer variables (Taha, 2003). The model in Easa (1987) uses cost functions with three cost levels for the use of borrow pits.

Easa (1988a) determines roadway levels which minimize earthwork cost using linear programming and constant unit costs of haul. Easa (1988b) optimizes earthwork allocation problems where project set-up costs are governed by a quadratic function which defines non-constant unit costs of haul for the use of borrow pits. So, in addition to linear programming models, a *Quadratic Programming* model – a model which has a quadratic objective function of several variables subject to linear constraints on these variables (Taha, 2003) – can also be used to optimize earthwork allocation problems.

Jayawardane *et al*. (1994) proposes a multi-staged solution approach, also referred to as a *Dynamic Programming* model, to optimize earthwork for road construction projects. Problems which exhibit overlapping sub-problems and an overall optimal sub-structure can be formulated as dynamic programming models (Taha, 1982). Dynamic programming primarily serves to enhance the computational efficiency of solving large problems and usually takes one of two forms: A ‘top-down’ or a ‘bottom-up’ approach (*ibid*.). In a ‘top-down’ approach a large problem is broken into sub-problems, which are solved separately, remembering their solutions in case they need to be solved again (*ibid*.). In a ‘bottom-up’ approach relevant sub-problems are solved in advance and then used to build up solutions to larger problems (*ibid*.).

The dynamic programming model used in Jayawardane *et al*. (1994) for optimizing earthworks is a bottom-up approach consisting of three stages: Simulation, mixed-integer linear programming and network scheduling. The first stage, that of simulation, generates realistic unit earthmoving costs corresponding to an optimal combination of plant for each haulage operation considered (*ibid*.). The simulated data then serves as input for the next stage, a mixed-integer linear program, with which the most economical distribution of cut and fill for the chosen combination of plant is determined (*ibid*.). In the third and final stage, the network scheduling stage, the most economical distribution of cut and fill is applied together with the sequential logic of the construction operations adopted in the second stage to obtain a construction schedule in the form of a network and a bar chart describing the earthwork allocation plan (*ibid*.). Jayawardane *et al*. (1994) concludes that the comprehensive model proposed can successfully accommodate constraints in plant availability, project completion time, availability of different soil strata at cut sections and borrow pits, various degrees of compaction at various layers, and various borrow pits and landfills.

Henderson *et al*. (2003) solves the problem of levelling a construction site by redefining the associated earthwork allocation problem as a shortest route cut and fill problem. The shortest route cut and fill problem is form of *Travelling Salesperson Problem*. Henderson *et al*. (2003) uses a *Simulated Annealing* algorithm to find optimal solutions to 90 hypothetical earthwork allocation problems. A travelling salesperson problem is an assignment problem with the additional condition that the assignments chosen must constitute a tour and the objective is to minimize the total distance travelled (Schrage, 1999). Simulated annealing is an optimization method using a stochastic local search technique which is analogous to the annealing of solids where, as the process advances increasingly better solutions are found, eventually converging to, or close to, a global optimum (Henderson *et al*., 2003). Solving the shortest route cut and fill problem equates to finding a route, beginning and ending at the same cut location, for a single earthmoving vehicle which minimizes the total distance travelled between cut and fill locations (*ibid*.).

A single-period transportation problem models the movement of identical units along arcs between nodes and as such it is not considered suitable as a model for dredge cut nesting problems. Other Operations Research problems which can be used to model dredge cut nesting problems with greater ease are reviewed Section 3.3. The applicability of a single-period transportation problem to model dredger routing problems is limited to dredger routing problems consisting of two nodes connected with one arc. Multi-period transportation problems can be used to model dredger routing problems. However, a travelling salesperson problem, as used in Henderson *et al*. (2003) to model a transportation problem, is considered a better choice for modelling dredger routing problems, which is explained in Section 3.4.

A dynamic programming solution approach, as used in Jayawardene *et al*. (1994) to solve earthwork allocation problems, is of direct relevance to solving the research problem since it is made up of two problems: Dredge cut nesting and dredger routing. The applicability of optimization methods such as linear-, mixed-integer linear-, quadratic programming and simulated annealing to optimizing the research problem is discussed in Section 3.5. To see if the problem of two-dimensional cut planning for cutter suction dredgers has been identified, modelled and/or optimized before, literature on dredging optimization is reviewed next.

3.2 Dredging Optimization

The literature reviewed in the previous section supports the view put forward in Jayawardene *et al*. (1994) that earthwork optimization, especially in road construction, has drawn considerable attention from researchers, particularly in the USA and Canada. Some commentary suggests the opposite with regard to the levels of attention the subject of dredging as a whole has received from researchers. The preface of Herbich (1975) quotes Houston as having said the following in 1968:

> "*On premise that a profession is known by its literature, dredging might well be eliminated. Its literature is almost nil.*"

In the foreword of Herbich (1975) Taylor observes that:

> "…*there is a paucity of comprehensive literature in the dredging industry.*"

More recently, in the preface of Bray *et al*. (1997), Murden states that:

> "…*the number of published manuscripts which fully address the entire scope of dredging technology continues to be limited.*"

Although prefaces and forewords tend to promote the work in which they reside, a perceived lack of published research of dredging can be the result of the dredging industry having been somewhat secretive for many years (Riddel, 2000). The dredging industry used to be insular and inward looking, and saw little advantage in sharing information about projects, problems or new techniques (*ibid*.). Although the dredging industry has changed, technical secrecy remains, which, it is claimed, is necessary for maintaining commercial competitiveness (*ibid*.). Commercial confidentiality continues to prevent the sharing of detailed information on production methods and rates (Riddell, 2000). However, Murden states in the preface of Bray *et al*. (1997) that:

> *"Since the inception of the World Organization of Dredging Associations in 1967 the number of technical papers on dredging presented at seminars, conferences and published in journals has significantly increased."*

Instead of classifying Operations Research models by model type (exact-, simulation-; and heuristic models) as done in Taha (1982) and Winston (1994), Denes (1991) refers to the optimization methods used, distinguishing between two main groups: Quantitative and qualitative methods. Quantitative optimization methods aim to provide values of previously unknown variables which result in optimal or near-optimal values of an objective function. Quantitative optimization methods are associated with exact formulations of problems. Linear programming is a quantitative optimization method according to Denes (1991), while Taha (2003) does not group linear programming with models which give near-optimal solutions. Qualitative optimization methods on the other hand, aim to provide descriptive solutions which are recommendations for best practices. Qualitative optimization methods rely on the collection and statistical analysis of field data or data generated with physical or computer simulation models. Literature on the qualitative optimization of dredging problems provides recommendations for:

- reducing environmental impacts of dredging operations as done in Clarke *et al*. (2002) and Barth *et al*. (2004),
- reducing maintenance dredging in ports and waterways as done in De Meyer *et al*. (1987),
- increasing storage capacity of disposal areas as done in Moritz *et al*. (1995), Van Mieghem *et al*. (1997), and McKee *et al*. (2005),
- reducing risks and costs of dredging projects as done in Henshaw *et al*. (1999), Zhu *et al*. (1999), Creed *et al*. (2000), and Demir *et al*. (2004),
- improving dredger performance as done in Denes (1993), Deketh (1995), and Blasquez *et al*. (2001).

Other dredging research presents models simulating real dredging systems, but without evaluating ‘what if’ scenarios or optimizing the modelled systems. Examples of simulation models of real dredging systems are those of soil cutting mechanisms (He *et al*. 1998, Miedema, 2004), sediment transport in pipes (Luca, 1995, Matousek, 1998), suction pipe dynamics (Ten Heggeler *et al*. 2001, Liu *et al*., 2003) and hopper loading processes (Paris *et al*., 1996).

It can be said that there appears to be no shortage of literature which presents models of varying complexity of real dredging problems and systems. Some simulation models, for instance those presented in Blasquez *et al*. (2001) and Clarke *et al*. (2002), can be suitable for quantitative optimization. However, none of the literature summarized so far in this section identifies, models or optimizes the research problem. Literature on the quantitative optimization of dredging problems, as is done for earthwork allocation problems reviewed in Section 3.1, can be subdivided according to whether stochastic or deterministic models are used.

Stochastic models are exact formulations of problems where certain data is modelled as a random variable, the probability distribution of which depends on parameter values (Winston, 1994). The outcomes of stochastic models depend on a variety of draws from the probability distributions used and therefore contain uncertainties, which is why they are sometimes described as models with risk (*ibid*.). Dredging problems optimized quantitatively using stochastic models include the planning of maintenance dredging (Lund, 1990, Lansey *et al*., 1993, Mayer *et al*. 2002), dredged-material disposal (Stansbury *et al*., 1999), and sand nourishment (Bruun, 1991, Van Noortwijk *et al*., 2000). However, none of the stochastic models in the literature referred to, model the operation of individual dredgers. Instead these stochastic models provide optimal solutions as to when or how to carry out dredging activities in general.

Deterministic models are exact formulations of problems which do not contain random or stochastic components and as such their outcomes are free of risk: Solutions to the problems they model can be calculated according to some pre-specified logic (Winston, 1994). Dredging problems optimized quantitatively using deterministic models include the following:

- bid proposals (Mayer *et al*., 1984),
- managing dredged-material disposal (Mayer *et al*., 1984, Ford, 1984 and 1986, Schroeder *et al*., 1995),
- allocating dredging fleets (Denes, 1991),
- planning maintenance dredging (Mayer *et al*., 2002),
- loading of hopper dredgers (Howell *et al*. 2002).

Mayer *et al*. (1984) uses linear programming to optimize a bidding proposal for a hypothetical dredging project thereby maximizing present worth of anticipated project revenue. In addition, Mayer *et al*. (1984) uses linear programming to optimize a single-period dredged-material allocation problem modelled as a transportation problem as done in Mayer *et al*. (1981) for earthwork allocation problems. Howell *et al*. (2002) states economical loading of hopper dredgers can be optimized by linear programming but does not formulate the linear program to be used.

Denes (1991) models the problem allocating a fleet of dredgers to a number of projects as an *Assignment Problem* with the objective to minimize the total associated cost. The total cost of undertaking a dredging project is made up of fixed cost elements, such as mobilization and labour, and variable cost elements, such as fuel, supplies and repair costs.

The linear program of an assignment problem has an objective function, Equation 3.6, which is very similar to that of a transportation problem:

$$minimize\ Z = \sum_{i=1}^{M} \sum_{j=1}^{N} c_{ij} x_{ij} \qquad (3.6)$$

where: Z = total cost; c_{ij} = cost of assigning agent i to task j; x_{ij} = agent i assigned to task j; M = total number of agents; and N = total number of tasks.

The constraints of the linear program representing an assignment problem are:

$$\sum_{i=1}^{M} x_{ij} = 1, \quad i = 1, 2, ..., m \qquad (3.7)$$

$$\sum_{j=1}^{N} x_{ij} \leq 1, \quad j = 1, 2, ..., n \qquad (3.8)$$

$$x_{ij} \geq 0, \quad \text{for all } i \text{ and } j \qquad (3.9)$$

$$x_{ij} = (0, 1), \quad \text{for all } i \text{ and } j \qquad (3.10)$$

Equation 3.7 specifies that each task is assigned one agent; Equation 3.8 stipulates that each agent is not assigned to more than one task; and Equation 3.9 ensures that numbers of assigned agents are not negative. Lastly, Equation 3.10 specifies that values for agents assigned to tasks can only be 0 or 1 since an agent is either assigned to a task or not. The linear program of an assignment problem can include additional constraints, which, for example, specify that the total number of agents must be equal or greater than the number of tasks. Table 3.2 gives objective function variables of the assignment and the transportation problem for comparison.

Table 3.2 Assignment and Transportation Problem variables

Variable	Assignment Problem Definition	Transportation Problem Definition
Z	Total cost	Total cost
M	Total number of agents	Total number of sources
N	Total number of tasks	Total number of destinations
c_{ij}	Cost of assigning agent i to task j	Unit transportation cost from i to j
x_{ij}	Agent i assigned to task j	Units transported from i to j

Through substitution of agents with dredgers, and tasks with dredging projects Denes (1991) solves a hypothetical dredger allocation problem. The hypothetical cut planning problem given in Section 2.2 can not be directly modelled as an assignment or transportation problem, at least not as a single-period one. However, a special form of assignment problem, the travelling salesperson problem, can be used to model dredger routing problems, which is discussed in more detail in Section 3.4.

Ford (1984) presents a dredged-material disposal management model which can be used to determine the minimum-net-cost operation policy for systems of disposal sites over future periods by optimizing the associated dredged-material allocation problem. The associated dredged-material allocation problem is modelled as a multi-period transportation problem. A basic version of the equation used in Ford (1984) to calculate volumes of dredging material in each disposal area over a single period of time is as follows:

$$V_{End} = V_{Start} + F \times (V_{Dredged\ In}) + (V_{Transferred\ Out} - V_{Transferred\ In}) - V_{Reused\ Out} \qquad (3.11)$$

where: V = total volume; and F = reduction factor.

In Equation 3.11: The total incoming volume of dredged material can be made up of volumes arriving from more than one source; the total volumes transferred into or out of a disposal area is the sum of volumes of dredged material arriving from or departing to other disposal areas within the system considered; and the total volume reused represents the sum of volumes taken out of a disposal area for reuse outside of the system considered. Despite an absence of errata or discussion in subsequent literature, it is thought that Ford (1984) meant to state Equation 3.11 as follows:

$$V_{End} = V_{Start} + F \times (V_{Dredged\ In}) - (V_{Transferred\ Out} - V_{Transferred\ In}) - V_{Reused} \qquad (3.12)$$

In Equation 3.12 the second plus sign from the left in Equation 3.11 is replaced with a minus sign. The dredged-material disposal system operation model of Ford (1984) has a linear objective function and linear constraints. The objective function to be minimized includes unit costs for transport, storage and transfer of dredged-material as well as unit benefits for the reuse of disposed dredged-material and these unit costs and benefits are considered constant over time. The objective function is subjected to two sets of constraints. The first set of constraints specifies the maximum volumes of dredged material which can be transported from a source to a disposal area. The second set of constraints stipulates that the maximum available storage capacity of each disposal area cannot be exceeded.

Since the objective function and constraints are linear, Ford (1984) states that linear programming can be used to optimize the multi period dredged-material disposal system operation model. However, Ford (1984) argues that for transportation problems other optimization methods have been found to be more efficient than linear programming. Ford (1984) uses a network-with-gains algorithm, allowing for reduction factors, combined with a flow-augmentation algorithm to optimize dredged-material allocation problems modelled as multi-period transportation problems.

The first steps of the algorithm used in Ford (1984) consist of setting all transported volumes to zero and identifying the minimum-cost transportation path. Then the algorithm increases the volume transported along the minimum-cost transportation path until a maximum allowable transported volume along one or more transportation paths is reached. The process of identifying the next minimum-cost transportation path and increasing the volume transported along it until the next limit is reached is repeated until either the maximum volumes of dredged-material have all been transported or the maximum allowable volume of dredged-material which can be transported within the network is reached. According to Ford (1984) the algorithm used finds feasible and global optima to the modelled dredged-material management problem if such a solution exists.

The dredged-material disposal management model presented in Ford (1984) is used in a computer program titled “Optimization of Long-Term Operation and Expansion of Multiple Disposal Sites Incorporating Dredging Sites, Disposal Sites, Transportation Facilities, and Management Restriction (D2M2)” in Schroeder *et al*. (1995). The D2M2 computer program is a module of the Automated Dredging and Disposal Alternatives Modeling System (ADDAMS) used by the United States Corps of Engineers (*ibid*.).

Mayer *et al*. (2002) proposes an adaptation of a *Classic Inventory Problem* to model and optimize the planning and cost of maintenance dredging operations. Classic inventory problems involve optimal decisions with respect to inventory management: When to replenish inventories and by how much, such that replenishment, storage and shortage costs are minimal for a given inventory system (*ibid*.).

Mayer *et al*. (2002) presents the *Sediment Inventory Model* where the total cost of maintaining adequate navigation depths in a waterway system is made up of maintenance dredging costs, inefficiency costs from having inadequate navigation depths, and offset costs, or benefits, from having excess navigation depths. Mayer *et al*. (2002) analyzes three models of maintenance dredging scenarios and finds quantitative optimal solutions for each by either setting the derivative of the objective function to zero to solve for one variable or by partial differentiation to solve for two or more variables.

Mayer *et al* (2002) concludes that the insights provided by the analysis is useful but that neither of the three models are fit for indiscriminate application to real problems or substitutes for good design practice. Mayer *et al* (2002) adds this remains the case even when the models are made more sophisticated by the inclusion of stochastic variables.

None of the literature on dredging optimization reviewed in this section identifies the two-dimensional cut planning problem for cutter suction dredgers. Therefore models for dredge cut nesting and dredger routing problems are set up as part of the work presented here. In summary, like with transportation and assignment problems, it is not obvious how an inventory problem could be used to model dredge cut nesting problems. As mentioned at the end of Section 3.1, a special form of assignment problem can be used to model dredger routing problems: The travelling salesperson problem. How dredger routing problems can be modelled as a travelling salesperson problem is discussed in more detail in Section 3.4, but first literature on a particular group of Operations Research problems with potential to model dredge cut nesting problems is reviewed.

3.3 Nesting Problems

The inclusion of a review of research on so-called nesting problems is the result of similarities between these problems and dredge cut nesting problems. These similarities were responsible for describing the first stage of two-dimensional cut planning for cutter suction dredgers as 'dredge cut nesting'. Nesting problems traditionally involve a non-overlapping placement of a set of shapes within some larger region(s), but the objective can vary (Nielsen *et al*., 2003). Usually the objective of a nesting process is to maximize utilization of stock material (Hopper *et al.*, 2001). Nesting problems, or cutting and packing problems, are considered combinatorial problems with very large search spaces (Lirov, 1992) and have received substantial amounts of attention from researchers the world over (Bischoff *et al*., 1995, Elmghraby *et al*., 2000).

Variants of the *Stock Cutting Problem*, a special form of nesting problem, are stated and treated in Operations Research as well as in other disciplines such as engineering, information and computer science, and mathematics (Elmghraby *et al*., 2000). To solve a stock cutting problem a number of geometrical patterns are selected and arranged so that the total cost of the underlying process is minimized (Lirov, 1992). The underlying process can be described as a general resources allocation problem where the objective is to subdivide a given quantum of a resource into a number of predetermined allocations so that the left-over amount is minimized (*ibid*.). Dyckhoff (1990) notes that stock cutting problems are also sometimes referred to as trim loss problems. According to Bischoff *et al*. (1995) and Elmghraby *et al*. (2000) the wide interest in nesting or cutting and packing problems can be attributed to the following aspects:

3.3.1 Applicability of Cutting and Packing Research

Cutting and packing problems are encountered in various industries. For example, in steel, glass and paper manufacture, where optimal solutions to real-world problems have been determined. In addition, many other industrial problems exist which possess a structure similar to cutting and packing problems. For example, capital budgeting, assembly-line balancing and processor scheduling.

3.3.2 Diversity of Real-world Cutting and Packing Problems

Common structures can be found in real-world cutting and packing problems. However, these problems often differ significantly with respect to specific goals or constraints and other aspects, such as the degree of integration into wider planning systems. Therefore standard models often need to be reformulated and algorithms adjusted to the specific needs of a given problem.

3.3.3 Complexity of Cutting and Packing Problems

The majority of cutting and packing problems are known to be combinatorial optimization problems which are termed NP-hard (Non-deterministic Polynomial-time hard), meaning they cannot be solved in polynomial time, but possibly in exponential time (Helsgaun, 2000). It is difficult to mathematically determine optimal solutions within reasonable time for NP-hard cutting and packing problems (Kim *et al.*, 2003b). In other words, many algorithms currently known for finding optimal solutions require a number of computational steps that grow non-polynomially with the problem size rather than according to a polynomial function (Hopper *et al.*, 2001). Consequently, the development of faster exact (or optimal) algorithms and heuristic (or approximation) algorithms providing solutions nearer to optima is a major research topic (*ibid*.). Hopper *et al.* (2001) argues that for the more complex cutting and packing problems, with many underlying combinatorial conditions, it is often not worthwhile to search for an exact algorithm and that therefore solution quality is sacrificed to gain computational efficiency by using heuristic algorithms.

Wang *et al*. (2002) observes that nesting problems typically take the form of traditional one- and two-dimensional stock cutting problems and three-dimensional container/pallet loading problems. For two-dimensional stock cutting problems Nielsen *et al*. (2003) distinguishes between *Decision Problems*, *Knapsack Problems*, *Bin Packing Problems* and *Strip Packing Problems*. In decision problems, as the name suggests, it has to be decided whether a given set of shapes fits within a given region or not (*ibid*.). In knapsack problems a set of shapes and a single region are given and a placement of a subset of shapes is sought after which maximizes the use of the region (*ibid*.). In bin packing problems sets of shapes and regions are given, and the number of regions needed to place all shapes is to be minimized. In strip packing problems the length of a strip of fixed width is to be minimized such that all the shapes of a given set are contained within the strip region (*ibid*.).

Decision-, knapsack- and bin packing problems involve two-dimensional packing of regular and/or irregular shapes within some regular and/or irregular region(s) without overlap (*ibid*.). The regions in which two-dimensional shapes are to be placed can consist of two-dimensional representations of animal hides (in the leather industry), rectangular plates (in the steel industry) or tree boards (in the furniture industry). Figure 3.3 depicts a schematic representation typical of irregular decision-, knapsack- and bin packing problems.

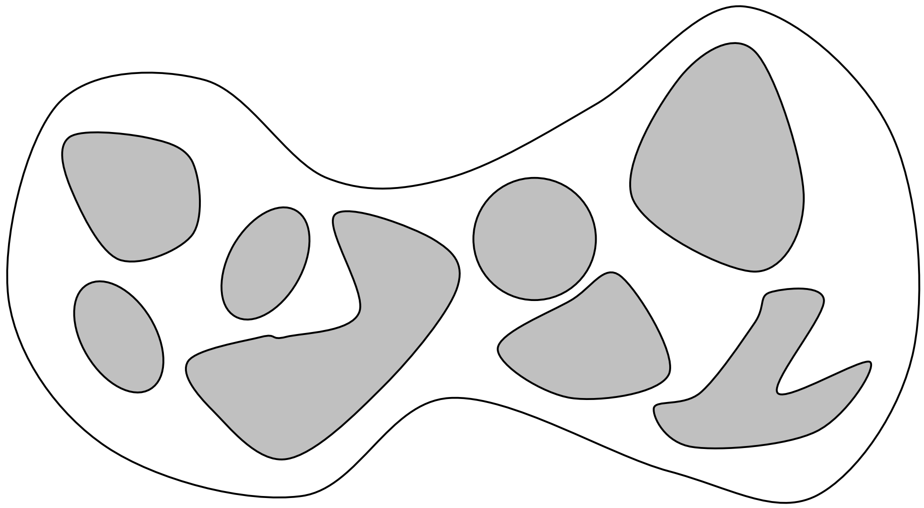


Figure 3.3 Decision-, knapsack- and bin packing problems (Nielsen *et al*., 2003).

In Figure 3.3 the grey regions represent irregular shapes to be nested in the larger region represented by the outer polygon. An example of a strip packing problem would be that of cutting a set of two-dimensional shapes out of a strip of cloth in the textile industry. Figure 3.4 depicts a typical strip packing problem.

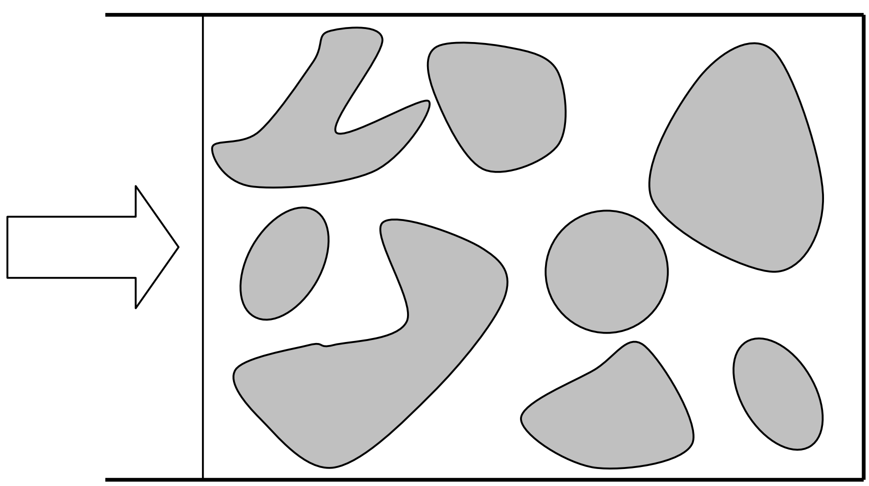


Figure 3.4 Strip packing problem (Nielsen *et al*., 2003).

In Figure 3.4 the grey areas again represent irregular shapes to be nested in the larger region which is a strip of material of fixed width. The arrow in Figure 3.4 indicates the direction into which the leftmost edge of the adjustable region is moved to compact the shapes to be nested without overlap. Figure 3.5 depicts a more comprehensive scheme for classifying nesting problems, as originally proposed in Dyckhoff (1990).

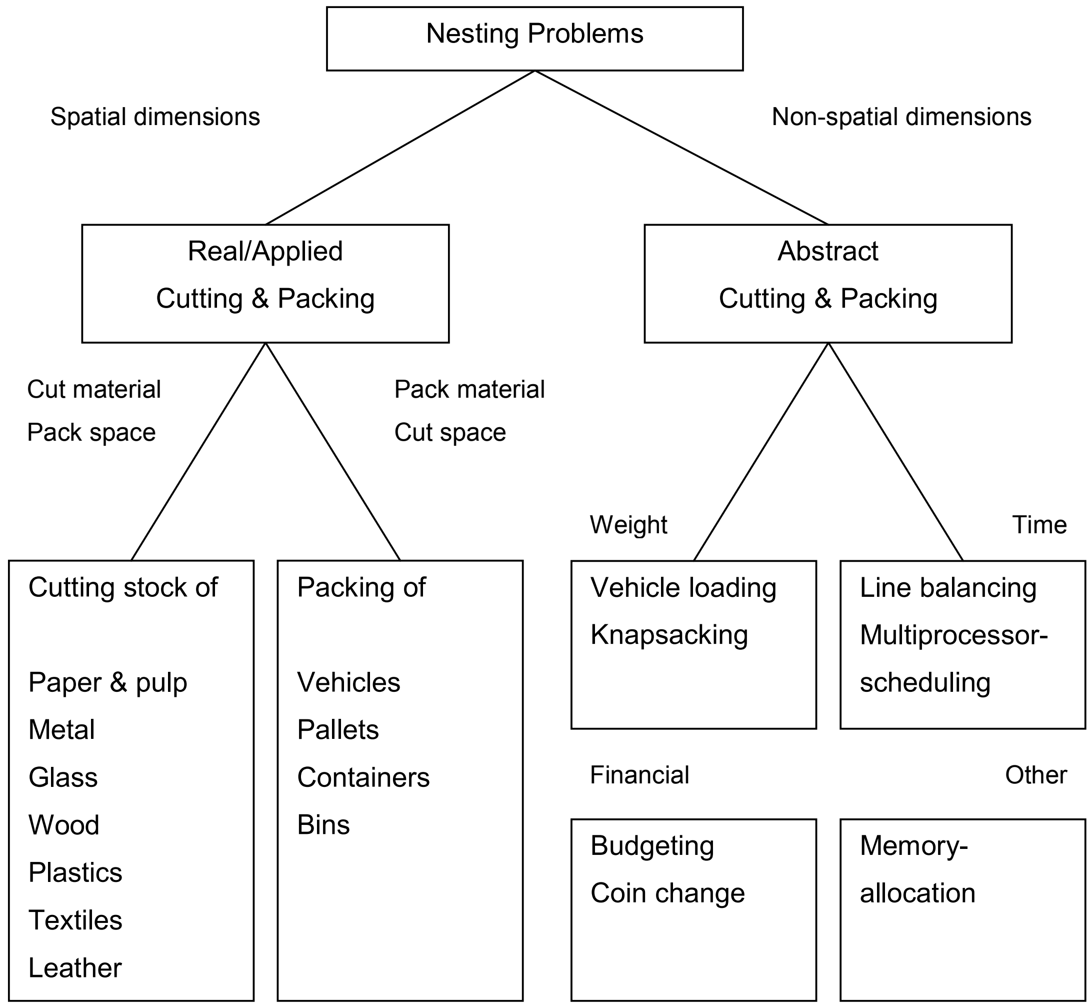


Figure 3.5 Nesting problem classification (Dyckhoff, 1990).

The classification scheme depicted in Figure 3.5 can be applied to cutting as well as packing problems since it takes into account the duality of material and space. Cutting can be seen as packing smaller pieces of material/space into larger pieces of material/space (Karelahti, 2002). Packing can be seen as cutting larger pieces of material/space into smaller pieces of material/space (*ibid*.). Dyckhoff's classification scheme, unlike the broad categorization of Nielsen *et al*. (2003), is not limited to two-dimensional spatial problems.

Figure 3.5 shows that the classification scheme proposed in Dyckhoff (1990) distinguishes between nesting problems involving spatial dimensions and those involving non-spatial dimensions. The first group, on the left, consists of real or applied cutting and packing or loading problems that are defined in Euclidean space up to three dimensions. The second group, on the right, covers abstract problems with non-spatial dimensions such as weight, time or financial dimensions (Hopper *et al*., 2001). Dyckhoff (1990) classifies knapsacking as an abstract nesting problem, whereas Nielsen *et al*. (2003) does not consider weight as being a decision variable of knapsack problems. The classification scheme proposed in Dyckhoff (1990) is used here to see if the first part of the research problem, the subdivision of a dredging area into smaller dredge cuts, can indeed be referred to as a dredge cut nesting problem.

The first part of the hypothetical example given in Section 2.2, where a larger rectangular dredging area was subdivided into smaller adjoining dredge cuts, essentially equates to the applied cutting or packing of two-dimensional material/space. The operation of cutter suction dredgers, as described in Section 2.1, involves the cutting of material in three dimensions, which emphasizes cutting rather than packing. Therefore, in the classification proposed by Dyckhoff (1990), as given in Figure 3.5, the dredge cut nesting problem fits best in the left-most bottom-most group of nesting problems, collectively known as stock cutting problems. One of the earliest reported formulations of the stock cutting problem was produced by the Russian economist Kantorovich for the paper industry in 1939 (Elmaghraby *et al*., 2000), and although it wasn't published in English until 1960, it became known for being the first real application of linear programming (Lirov, 1992). Dyckhoff (1990) states that stock cutting problems have four main variable characteristics, which are as follows:

1) Dimensionality:
   - number of dimensions.
2) Type of assignment:
   - all of the (large) stock objects must be used and a selection of (smaller) items is to be ordered or placed,
   - a selection of (large) stock objects is available, but wastage can be accepted as long as all (smaller) items are ordered or placed.
3) Assortment of stock:
   - one large stock object,
   - many identical large stock objects,
   - many different large stock objects (including, for example, residual stock).
4) Assortment of small items to be ordered or placed:
   - few small items of differing dimensions,
   - many small items of mostly differing dimensions,
   - many small items of mostly identical dimensions,
   - many small items of identical dimensions.

The elementary types of dimensionality of stock cutting problems are one-, two- and three-dimensional. However, a number of stock cutting problems with a complexity between one- and two-dimensional problems exist and these are referred to as 1.5-dimensional problems (Hinxman, 1979, Haessler *et al*., 1991). One-dimensional stock cutting problems were initially concerned with paper production but later on were also found to be applicable to the coil splitting and fibreglass industries (Lirov, 1992). Another example of a one-dimensional stock cutting problem is the cutting of steel bars where the length of the stock bars is fixed (Karelahti, 2002). Figure 3.6 depicts a schematic example of a one-dimensional stock cutting problem.

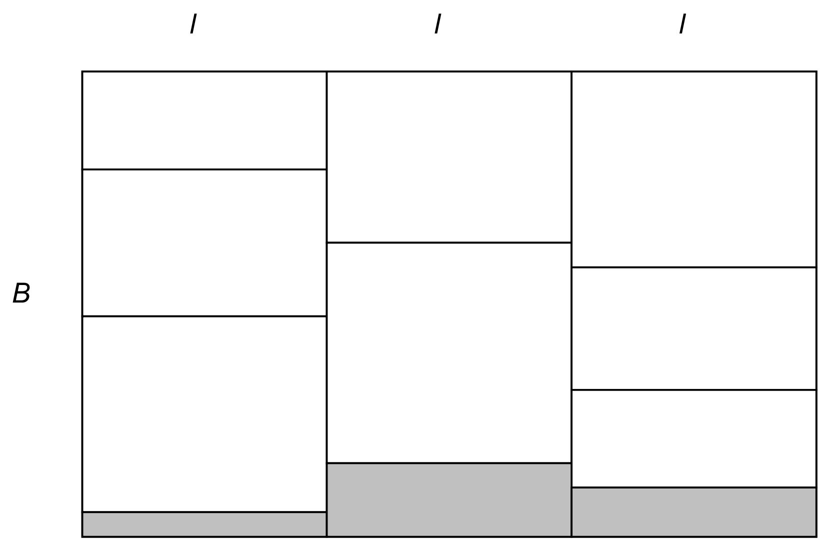


Figure 3.6 One-dimensional stock cutting problem (Karelahti, 2002).

In the one-dimensional stock cutting problem depicted in Figure 3.6 the width, *B*, of the stock reel and the length, *l*, of the shapes to be cut are both fixed and the grey areas represent trim loss. The one-dimensional problem consists of finding sums of varying widths of the shapes to be cut which are as near as possible to, but not greater than, the fixed width of the stock. In a 1.5-dimensional stock cutting problem the width of the stock reel is variable while the length of the shapes to be cut remains fixed. The cutting of steel reels of variable width, but of fixed length, is an example of a 1.5-dimensional stock cutting problem (Karelahti, 2002). Figure 3.7 depicts a schematic example of a 1.5-dimensional stock cutting problem.

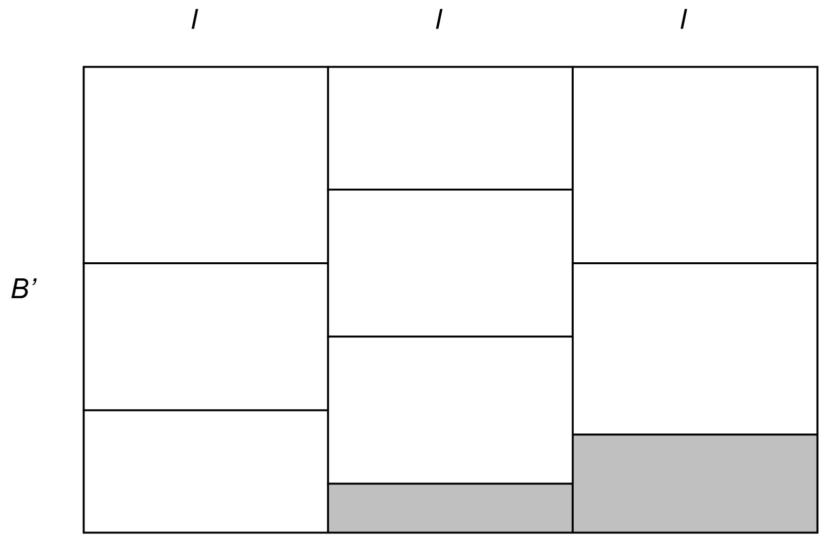


Figure 3.7 1.5-dimensional stock cutting problem (Karelahti, 2002).

In the 1.5-dimensional stock cutting problem depicted in Figure 3.7, the width, *B'*, of the stock reel is variable and the length, *l*, of the shapes to be cut is fixed. Again, the grey area represents trim loss. To minimize trim loss the sum of varying widths of the shapes to be cut has to come as near as possible, but not exceed the variable stock width. Figure 3.8 depicts a schematic example of a two-dimensional stock cutting problem.

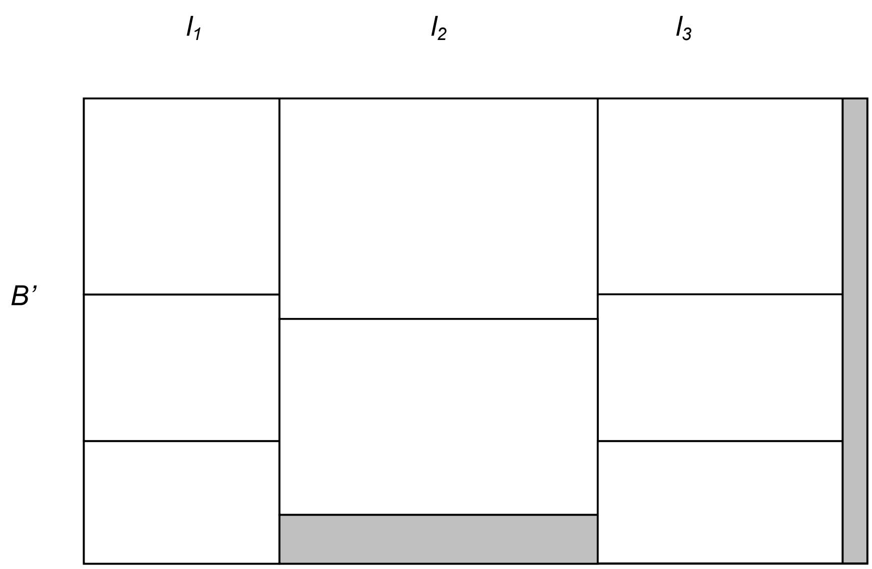


Figure 3.8 Two-dimensional Stock Cutting Problem (Karelahti, 2002).

In Figure 3.8 the width, *B'*, of the stock reel and the lengths, l_n, of the shapes to be cut are all variable. Two-dimensional stock cutting problems were first applied to glass cutting.

Subsequently two-dimensional stock cutting problems were also found useful in the clothing, leather and plastic film industries (Lirov, 1992). The formulation and solution of three-dimensional stock cutting problems has been used in the cargo loading and lumber cutting industries (*ibid*.). Figure 3.9 depicts an example of a three-dimensional stock cutting problem.

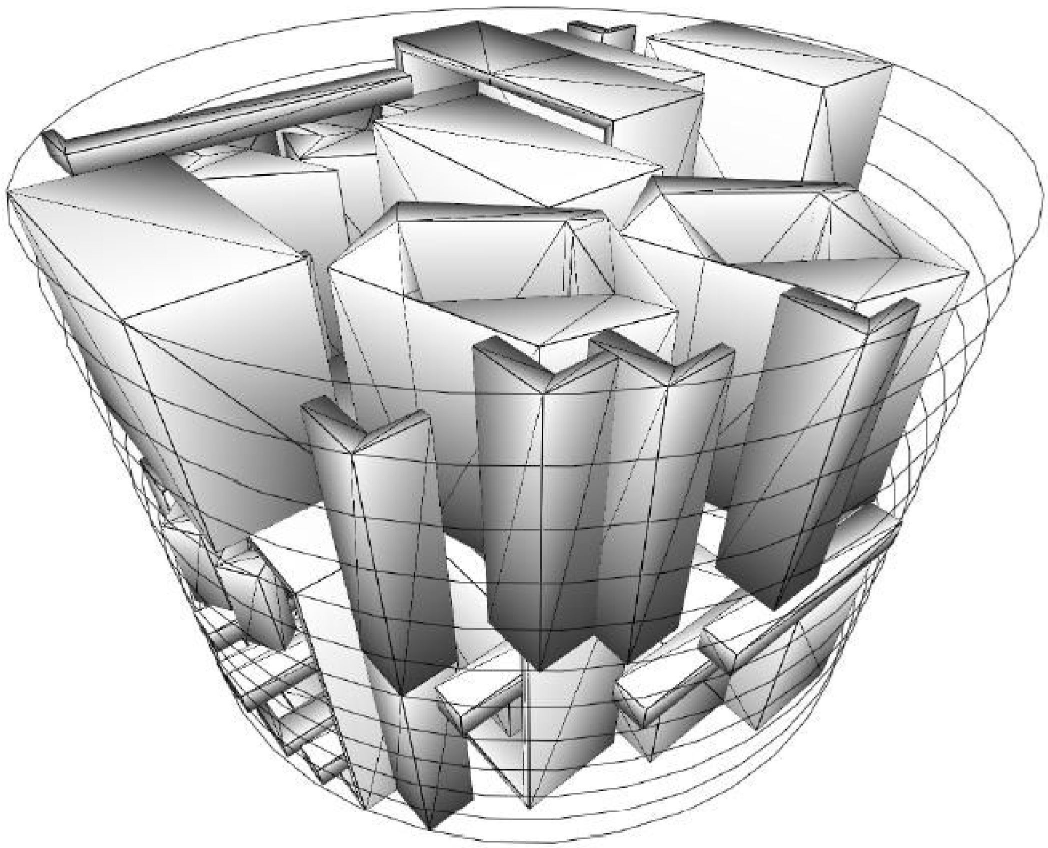


Figure 3.9 Three-dimensional stock cutting problem (Nielsen *et al*., 2003).

In Figure 3.9 three-dimensional items shaded in grey are to be cut out of cylindrical stock item, represented by the stacked circles. For three-dimensional stock cutting problems in the lumber cutting industry the orientation of shapes to be cut can be important because of the grain of the wood (Dowsland *et al*., 1995). Figure 3.9 highlights the duality between material and space in stock cutting problems: The objective to minimize trim loss equates to finding the densest packing configuration of smaller objects in a larger container.

The second main characteristic of stock cutting problems identified in Dyckhoff (1990) is the type of assignment required by a particular problem. The two types of assignment identified in Dyckhoff (1990) differ in whether wastage is allowed or not. The third and fourth main characteristics of stock cutting problems identified in Dyckhoff (1990) refer to the sizes and quantities of stock items and items to be cut. Further to Dyckhoff (1990), the hypothetical dredge cut nesting problem given in Section 2.2 can be defined as follows:

1) Two-dimensional: The dredging area and dredge cuts are considered in plan view.
2) All of a large stock object must be used, trim loss must be zero, and a selection of smaller items is to be ordered or placed: All of the dredging area must be excavated by dredging smaller adjoining dredge cuts in succession.
3) The stock object is a single large object: The dredging area.

With the above defining characteristics it is argued that the first part of the research problem, the dredge cut nesting problem, can indeed be treated as a two-dimensional stock cutting problem. What remains to be defined is the type of assortment of small items to be ordered or placed in the dredging area. The type of assortment representative of the dredge cut nesting problem considered gives additional information about the degree of irregularity of the problem. The degree of irregularity of a dredge cut nesting problem depends on whether a) the dredging area, and b) the dredge cuts to be nested are of regular or irregular shape. It is known that the plan view of the dredging area of the real dredging project to be modelled here as part of an engineering application is of irregular shape. It is also known that in the hypothetical example of dredge cut nesting given in Section 2.2 there were no restrictions on the lengths of individual dredge cuts and the that selection of their widths up to the maximum effective cutting width was also a matter of choice. In addition, before the hypothetical example was given, it was said that dredge cuts can be approximated by rectangles with lengths and widths equal to exact multiples or fractions, or a combination of both, of effective cut widths which can be achieved with cutter suction dredgers. Therefore, since both dredging areas and as well as assortments of dredge cuts can be irregular, two-dimensional dredge cut nesting problems can be highly irregular.

According to Dowsland *et al*. (1995) problems involving irregular shapes are difficult to solve. Highly irregular stock cutting problems are encountered, for example, in the manufacture of leather products. Often, where leather is used in the furniture-, car-, clothing- and shoe industry, the nesting problem consists of arranging a set of two-dimensional irregularly shaped parts on a two-dimensional irregularly shaped surface (Heistermann *et al*., 1995, Yuping *et al*., 2005).

Heistermann *et al*. (1995) notes that nesting on leather is further complicated by restrictions imposed by the resources to be used. In leather manufacturing the stock is represented by an animal hide. Parts of hides can vary in quality because of joints or the effects of injuries and hides can have holes caused by barbed-wire fences or tick bites (*ibid*.). In addition, hides are non-identical so each nest can only be cut once. Because each hide has a unique shape Heistermann *et al*. (1995) states it is not even practical to maintain a library of partial nests from which preliminary solutions can be chosen.

To solve leather nesting problems Yuping *et al*. (2005) models irregular leather hides and irregular shapes to be cut as polygons in two-dimensional space. Two-dimensional polygons representing hides are called sheets and those representing the shapes to be cut are called stencils. Yuping *et al*. (2005) evaluates a nest layout by calculating three quantities: The total area of *escape* (the total area of stencils outside sheet profiles); the total area of *non-placement* (the total sheet area not occupied by stencils); and the total area of *overlap* between stencils. The terms stencil overlap and overlap will used synonymously from here onwards. Yuping *et al*. (2005) calculates areas of escape, non-placement and overlap using a polygon comparison algorithm developed by Weiler (1980). Yuping *et al*. (2005) models irregular leather nesting problems with the following objective function:

$$\textit{minimize } Z = \alpha_{escape}A_{escape} + \alpha_{non\text{-}placement}A_{non\text{-}placement} + \alpha_{overlap}A_{overlap} \qquad (3.13)$$

where: Z = total cost; α_{escape} = escape weight factor; $\alpha_{non\text{-}placement}$ = non-placement weight factor; $\alpha_{overlap}$ = overlap weight factor; A_{escape} = total escape area; $A_{non\text{-}placement}$ = total non-placement area; and $A_{overlap}$ = total overlap area.

Yuping *et al*. (2005) states that feasible solutions to leather nesting problems must have zero escape and zero overlap, which can be achieved by selecting appropriate values for the weight factors, α_n, in Equation 3.13. For the hypothetical leather nesting problem solved in Yuping *et al*. (2005) weight factors of 50 were used for escape and overlap and a weight factor of 4 was used for non-placement. Yuping *et al*. (2005) defines escape area, A_{escape}, with Equation 3.14:

$$A_{escape} = \sum_{i=1}^{N} \left[s_{escape\,i} \right]^2 \qquad (3.14)$$

where: N = total number of stencils; and $s_{escape\,i}$ = area of stencil i outside the sheet profile(s).

To avoid infeasible nest solutions Yuping *et al*. (2005) increases the severity of the penalty for escape by squaring stencil areas outside the sheet profile(s) in Equation 3.14. Yuping *et al*. (2005) defines non-placement area, $A_{non\text{-}placement}$, with Equation 3.15:

$$A_{non-placement} = L_{area} - \sum_{i=1}^{N} s_i + \sum_{i=1}^{N} s_{escape\ i} \tag{3.15}$$

where: L_{area} = total area of sheet(s); N = total number of stencils; s_i = area of stencil i; and $s_{escape\ i}$ = area of stencil i outside the sheet profile(s).

Finally, Yuping *et al*. (2005) defines overlap area, $A_{overlap}$, with Equation 3.16:

$$A_{overlap} = \sum_{i=1}^{N} \sum_{j=i+1}^{N} \left[s_{overlap\ ij} \right]^2 \tag{3.16}$$

where: N = total number of stencils; and $S_{overlap\ ij}$ = area of overlap between stencil i and j.

To avoid infeasible nest solutions Yuping *et al*. (2005) also increases the severity of the penalty for overlap by squaring overlapping stencil areas in Equation 3.16. To search for optimal solutions to irregular leather nesting problems Heistermann *et al*. (1995) and Sharma *et al*. (1997) use a *Genetic Algorithm* while Yuping *et al*. (2005) uses an *Adaptive Simulated Annealing* algorithm. Genetic algorithms are optimization methods which use a stochastic local search technique which is analogous to evolution theory where, as the process advances, increasingly fitter or better solutions are found, eventually converging to, or close to, a global optimum (Knaapen *et al*., 2002). Adaptive simulated annealing is a special form of simulated annealing, which allows for the occasional widening of the stochastic local search technique employed and converges to global optima faster than simulated annealing (Yuping *et al*., 2005).

Because genetic and adaptive simulated annealing algorithms have both been used successfully to optimize irregular two-dimensional stock cutting problems (Heistermann *et al*., 1995, Yuping *et al*., 2004) they are natural candidates for optimizing irregular dredge cut nesting problems. Genetic and adaptive simulated annealing algorithms are discussed in more detail in Section 3.5. First literature on routing problems is reviewed, to find out if the dredger routing problem can be modelled as a travelling salesperson problem.

3.4 Routing Problems

One of the more well known routing and combinatorial optimization problems is the travelling salesperson problem (Schrage, 1999, Voudouris *et al*., 1999). Choi *et al*. (2003) states that because of their structure travelling salesperson problems are difficult to solve. Schrage (1999) describes the travelling salesperson problem as being an assignment problem with the additional constraint that the assignments chosen must constitute a tour and where the objective is to minimize the total distance travelled. Therefore, a travelling salesperson is an optimization problem of trying to find the shortest Hamiltonian cycle (Mulder *et al*., 2003). Hamiltonian cycles are named after Sir William Rowan Hamilton, who devised the Icosian game or Knight's tour puzzle, in which a graph cycle or closed loop is sought which connects all nodes and visits each node exactly once (Skiena, 1990, Marcotte *et al*., 2004).

By convention, the trivial graph on a single node is considered to posses a Hamiltonian cycle, while the connected graph of two nodes is not. A graph possessing a Hamiltonian cycle is said to be a Hamiltonian graph. Garey *et al*. (1983) states that the problem of finding a Hamiltonian cycle is NP-hard and that the only known way of determining whether a given graph has a Hamiltonian cycle is to undertake an exhaustive search. Since the problem of finding a Hamiltonian cycle is NP-hard, the travelling salesperson problem is also NP-hard. For this reason new optimization methods are often tested on travelling salesperson problems (Voudouris *et al*., 1999, Tsai *et al*., 2003). Figure 3.10 depicts a solution to a travelling salesperson problem.

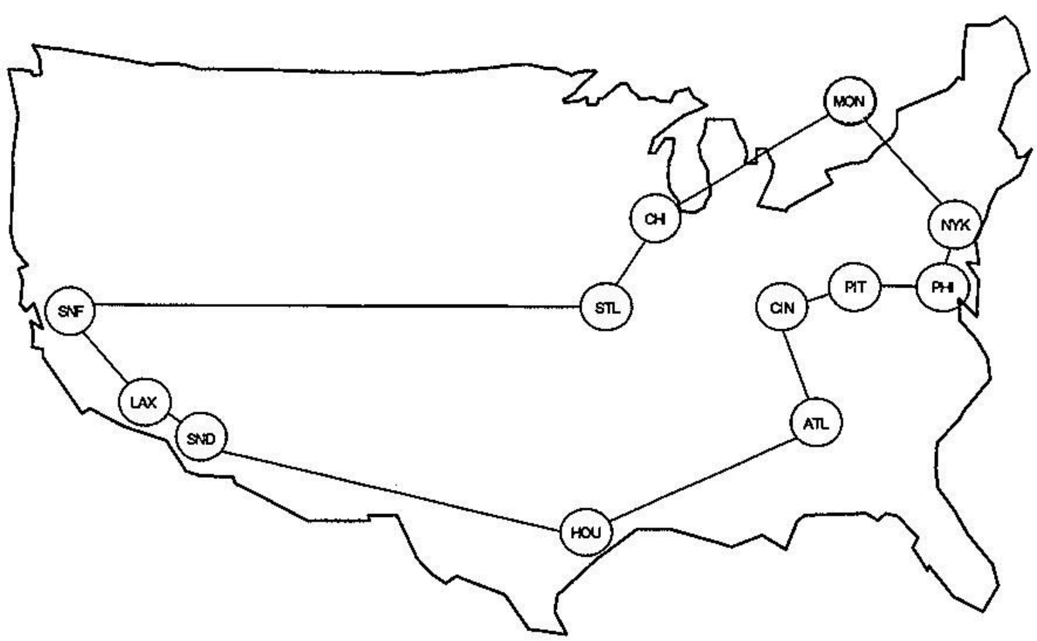


Figure 3.10 Travelling salesperson problem (Schrage, 1999).

The solution tour depicted in Figure 3.10, represented by the lines connecting twelve cities, is a closed tour where each city is visited exactly once. Choi *et al*. (2003) states that many scientific and engineering problems can be modelled as travelling salesperson problems. Likas *et al*. (2002) mentions that fields of study where problems are modelled as travelling salesperson problems include economy, complex systems administration, decision-making, mechanics, physics, chemistry and biology.

The field of chemistry is noted in Choi *et al*. (2003) as an example where the travelling salesperson problem has received substantial attention from researchers because of its relationship to batch scheduling problems. Choi *et al*. (2003) states that multi-product batch scheduling problems can be characterized as travelling salesperson problems because transition costs or time penalties are incurred when transforming materials from one state into another.

As mentioned in Section 3.1, Henderson *et al*. (2003) models a shortest route cut and fill problem as a travelling salesperson problem to optimize earthworks. Gimadi *et al*. (2004) model a vehicle routing and loading problem as a travelling salesperson problem. In the problem modelled in Gimadi *et al*. (2004) the total profit realized from purchasing and selling commodities loaded by a vehicle (with limited capacity) at locations in a closed tour has to be maximized. Bosch *et al*. (2003) uses instances of travelling salesperson problems to create continuous line drawings of target pictures of Marilyn Monroe and part of the Mona Lisa.

Helsgaun (2000) states that travelling salesperson problems, where distance measured between nodes is Euclidean, are also referred to as Euclidean travelling salesperson problems. The value of a tour edge between nodes in travelling salesperson problems can also be expressed as a cost. When the cost of a tour edge is equal for both directions of travel then the travelling salesperson problem is said to be symmetric, otherwise it is said to be asymmetric (*ibid*.). Euclidean travelling salesperson problems are symmetric. To model a Euclidean *N*-city travelling salesperson problem Taha (2003) defines the problem's decision variable as follows:

$$x_{ij} = \begin{cases} 1, \text{ if city } j \text{ is reached from city } i \\ 0, \text{ otherwise} \end{cases} \quad (3.17)$$

Taha (2003) states the objective function of a symmetric travelling salesperson problem as follows:

$$minimise\ Z = \sum_{i=1}^{N}\sum_{j=1}^{N} d_{ij} x_{ij} \qquad d_{ij} = \infty \text{ for } i = j \qquad (3.18)$$

where; N = total number of cities; and d_{ij} = distance from city i to city j.

Subject to the constraints:

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad i = 1, 2, 3, \ldots, N \qquad (3.19)$$

$$\sum_{i=1}^{N} x_{ij} = 1 \qquad j = 1, 2, 3, \ldots, N \qquad (3.20)$$

$$x_{ij} = (0, 1) \qquad \text{for all } i \text{ and } j \qquad (3.21)$$

Solution forms a tour. (3.22)

Constraints 3.19 and 3.20 ensure that each city is arrived at, and departed from only once. If locations on dredge cuts through which a dredger has to pass can be defined by nodes, then the decision variable stated in Equation 3.17, the objective function stated in Equation 3.18 and the constraints stated in Equations 3.19, 3.20, 3.21 and 3.22 can all be directly applied to a dredger routing problem. Methods for optimizing two-dimensional stock cutting and symmetric travelling salesperson problems are discussed in more detail next.

3.5 Optimization Methods

In Section 3.3 it is argued that the dredge cut nesting problem can be modelled as an irregular two-dimensional stock cutting problem and Section 3.4 concludes with stating that the dredger routing problem can be modelled directly as a travelling salesperson problem. The next step is to search for optimal solutions to the modelled problems. For this a suitable optimization method has to be identified. As part of classifying methods used for solving combinatorial optimization problems, Voudouris *et al*. (1999) distinguishes between heuristics and meta-heuristics, defining heuristics as optimization methods using local search algorithms and meta-heuristics as optimization methods using tabu search, genetic and simulated annealing algorithms.

Voudouris *et al*. (1999) notes that many heuristic methods use local search, also sometimes referred to as neighbourhood search or hill climbing, to solve combinatorial optimization problems. Hill climbing is an iterative method where trial and error is used for finding good approximations of optimal solutions (*ibid*.). A hill climbing algorithm repeatedly compares solutions, the very first being arbitrary, with neighbouring solutions, continually storing the better solutions until no further improvement is possible (*ibid*.). Accepting a better solution can be done in various ways: For example, first improvement local search accepts a better solution when it is found, whereas best improvement (greedy) local search replaces a current solution with the most improved solution of searched neighbourhood solutions (*ibid*.). In general, the larger the neighbourhood searched around a particular solution is, the more time is required to search it, but the better the final solution arrived at is (*ibid*.). According to Voudouris *et al*. (1999) heuristics can get stuck in local optima, which can give good solutions but are not global optima.

Voudouris *et al*. (1999) states that meta-heuristics aim at enhancing the performance of heuristics by allowing for the possibility of escaping from local optima so that global optima can be found (*ibid*.). Tabu search, genetic and simulated annealing algorithms are meta-heuristics which make use of such stochastic local search techniques (*ibid*.). Tabu search algorithms contain built-in memory mechanisms which prevent returning to recently executed changes to solutions for a number of iterations (Hopper *et al*. 2001). Genetic and simulated annealing algorithms do not contain such mechanisms. However, tabu search, genetic and simulated annealing algorithms all allow for selecting a solution worse than the current one, and it is this common feature which reduces the possibility of getting stuck in local optima (Voudouris *et al*. 1999). The selection of a heuristic or a meta-heuristic optimization method is problem dependent (*ibid*.).

In Sections 3.5.3 and 3.5.4 genetic and simulated annealing algorithms are discussed in more detail. First, two sections are presented in which literature on the optimization of stock cutting and travelling salesperson problems is reviewed.

3.5.1 Stock Cutting Problem Optimization

Elmghraby *et al*. (2000) notes that more than 800 papers have been published on solving stock cutting problems of varying complexity. Sharma *et al*. (1997) distinguished between two main approaches for solving stock cutting problems: One where stencils are placed on the sheet(s) one at a time and the other where all stencils are placed on the sheet(s) simultaneously. Sharma *et al*. (1997) states that simultaneous placement of all stencils leads to better solutions. Lirov (1992) points out that when stock cutting problems are optimized using linear programming two difficulties are encountered: How to generate the set of patterns to be nested and how to compute an integer solution from the generally fractional optimal solution returned by linear programming solvers. The second difficulty, however, can be overcome by a rounding algorithm proposed in Johnston (1986).

Bennell *et al*. (2001) states that researchers have had considerable success in solving two-dimensional stock cutting problems with irregular stencils and regular sheets by using linear programming compaction methods. However, Bennell *et al*. (2001) notes that good starting solutions are required since linear programming compaction methods have difficulty in allowing for significant changes to stencil positions such as the movement of a stencil from one end of a sheet to another. A good starting solution can be determined with a heuristic model which mimics the strategies employed by human experts (*ibid.*).

Degraeve *et al*. (2001) proposes two mixed-integer linear programming models for solving two-dimensional stock cutting problems in the garment industry and demonstrates that both outperform an earlier proposed mixed-integer linear programming model. The stock cutting problem solved in Degraeve *et al*. (2001) involves the stacking and cutting of multiple layers of fabric of fixed length and width into groups of stencils of equal length. Degraeve *et al*. (2001) overcomes the problem posed by a non-linear variable, which defines the number of copies of a group of stencils, by applying a discrete expansion to it and by linearizing the product of this variable with the number of fabric layers.

Chen *et al*. (2002) proposes rectilinear representation of irregularly shaped stencils to be nested on a rectangular sheet in order to limit spatial calculations to integer coordinates and to make checking for overlap between stencils easier. After rectilinear representation of stencils, Chen *et al*. (2002) applies a two-stage heuristic method to find suitable combinations of two stencils. In the first stage, the fitness of each combination of two stencils in varying positions is rated by summing the straight edges of combined shapes, a value of 4 – signifying a rectangle – being best. In addition, the fitness of each combination of stencils is rated in terms of wasted sheet area. After identifying the best combination of stencils the second stage of the optimization method is effected, which consists of a ‘left-most bottom-most’ compaction procedure to nest combinations of stencils. Chen *et al*. (2002) reports that the heuristic optimization method presented finds better solutions to two-dimensional stock cutting problem than the genetic algorithm optimization method proposed in by Sakait *et al*. (1998).

To overcome the complexity of calculating overlap of irregularly shaped stencils Kim *et al*. (2003b) includes a polygon clipping algorithm in a heuristic optimization method for solving two-dimensional stock cutting problems. The polygon clipping algorithm used in Kim *et al*. (2003b) calculates areas of union, intersection, and difference between two polygons, and is very similar to the algorithm proposed in Weiler (1980). As mentioned in Section 3.3.3, Yuping *et al*. (2005) uses the Weiler algorithm to calculate overlap, non-placement and escape for leather nesting problems .

Milenkovic *et al*. (1992) uses a dynamic programming model with a top-down solution approach to optimize two-dimensional stock cutting problems in the garment industry. The dynamic programming model proposed in Milenkovic *et al*. (1992) condenses constraints of previously solved sub-problems into constraints for solving the latest sub-problem, which in turn are used to obtain constraints for following sub-problems. Elmaghraby *et al.* (2000) also uses a dynamic programming model with a hierarchical structure, the highest level of which is an expert system, to optimize two-dimensional stock cutting problems. Expert systems contain subject-specific intelligence and information found in the intellect of experts translated into a set of rules with which specific problems are analyzed and are designed to carry knowledge to other members of an organization for problem-solving purposes (*ibid*.). Computer programs of expert systems usually recommend a course of user action in order to improve solutions to a problem (*ibid*.). Expert systems use what appear to be reasoning capabilities to reach conclusions and are valuable to organizations where high levels of experience and expertise are not easily transferable (*ibid*.). The computer program of the expert system proposed in Elmaghraby *et al.* (2000) consists of a graphic interface where the user is required to give an accurate description of the cutting problem to be solved. The computer program then decides which solution approach is best to adopt to solve the problem given by the user.

Hopper *et al*. (2001) reviews the application of meta-heuristic algorithms, in particular genetic algorithms, to two-dimensional packing problems and cites over 70 references of which 15 are other reviews and surveys of packing problems. Smith (1985) gives one of the earliest proposals for using genetic algorithms to solve regular two-dimensional bin packing problems. Later applications of genetic algorithms to optimize strip packing problems can be found in Jakobs (1996) and Liu *et al.* (1999). Smith (1985) compares the performance of genetic algorithms with heuristic and dynamic programming optimization methods and concludes that genetic algorithms achieve very similar packing densities, but in less time. It must be noted, however, that the optimization methods used in Smith (1985), Jakobs (1996) and Liu *et al.* (1999) only permit rotation of stencils in steps of 90 degrees.

Out of 36 papers presenting methods for optimizing cutting and packing problems compared in Hopper *et al*. (2001) 30 papers propose methods which either do not allow, or only allow for restricted rotation of parts to be nested, while 6 papers present solution approaches which allow for free rotation of stencils. Of the 6 solution approaches which allow free rotation reviewed by Hopper *et al*. (2001), one uses a genetic algorithm, three use hybrid genetic algorithms and the remaining two use simulated annealing algorithms to search for global optima.

The end of Section 3.3 suggests that genetic and adaptive simulated annealing algorithms are natural candidates for optimizing irregular dredge cut nesting problems as both algorithms have been successfully used to optimize irregular two-dimensional stock cutting problems (Heistermann *et al*., 1995, Yuping *et al*., 2004). The literature reviewed in this section can be said to support this view: Genetic and simulated annealing algorithms can arrive at solutions which are global optima and do not give rise to complications in representing stencils as experienced when using linear programming or require the formulation of a specific set of nesting rules as with expert systems. Literature on methods for optimizing travelling salesperson problems is reviewed next.

3.5.2 Travelling Salesperson Problem Optimization

Helsgaun (2000) states travelling salesperson problems have been proven to be NP-hard problems and that attempts to construct a general algorithm for finding optimal solution tours in polynomial time are unlikely to succeed. Helsgaun (2000) divides algorithms used for solving travelling salesperson problems into two groups: Exact algorithms; and heuristic (or approximation) algorithms. Schrage (1999) notes that the difficulty with optimizing travelling salesperson problems with linear programming lies in the fact that solutions to large models tend to contain sub-tours. A sub-tour is a tour of a subset of assignments which is not connected to the main tour. Constraints can be added to break sub-tours, but the number of constraints required grows disproportionately as the number of assignments increase.

Lin *et al*. (1973) states that solving large travelling salesperson problem models is best approached using heuristic algorithms. Voudouris *et al*. (1999) examines how guided local search, a form of tabu search, combined with fast local search can be applied to travelling salesperson problems. Guided local search is a meta-heuristic optimization method applicable to a wide range of combinatorial optimization problems (*ibid*.). Guided local search has been successfully applied to optimize practical problems such as workforce scheduling and vehicle routing (*ibid*.). Guided local search sits on top of local search, the main aim of which is to efficiently and effectively explore large search spaces of combinatorial optimization problems (*ibid*.). Voudouris *et al*. (1999) combines guided local search with fast local search to optimize travelling salesperson problems to limit the amount of neighbouring solutions explored.

In Voudouris *et al*. (1999) the guided local search technique increases the cost of solutions to travelling salesperson problems by applying a set of penalty terms, similar to what Yuping *et al*. (2005) does for irregular leather nesting problems. In Voudouris *et al*. (1999) the cost function of a travelling salesperson problem, instead of the penalties, is passed on to the local search technique for optimization. The local search algorithm is confined by the penalty terms of the cost function and therefore focuses on promising regions of the search space, hence the name fast local search (*ibid*.). Each time fast local search gets caught in a (local) minimum, the penalties are modified by guided local search and the fast local search technique is called again to optimize the modified cost function (*ibid*.). Fast local search breaks down the current neighbourhood into sub-neighbourhoods, each of which is assigned a 0 or 1 (*ibid*.). The fast local search scans sub-neighbourhoods in a given order, searching only active sub-neighbourhoods, those which are assigned a 1 denoting they are active (*ibid*.).

In the fast local search technique of Voudouris *et al*. (1999), all sub-neighbourhoods are initially active, but when a sub-neighbourhood is examined and does not contain any improving solution changes, to a tour for example, it becomes inactive. Only sub-neighbourhoods from which improving solution changes are accepted remain active (*ibid*.). When an improved solution is accepted, sub-neighbourhoods from which other solution improvements are expected are re-activated (*ibid*.). Improved solutions are accepted until the fast local search process dies out as a result of all sub-neighbourhoods gradually becoming inactive and at that moment the solution found is returned as an approximate local minimum (*ibid*.).

Although, according to Voudouris *et al*. (1999), fast local search techniques do not generally find very good solutions, when they are combined with guided local search they accelerate the optimization of combinatorial problems because fast local search focuses on removing the penalized features from the solution instead of considering all possible solutions. Voudouris *et al*. (1999) finds that a particular variant of the proposed combined guided and fast local search technique performs better than variants of other search methods such as simulated annealing and tabu search.

Likas *et al*. (2002) presents a modified nearest neighbour search strategy for optimizing travelling salesperson problems, where search strategies encoded in a string are used for tour construction. At given points in time, the search strategy can be formulated to overrule the nearest neighbour selection rule by specifying the selection of, for example, the second or third nearest available neighbour or city instead (*ibid*.). The algorithm used in Likas *et al*. (2002) consists of two stages. First, randomized local search with a specified search strategy is used to find the city from where to best start valid tours and this city is then considered as the only initial city in the second stage. In the second stage, the travelling salesperson problem is further optimized by repeatedly evaluating minor changes to current best states. Likas *et al*. (2002) effects minor changes to current best states in a manner analogous to the mutation operation of genetic algorithms. Likas *et al*. (2002) tests the modified greedy heuristic approach on a number of travelling salesperson problems, ranging from 10 up to 2428 cities. For 428 smaller travelling salesperson problems Likas *et al*. (2002) finds optimal solutions 95% of the time while for 325 larger problems optimal solutions are found 71% of the time.

Helsgaun (2000) states the Lin-Kernighan heuristic is considered to be one of the most effective methods for generating optimal or near-optimal solutions for symmetric travelling salesperson problems. However, Helsgaun (2000) adds that the design and implementation of an optimization method including the Lin-Kernighan algorithm is not straightforward. Helsgaun (2000) states that the Lin-Kernighan heuristic uses a variable tour edge exchange algorithm, where tour edge connects two nodes of a travelling salesperson problem.

The original Lin-Kernighan algorithm starts by considering a random initial tour and then executes a search strategy which attempts to find two sets of valid tour edges, for increasing values of lambda (signifying the number of tour edges to be exchanged), which when exchanged, possibly reversing some tour edges, result in a shorter valid solution tour (*ibid*.). The improved tour becomes the current solution tour and the process of finding an improved solution tour is repeated (*ibid*.).

To shorten solution times the Lin-Kernighan algorithm contains additional heuristic rules such as: A previously broken tour edge must not be added; a previously added tour edge must not be broken; the search for a tour edge to be added is limited to being within 5 nodes of the current tour node under consideration; and the algorithm is stopped when the current tour is the same as the previous solution tour (*ibid*.). Lin-Kernighan algorithms are usually executed more than once with different random initial tours to obtain better final results (*ibid*.).

Helsgaun (2000) proposes a modified Lin-Kernighan algorithm for optimizing symmetric travelling salesperson problems. The algorithm proposed in Helsgaun (2000) differs from the original Lin-Kernighan algorithm by adopting a different search strategy: It uses larger and more complex search steps, and uses sensitivity analysis to direct and restrict the search. Helsgaun (2000) states that although the modified Lin-Kernighan algorithm presented is an approximation algorithm it finds global optima for travelling salesperson problems of up to 13,509 cities.

Other optimization methods applied to travelling salesperson problems have used neural networks (Hasegawa *et al*., 2002, Cochrane *et al*., 2003, Mulder *et al*., 2003) and ant colony systems (Tsai *et al*., 2003). Hasegawa *et al*. (2002) demonstrates that extending tabu search to include neural network optimization techniques can be used to solve large travelling salesperson problems of up to 85,900 cities, and results in finding better solutions than the use of conventional tabu search alone.

Cochrane *et al*. (2003) proposes a self-organizing neural network optimization technique, which is tested on various travelling salesperson problems, the largest of which involves 85,900 cities. The neural network optimization technique Mulder *et al*. (2003) proposes for solving a hypothetical million city travelling salesperson problem does not give solutions which are better than obtained with other heuristic optimization methods derived from the Lin-Kernighan algorithm.

Tsai *et al*. (2003) describes ant colony systems as meta-heuristic optimization methods for solving combinatorial optimization problems which simulate the ability of ant colonies to determine the shortest paths to food. Tsai *et al*. (2003) concludes ant colony systems are best coupled with nearest neighbour search algorithms.

Henderson *et al*. (2003) solves shortest route cut and fill problems by applying a nearest neighbour algorithm which begins at an arbitrary cut location, then moves to the nearest fill location, from where a move is made to the nearest remaining cut location, and so on until the project site is levelled. Henderson *et al*. (2003) compares solutions obtained with the nearest neighbour algorithm to those obtained with a simulated annealing algorithm and concludes the simulated annealing algorithm performs better than the nearest neighbour algorithm. Hurkens *et al*. (2004) states that even for small Euclidean travelling salesperson problems solutions determined with nearest neighbour algorithms can be of low quality.

In summary, genetic algorithms do not appear to be the preferred choice of heuristic algorithm for researchers to optimizing travelling salesperson problems. Simulated annealing has been used to solve travelling salesperson problems in Henderson *et al*. (2003), but according to Hurkens *et al*. (2004) simulated annealing algorithms can only slightly improve on low quality solutions arrived at with nearest neighbour algorithms. As stated at the end of Section 3.3, it is thought genetic and simulated annealing algorithms can be successful at solving dredge cut nesting problems. However, it is not sure if the same can be said for solving dredger routing problems. To see if genetic and simulated annealing algorithms can be used to solve dredge cut nesting as well as dredger routing problems, these algorithms are looked at in more detail in Sections 3.5.3 and 3.5.4, which are next.

3.5.3 Genetic Algorithms

Sharma *et al*. (1997) defines genetic algorithms as search algorithms which use tools inspired by natural selection and genetics. These tools consist of concepts such as inheritance, mutation, selection, and crossover. Sharma *et al*. (1997) states that in genetic algorithms candidate solutions to an optimization problem are referred to as phenotypes (individuals) and abstract representations of these are referred to as genotypes (chromosomes or genomes) where candidate solutions are often represented by binary strings of 0s and 1s.

A genetic algorithm usually starts with a large and diverse initial population of random solutions from which subsequent populations are generated. This is done by: Evaluating the fitness of each individual in the current population; followed by stochastic selection of individuals from the current population according to their fitness; and lastly by modifying the selected individuals through crossover and mutation to form the next population of individuals. The gradual improving fitness of generated populations eventually allows for the identification of an optimal or near-optimal solution (*ibid*.). Figure 3.11 depicts how a genetic algorithm can be used to arrive at new population generations.

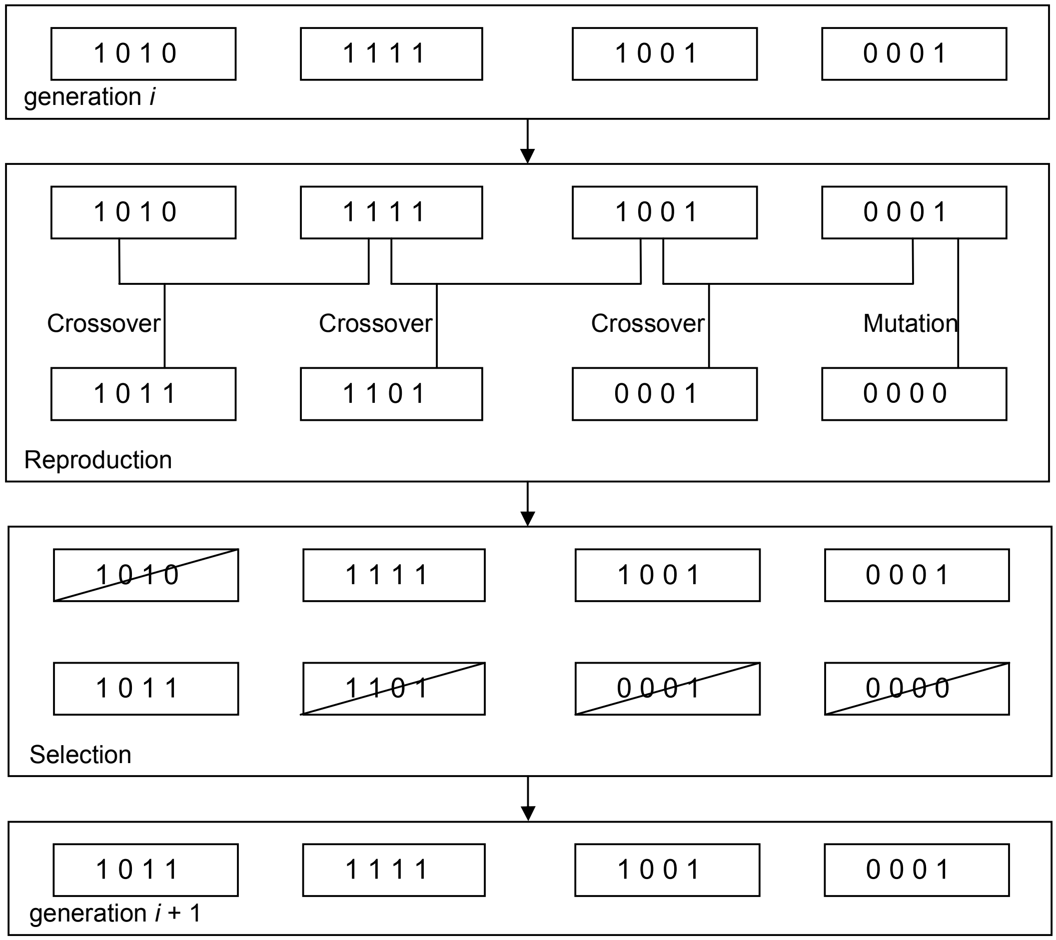


Figure 3.11 Principle workings of genetic algorithms (Knaapen *et al*., 2002).

Knaapen *et al*. (2002) states genetic algorithms are robust optimization methods which find solutions near to global optima because the selection of worse solutions is possible and therefore allow for escaping from local optima. Gradient or hill ascent or descent methods on the other hand only accept improved solutions, which is why they can get stuck in local optima. Hinterding *et al*. (1994) and Wagner (1999) use genetic algorithms to solve one-dimensional stock cutting problems. Heistermann *et al*. (1995) and Sharma *et al*. (1997) use genetic algorithms to solve irregular two-dimensional stock cutting problems. Heistermann *et al*. (1995) and Sharma *et al*. (1997) both represent stencils and sheets as polygons which can be convex or non-convex. While Heistermann *et al*. (1995) opt for placing stencils sequentially, Sharma *et al*. (1997) place stencils simultaneously.

Both methods in Heistermann *et al*. (1995) and Sharma *et al*. (1997) can accommodate holes in stock sheets and areas of varying quality as found in, for instance, leather stock sheets. The genetic algorithm solution approaches proposed in Heistermann *et al*. (1995) and Sharma *et al*. (1997) do however rely on other algorithms to, for example: Decompose non-convex stencils into convex parts; translate and rotate stencil polygons; and determine if two polygons intersect.

Heistermann *et al*. (1995) reports that trials with a computer program using a simulated annealing algorithm to solve irregular two-dimensional stock cutting problems gave worse results than when a genetic algorithm was used. Heistermann *et al*. (1995) states that simulated annealing algorithms require more run time and/or computer resources and lack flexibility in comparison to genetic algorithms. Sharma *et al*. (1997) and Heistermann *et al*. (1995) both make good cases for using a genetic algorithm solution approach to optimize irregular two-dimensional stock cutting problems, especially since Heistermann *et al*. (1995) states that the genetic algorithm method it proposes has been in industrial use for leather manufacturing since 1992.

The industrial software package for leather nesting described in Heistermann *et al*. (1995) consists of 115,000 lines of code in the standard C language. However, as stated at the end of Section 3.5.2, genetic algorithms are not the preferred solution approach for travelling salesperson problems. Since Yuping *et al*. (2005) and Henderson *et al*. (2003), respectively, use simulated annealing algorithms to solve irregular two-dimensional stock cutting problems and symmetric travelling salesperson problems, it is thought a simulated annealing algorithm can be used to solve dredge cut nesting as well as dredger routing problems. Simulated annealing algorithms are reviewed in more detail in Section 3.5.4, which is next.

3.5.4 Simulated Annealing Algorithms

Metropolis *et al*. (1953) first introduced the idea of simulated annealing. The *Metropolis* algorithm simulates the process of cooling material in a heat bath, known as annealing. When a solid system of atoms is heated past its melting point and then cooled, the structural properties of the particle system during cooling, or how it crystallizes, depend on the rate of cooling applied. The energy of atoms is temperature dependent: The higher the temperature of a material, the higher the energy of the atoms in that material. The higher energies of atoms at higher temperatures enable them to restructure themselves with greater ease than at lower temperatures where they have lower energies.

When temperature is reduced the energy of atoms is also decreased. If cooled slowly enough large crystals tend to form, but if cooled too quick, or quenched, risks of crystal formation which contain imperfections increase. A gradual fall to lower energy states is said to allow for the formation of a more regular crystalline structure. In thermodynamics, the probability of a system in equilibrium assuming a higher energy state is given by:

$$P(\Delta E) = exp(-\Delta E / kT) \tag{3.23}$$

where: P = probability; ΔE = energy increase; k = Boltzmann's constant and T = temperature.

The Metropolis algorithm compares the energies of two successive states of a given system of particles, the new state being a modified version of the preceding state. If the energy of the modified state is lower then the system is automatically moved to the new state. However, if the energy of the modified state is higher then the new state is accepted only when the probability returned by Equation 3.23 satisfies an acceptance criterion. The Metropolis algorithm repeatedly compares the energies of old and new states of a system of particles at ever decreasing temperatures until a steady, or frozen, state is arrived at. The final steady state arrived at is a state at which the system of particles has a very low, possibly the lowest, energy level.

Three decades after the Metropolis algorithm was proposed Kirkpatrick *et al*. (1983) used it for optimizing Operations Research problems by modelling variables of these problems as the particles in a system of atoms: At high temperatures system variables are given wide ranges of random values which they can assume, and as temperatures are gradually reduced the ranges of values which can be assumed are also reduced. Kirkpatrick *et al*. (1983) use Equation 3.24 to give the probability of accepting a worse state in the simulated annealing process.

$$P = exp(-\Delta \underline{L} / T) \tag{3.24}$$

where: P = acceptance probability; $\Delta \underline{L}$ = change in objective function value; and T = temperature.

Kirkpatrick *et al*. (1983) calls Equation 3.24 the *Acceptance Function*. In Equation 3.24 Boltzmann's constant of Equation 3.23 is left out as it serves to cope with varying materials, which is not required in the modelling of Operations Research problems. Despite the omission of Boltzmann's constant, simulated annealing is often referred to as *Boltzmann* annealing. Kirkpatrick *et al*. (1983) defines the *Acceptance Criterion* of simulated annealing algorithms as follows:

$$P > r \tag{3.25}$$

where: P = acceptance probability; and r = random number between 0 and 1.

Equations 3.24 and 3.25 give simulated annealing algorithms the capability of accepting worse solutions, which, as with genetic algorithms, allows for escaping local optima. Figure 3.12 depicts an overview of the simulated annealing algorithm after Heaton (2005).

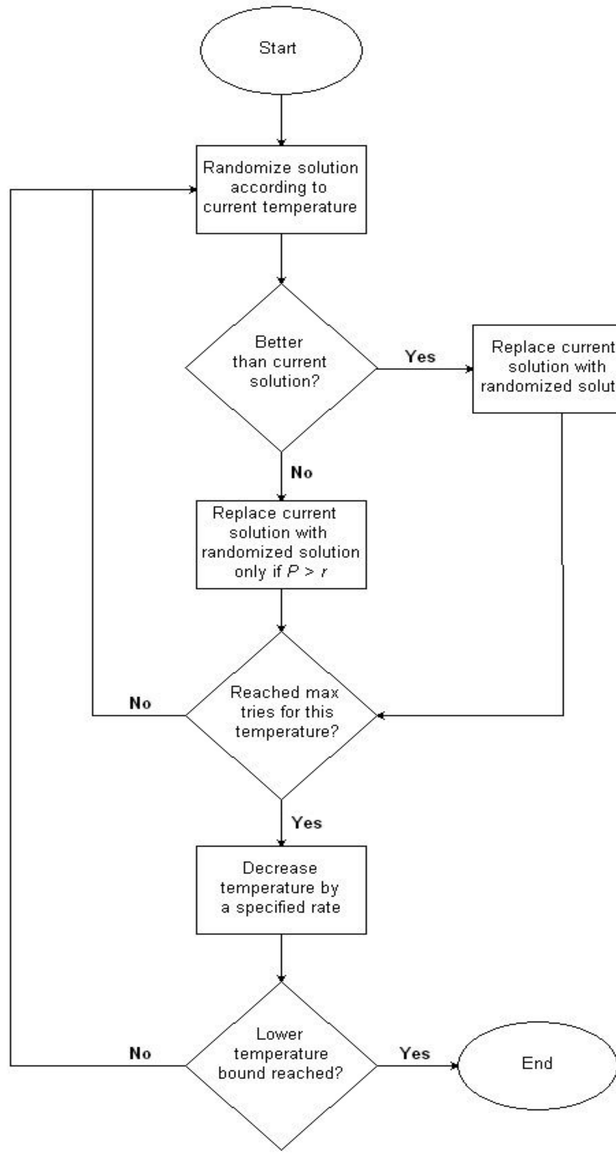


Figure 3.12 Simulated Annealing algorithm overview (Heaton, 2005).

The randomized process used for modifying current solutions of a modelled problem mentioned in Figure 3.12 is temperature dependent. Henderson *et al*. (2003) states this process combines the use of a neighbourhood function and a probability function.

For the optimization of NP-hard combinatorial problems Henderson *et al*. (2003) gives the following pseudo-code of the simulated annealing algorithm:

Select initial solution, ω, from solution space, Ω;
Set temperature change counter, *p*, to zero;
Select number of temperature changes, *M*;
Select temperature cooling schedule, *T*(*p*);
Select initial temperature $T(0) \geq 0$;
Select repetition schedule, *N*(*p*), i.e. number of iterations executed at each temperature, *T*(*p*);
Repeat……………………………………...*[start outer loop]*
  Set repetition counter, *q*, to zero
  Repeat…………………………………... *[start inner loop]*
    Generate modified solution, ω', using probability generation function, *g*(*T*), and neighbourhood function, $\eta(\omega)$;
    Calculate cost difference, δ, from objective function values: $f(\omega')$ minus $f(\omega)$
    If $\delta \leq 0$, then $\omega \leftarrow \omega'$
    If $\delta > 0$, then $\omega \leftarrow \omega'$ subject to probability, $e^{-\delta / T(p)}$, satisfying acceptance criterion
    $q \leftarrow q + 1$
  Until $q = N(p)$………………………………*[end inner loop]*
  $p \leftarrow p + 1$, $T \leftarrow T(p)$
Until stopping criterion is met or $p = M$……*[end outer loop]*

The simulated annealing algorithm has two loops: An outer and an inner loop. The outer loop carries out the temperature cooling schedule: It executes the pre-defined number of annealing temperature reductions and continues until a stopping criterion is met or the lower temperature bound is reached. A stopping criterion, for example, can relate to the occurrence of a certain value of an objective function, for example, the optimum value if it is known. The inner loop carries out the repetition schedule: It executes the pre-defined number of iterations at each annealing temperature, thereby defining the length of each annealing temperature interval.

Desai *et al*. (1995) states that the simulated annealing algorithm statistically promises to deliver a global optimum providing, as Henderson *et al*. (2003) states, the cooling schedule is not too fast. The slowest cooling schedules for simulated annealing algorithms for which they are guaranteed to find a global optimum of non-convex cost-functions depend on which probability generation function is used to identify neighbouring solutions (Ingber, 1989).

Geman *et al*. (1984) proves that for simulated annealing algorithms which use the Boltzmann distribution as their probability generation function a global optimum can be obtained if the annealing temperature T is not annealed faster than:

$$T(k) = T(0) / \ln M \tag{3.26}$$

where: $T(0)$ = initial temperature; and M = number of temperature changes.

Ingber (1989) points out that simulated annealing algorithms can also make use of other "*reasonable*" probability generation functions, which do not necessarily reflect principles underlying the ergodic nature of statistical physics. For example, fast annealing, or *Cauchy* annealing uses the Cauchy distribution as its probability generation function (*ibid*.). Szu *et al*. (1987) concludes that using the Cauchy distribution has advantages over use of the Boltzmann distribution and demonstrates statistically that when using the Cauchy distribution as a probability generation function a global optimum is found if the annealing temperature T is not annealed faster than:

$$T(k) = T(0) / M \tag{3.27}$$

where: $T(0)$ = the initial temperature; and M = the number of temperature changes.

Equations 3.26 and 3.27 show that Cauchy annealing has an annealing schedule which is exponentially faster than Boltzmann annealing. Ingber (1989) states that many optimization problems have multiple parameters in varying dimensions, each of which have their own distinct finite range of values, and which exhibit varying annealing-time dependent sensitivities. Boltzmann and Cauchy annealing have probability generation functions which sample infinite ranges and therefore cannot account for problem parameters with varying sensitivities (*ibid*.). Ingber (1989) therefore proposes a modified simulated annealing algorithm, called *Adaptive Simulated Annealing*, to overcome the limitations of Boltzmann and Cauchy annealing when dealing with multiple parameters in varying dimensions and of different sensitivities.

The adaptive simulated annealing algorithm considers each problem parameter with a finite range in a dimension at a given annealing-time separately by allocating it its own parameter annealing temperature (*ibid*.). The modified value of a parameter variable at annealing-time plus one is calculated by adding a fraction of its finite range to its previous value. To arrive at a modified value of a problem parameter Ingber (1989) first defines their finite ranges:

$$\alpha_n^i \in [A_i, B_i] \tag{3.28}$$

where: α_n^i = problem parameter in dimension *i* at annealing-time *n*; A_i = the lower limit of the finite range of problem parameter values; and, B_i = the upper limit of the finite range of problem parameter values.

Next Ingber (1989) defines the finite range of a random variable, used for calculating the parameter range fraction with which a problem parameter's value is modified:

$$y^i \in [-1,1] \tag{3.29}$$

The random variable, y^i, is arrived at with the help of another random variable, u^i, from the uniform distribution ranging from zero to one, defined by Ingber (1989) as follows:

$$u^i \in U[0,1] \tag{3.30}$$

To calculate the value of the random variable, y^i, with which the finite range of a problem parameter is multiplied to arrive at a modified value of the problem parameter Ingber (1989) states the following:

$$y^i = \mathrm{sgn}\left(u^i - \frac{1}{2}\right) t_i \left[\left(1 + \frac{1}{t_i}\right)^{|2u^i - 1|} - 1\right] \tag{3.31}$$

where; t_i = annealing temperature of the problem parameter in dimension *i*; and the sgn function is -1 for all negative values, 0 for 0, and 1 for all positive values.

Ingber (1989) then proposes the following annealing schedule for a parameter temperature, t_i:

$$t_i(n) = t_{0i} \exp(-c_i n^{1/D}) \tag{3.32}$$

where; t_{0i}= initial parameter temperature in dimension i; c_i = parameter tuning factor; n = parameter annealing-time; and, D = number of dimensions of parameter space.

Ingber (1989) states that Equation 3.32 is also used for annealing the cost temperature, values of which influence the outcome of Equation 3.24 – the acceptance function. When using Equation 3.32 for annealing the cost temperature a cost tuning factor is used, which can have a value different from that used for the parameter tuning factor. Equations 3.30 and 3.31 make up the probability generation function of the adaptive simulated annealing algorithm. Table 3.3 gives rounded parameter range multiplication factor values of y^i, calculated with Equations 3.30 and 3.31, for increasing values of parameter annealing-times and the corresponding parameter annealing temperatures for a one-dimensional parameter space. The parameter tuning factor and initial parameter annealing temperature are both equal to unity.

Table 3.3 Adaptive Simulated Annealing parameter range multiplication factors

Parameter Annealing Time	Parameter Annealing Temperature	Random variable, u_i										
n	*t(n)*	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0	1.00000	-1.00	-0.74	-0.52	-0.32	-0.15	0	0.15	0.32	0.52	0.74	1.00
1	0.36788	-1.00	-0.68	-0.44	-0.25	-0.11	0	0.11	0.25	0.44	0.68	1.00
2	0.13534	-1.00	-0.61	-0.35	-0.18	-0.07	0	0.07	0.18	0.35	0.61	1.00
3	0.04979	-1.00	-0.52	-0.26	-0.12	-0.04	0	0.04	0.12	0.26	0.52	1.00
4	0.01832	-1.00	-0.44	-0.19	-0.07	-0.02	0	0.02	0.07	0.19	0.44	1.00
5	0.00674	-1.00	-0.36	-0.13	-0.04	-0.01	0	0.01	0.04	0.13	0.36	1.00
6	0.00248	-1.00	-0.30	-0.09	-0.02	-0.01	0	0.01	0.02	0.09	0.30	1.00
7	0.00091	-1.00	-0.25	-0.06	-0.01	0.00	0	0.00	0.01	0.06	0.25	1.00
8	0.00034	-1.00	-0.20	-0.04	-0.01	0.00	0	0.00	0.01	0.04	0.20	1.00
9	0.00012	-1.00	-0.17	-0.03	0.00	0.00	0	0.00	0.00	0.03	0.17	1.00

Table 3.3 shows that as parameter temperatures are reduced the possibility that a problem parameter is modified by a maximum range value remains throughout. However, the magnitude of modifications to problem parameters for values of the random variable, u_i, other than 0, 0.5 and 1 gradually reduces as simulated annealing progresses. Figure 3.13 depicts a plot of the values given in Table 3.3.

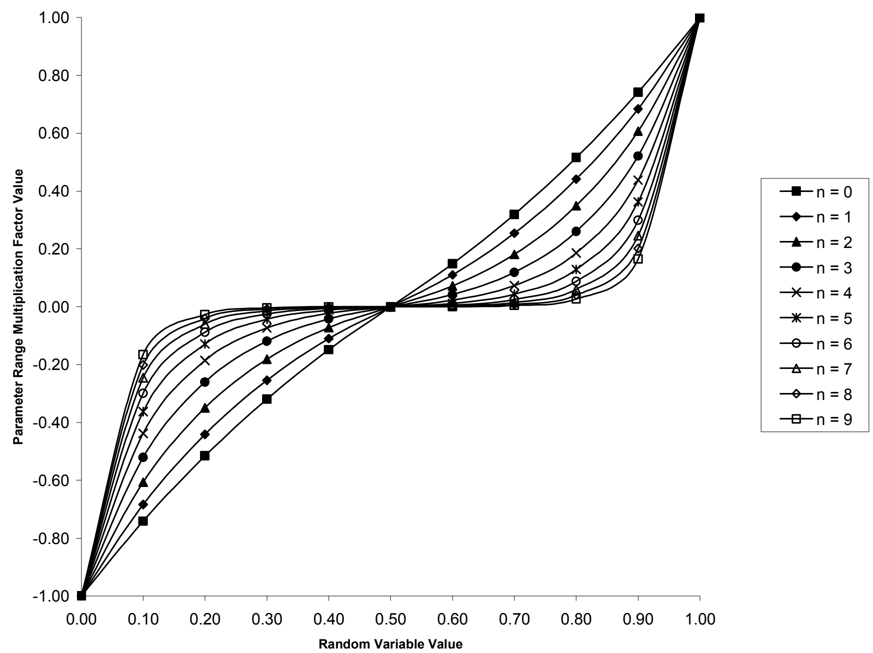


Figure 3.13 Parameter range multiplication factors.

Ingber (1989) states that for the parameter temperature annealing schedule given in Equation 3.32, statistically a global minimum can be found. To maintain this statistical guarantee Ingber (1989) advises to control the tuning factor, c_i, of Equation 3.32 as follows:

$$c_i = m_i \exp(-n_i / D) \qquad (3.33)$$

where; m_i and n_i are control coefficients.

Ingber (1989) states that it has proven useful to anneal the acceptance function (Equation 3.24) in a way similar to the generation function, as done with Equation 3.32, but using the number of acceptance events as a value for the annealing-time instead of the number of generated states, as done for annealing parameter temperatures. An acceptance event is defined as accepting a better solution with an improved objective function value. Ingber (1989) also periodically rescales parameter annealing-times, essentially re-annealing, when a pre-specified number of acceptance events have occurred. The rescaling of parameter annealing-times stretches out the range over which relatively insensitive parameters are being searched in relation to ranges of more sensitive parameters (*ibid*.).

For re-annealing parameter temperatures Ingber (1989) calculates parameter sensitivities, s_i, at the current value of the objective function as follows:

$$s_i = (A_i - B_i)\partial \underline{L} / \partial \alpha^i \tag{3.34}$$

where: A_i and B_i are the lower and upper limits of problem parameter range values, respectively; $\partial \underline{L}$ = the change in objective function value resulting from $\partial \alpha^i$; and $\partial \alpha^i$ = a small change in problem parameter value in dimension *i*.

Ingber (1989) then re-anneals the parameter temperature annealing-time using linear rescaling in reference to the largest parameter sensitivity found, calculating rescaled parameter annealing-times as follows:

$$n'_i = ((\ln[t_{i0} / t_{in})(s_{\max} / s_i)]) / c_i)^D \tag{3.35}$$

where; n'_i = the rescaled annealing-time associated with the parameter temperature; t_{in} = the current parameter temperature; $s_{\max}$ = the largest parameter sensitivity; and s_i = the parameter sensitivity.

Figure 3.14 depicts an overview of the adaptive simulated annealing algorithm after Chen *et al*. (1999). In Figure 3.14 shows the inner and outer loop of the adaptive simulated annealing algorithm: The inner loop continues until the upper limit of the repetition schedule is reached and the outer loop executes the temperature cooling schedule until a stopping criterion is satisfied.

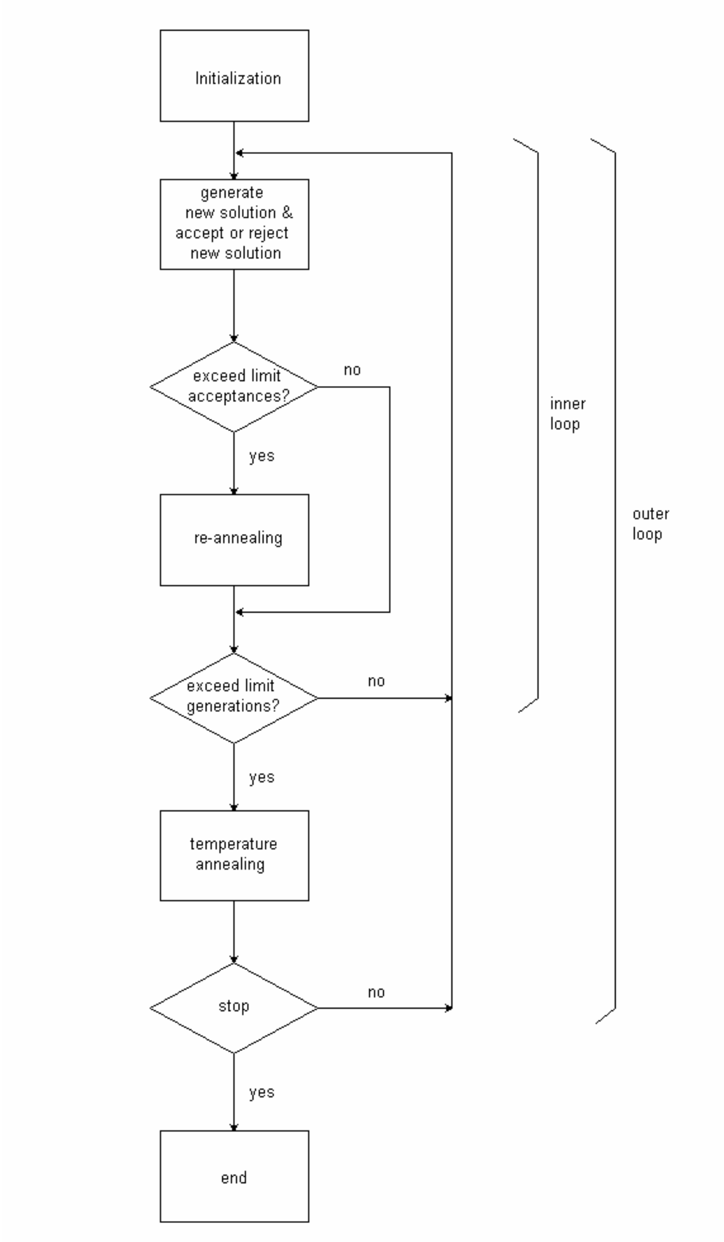


Figure 3.14 Adaptive Simulated Annealing algorithm overview (Chen *et al*., 1999).

The last part of this section concludes the literature review by summarizing applications of the adaptive simulated annealing algorithm to a wide variety of Operations Research problems. Yuping *et al*. (2005) solves irregular leather nesting problems using the adaptive simulated annealing algorithm. Yuping *et al*. (2005) finds that, to solve an irregular nesting problem proposed in Jain *et al*. (1998), the implementation of an adaptive simulated annealing algorithm performs better than that of a genetic algorithm. Yuping *et al*. (1998) states optimal solutions were found more than twenty times faster with adaptive simulated annealing.

Rosen (1992) tests adaptive simulated annealing on finding solutions to a non-differential sample function with a two-dimensional parameter space. Rosen (1992) finds that, for the sample function tested, adaptive simulated annealing vastly outperforms Boltzmann and Cauchy annealing: On average Boltzmann and Cauchy annealing, respectively, find final solutions 90 billion times and 800 million times higher than adaptive simulated annealing, while best final solutions arrived at with Boltzmann and Cauchy annealing, respectively, are 100 million and 17 times higher than those found with adaptive simulated annealing.

Dykes *et al*. (1994) implements a parallel use of adaptive simulated annealing to optimize non-linear, multi-dimensional functions with many local minima, which include the five functions of De Jong's (1981) test suite. De Jong's (1981) test suite is typically used for benchmarking genetic algorithms. Dykes *et al*. (1994) concludes that adaptive simulated annealing is a powerful tool for optimizing difficult functions and that parallelization substantially improves performance, especially when optimizing large parameter spaces of up to 30 dimensions.

Morril *et al*. (1995) uses adaptive simulated annealing to optimize a radiation therapy treatment plan. Morril *et al*. (1995) optimizes treatments for three clinical cases with two cost functions: The first is a linear cost function (minimum target dose) with non-linear dose-volume constraints for normal tissue; and the second is a function of the weighted product of normal tissue complication probabilities and tumour control probability. For both cost functions Morril *et al*. (1995) finds that adaptive simulated annealing can be used for optimizing radiation treatment planning in clinically useful execution times, arriving at results within 3 to 10 percent of the optimal solution found by mixed integer programming.

Wang *et al*. (1997) applies adaptive simulated annealing to optimize the structure determination of bio molecules. Wang *et al*. (1997) optimizes the energy surface of the Met-enkephalin molecule which is subject to a total of 19 variables. Wang *et al*. (1997) carries out 55 independent adaptive simulated annealing runs to experiment with varying initial configurations and cooling schedule settings. Wang *et al*. (1997) concludes that the adaptive simulated annealing is an efficient and robust optimization technique which performs equally well or better than two applications of Boltzmann annealing presented in previous studies on the optimization of surface energies of bio molecules.

Chen *et al*. (1999) notes that many signal processing problems depend on multiple parameters and have non-smooth cost functions making them difficult to solve by gradient ascent/descent optimization techniques because of the presence of local optima and difficulties in calculating gradients. Chen *et al*. (1999) uses adaptive simulated annealing to optimize the signal processing problem of infinite-impulse-response filter design. To reduce the time of finding optimal solutions in comparison to a well tuned genetic algorithm Chen *et al*. (1999) opts for the adaptive simulated annealing algorithm, which converges faster than Boltzmann and Cauchy annealing algorithms. Chen *et al*. (1999) concludes that, in general, simulated annealing algorithms are easier to program and require less tuning than genetic algorithms.

Chen *et al*. (2001) uses adaptive simulated annealing to optimize the signal processing problem of maximum-likelihood joint channel and data estimation. Chen *et al*. (2001) concludes that the efficiency of adaptive simulated annealing is equal to that of better known genetic algorithms and that, therefore, adaptive simulated annealing is a viable alternative to these genetic algorithms for solving signal processing problems with multimodal and non-smooth cost functions.

Zhang *et al*. (2002) uses adaptive simulated annealing to optimize the placing of macro-cells on an analogue integrated circuit, essentially a nesting problem. Zhang *et al*. (2002) notes that on application-specific integrated circuits, analogue circuits occupy smaller areas than digital components. However, analogue circuits require an inversely large proportion of design time and are often responsible for design errors and expensive design changes (*ibid*.). Zhang *et al*. (2002) states that analogue circuit design is more knowledge intensive and generally has more degrees of freedom than digital circuit design. Zhang *et al*. (2002) determines an objective function for analogue circuit design and uses adaptive simulated annealing to optimize the layout of three integrated circuits. For each of the three integrated circuit layouts Zhang *et al*. (2002) finds that adaptive simulated annealing outperforms Boltzmann and Cauchy annealing by finding better objective function values in less time.

Garg *et al*. (2002) optimizes the joint trajectory between the initial and final positions of the end effector of manipulator robots such that actuator torques applied at robot arm joints are minimal. Garg *et al*. (2002) uses a genetic algorithm and adaptive simulated annealing for optimization and finds that, for both single robotic manipulators and two cooperating robotic manipulators, adaptive simulated annealing converged faster to global optima than the genetic algorithm.

Ingber (1992) compares the performance of adaptive simulated annealing with genetic algorithms by solving the five functions of De Jong’s (1981) test suite. Ingber *et al*. (1992) finds that for De Jong’s (1981) test suite adaptive simulated annealing converges faster to global optima than genetic algorithms, and with smaller variances.

The majority of the literature reviewed here which compares the performance of adaptive simulated annealing with that of genetic algorithms is in favour of using adaptive simulated annealing. In addition to better performance, it is thought adaptive simulated annealing is to be chosen here for optimizing dredge cut nesting and dredger routing problems because of the following:

- The statement in Chen *et al*. (1999) that simulated annealing algorithms are easier to program and require less tuning than genetic algorithms,
- The reportedly successful application of adaptive simulated annealing to solve irregular nesting problems in Yuping *et al*. (2005),
- The successful application of simulated annealing to a variant of the travelling salesperson problem in Henderson *et al*. (2003).

Based on the above three findings of other research the use of adaptive simulated annealing as a solution approach is opted for. The hypothesis, objective and scope of the research undertaken here are presented next.

4 Hypothesis, Objective and Scope

The hypothesis of the research presented here is the following:

> *Two-dimensional cut planning for cutter suction dredgers can be modelled as a combination of a modified stock cutting problem and a modified travelling salesperson problem and optimized with adaptive simulated annealing in a computer-based solution approach.*

The objective of the research presented here is to contribute to improving the operational efficiency of cutter suction dredgers by providing a tool with which the optimization of two-dimensional cut planning for such dredgers can be automated. An optimal two-dimensional cut plan for a cutter suction dredger is a plan for excavating a dredging area in which the amount of downtime resulting from non-productive dredger movements in between dredge cuts is minimal.

The scope of the research undertaken here does not aim to determine the most computationally efficient method for solving the research problem and applies to dredging areas which are assumed to be homogenous throughout and which have unrestricted access. It is assumed dredging areas considered are homogenous throughout because models developed here do not take into account varying soil characteristics. It is possible, for example, that higher dredging production rates can be achieved when certain soil strata are excavated in directions other than those suggested by the solutions arrived at with the models used here. In addition, with exception of a reduction in dredging production experienced when dredging head on into previously excavated areas, it is assumed dredging production rates are constant for any width of cut dredged.

It is assumed dredging areas have unrestricted access because in principle the models developed here do not take into account specific site conditions related to, for instance, pre-existing ground and sea bed levels, obstructions, milestone activities, coordination of dredging with other construction activities and temporal variation in sea states, which can affect access to dredging areas and their surroundings. The methods and materials of the research undertaken here are presented next.

5 Materials and Methods

This section lists the experimental equipment, presents the dredge cut nesting and dredger routing models used, and details the experimentation carried out to test the research hypothesis.

5.1 Experimental Equipment

The equipment used for running the dredge cut nesting models consisted of four desktop Dell computers with Microsoft Windows XP operating systems, 1.73 gigahertz Intel Pentium 4 processors and 1 gigabyte of random access memory each. The dredger routing models were run on a portable Hewlett-Packard computer with a Microsoft Windows Vista operating system, a 2.00 gigahertz AMD Sempron processor, and 2 gigabytes of random access memory. The dredge cut nesting and dredger routing models were coded in the programming language Microsoft Visual C# 2005 Express Edition. At the time of writing Microsoft's Visual C# 2008 Express Edition was available for download at www.microsoft.com/express/vcsharp/. The non-standard classes and methods of the dredge cut nesting program consist of around 4,000 lines of code while those of the dredger routing model program total around 3,500 lines of code.

5.2 Dredge Cut Nesting Model

Two-dimensional dredge cut nesting problems are modelled here as a modified two-dimensional stock cutting problem. The modification consists of how problem decision variables are treated for dredge cut nesting in comparison to how they are treated for conventional stock cutting. In conventional two-dimensional stock cutting problems, stock items to be cut are referred to as sheets and the items to be cut from the stock are referred to as stencils. Solutions of two-dimensional stock cutting problems are quantified in terms of three decision variables: Escape, non-placement and overlap. Escape is generally defined as the union area of stencils outside the sheet(s); non-placement as the total sheet area not occupied by stencils; and overlap as the total area of overlap between stencils. Traditionally, two-dimensional stock cutting problems require the minimization of escape (if permitted), non-placement, and overlap (if permitted), where feasible solutions are those which have zero escape and zero overlap since partially cut stencils are not allowed.

For dredge cut nesting problems the sheet represents the area to be dredged and stencils represent individual dredge cuts or combinations thereof. Like stock cutting problems, dredge cut nesting problems require minimization of escape, non-placement and overlap, but dredge cut nesting problems differ fundamentally from conventional two-dimensional stock cutting problems in three respects: 1) While feasible solutions of two-dimensional stock cutting problems can exhibit non-zero non-placement, feasible solutions of dredge cut nesting problems must have zero non-placement; 2) Non-zero non-placement in a solution of a dredge cut nesting problem equates to parts of the dredging area remaining undredged and therefore should not be permitted; 3) Feasible solutions of two-dimensional stock cutting problems require zero escape and zero overlap, while feasible solutions of dredge cut nesting problems do not necessarily require this.

Non-zero escape and non-zero overlap can be permitted in solutions of dredge cut nesting when it is considered impractical to provide sets of different sized stencils from which a subset can be selected to match any dimension of an irregularly shaped dredging area. Provision of such stencil sets is not prioritized here since selected stencils (representing dredge cuts), or parts thereof, which overlap need not be dredged. Equally so, stencils, or parts thereof, which are outside dredging areas can also be left undredged. Table 5.1 summarizes the main differences between conventional two-dimensional stock cutting problems and two-dimensional dredge cut nesting problems.

Table 5.1 Dredge Cut Nesting and Stock Cutting Problem variables

Decision Variable	Two-dimensional Dredge Cut Nesting Problems	Two-dimensional Stock Cutting Problems
Escape	Permitted	Not permitted
Non-placement	Not permitted	Permitted
Overlap	Permitted	Not permitted

In the dredge cut nesting model, sheets and stencils are represented by polygons in two-dimensional space. Polygons representing dredge cut stencils are rectilinear and can be convex or non-convex. Dredge cut stencils can be of any shape and made up of fractions and/or multiples of squares with side lengths equal to or less than the maximum effective cut width which can be achieved with the cutter suction dredger considered. Polygons representing dredging areas are also rectilinear and can be convex or non-convex. Polygons representing dredging areas are called inner sheets, hereafter also referred to as sheets. Polygons representing the union of dredging areas and escape regions, if provided for, are referred to as outer sheets. In the dredge cut nesting model inner and outer sheets have fixed positions. Polygons representing dredge cut stencils have three degrees of motion freedom: Horizontal translation; vertical translation; and rotation. Figure 5.1 depicts the three degrees of motion freedom of stencils.

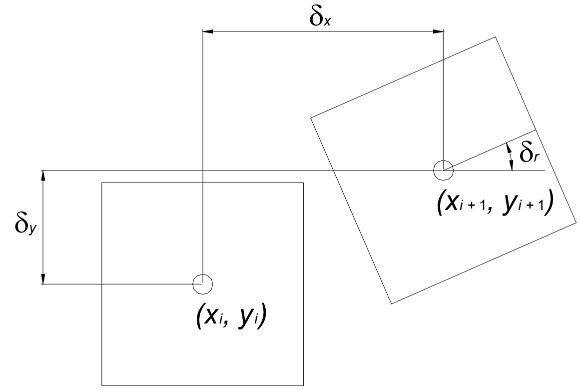


Figure 5.1 Stencil motion freedom (Yuping *et al.*, 2005).

In Figure 5.1 horizontal and vertical stencil translation, δ_x and δ_y respectively, and stencil rotation, δ_r, are related to a fixed reference point of the stencil. In the adaptive simulated annealing algorithm employed in the dredge cut nesting model, each degree of motion freedom of each stencil is assigned its own parameter temperature. Therefore, the dimensions of the parameter space of dredge cut nesting problems equal three times the total number of dredge cut stencils used. For example, a dredge cut nesting problem with 10 stencils has 30 parameter space dimensions and therefore has 30 parameter temperatures. In the adaptive simulated annealing algorithm nesting solutions are modified by repeatedly selecting randomized values for each of the three degrees of stencil motion freedom using the probability generation function for which Equations 3.29, 3.30 and 3.31 are used (see Section 3.5.4). The limits of the vertical and horizontal disturbance range for translation of stencils are set to +/- the maximum dimension of the bounding box of the inner or outer sheet used, whichever is greater. The limits of the disturbance range for rotation of stencils are set at +/- 360 degrees.

When the adaptive simulated annealing algorithm calls for a parameter sensitivity analysis an identical small perturbation is applied separately to each parameter of each stencil to find resulting objective function cost differences with which individual parameter sensitivities are calculated, which are subsequently used for re-annealing parameter temperatures with Equations 3.34 and 3.35 (see Section 3.5.4).

The decision variables of the dredge cut nesting problem are escape, non-placement and overlap, which are mathematically defined as follows:

$$A_{escape} = \left(\bigcup_{i=1}^{N} s_i \right) \cap \left(\bigcup_{j=1}^{M} e_j \right) \tag{5.1}$$

where: N = total number of stencils; s_i = area of stencil i ; M = total number of escape regions; and e_j = area of escape region j.

$$A_{non-placement} = \left(\bigcup_{k=1}^{K} L_k \right) - \left[\left(\bigcup_{i=1}^{N} s_i \right) - A_{escape} \right] \tag{5.2}$$

where: K = total number of sheets; L_k = area of sheet k; N = total number of stencils; and s_i = area of stencil i

$$A_{overlap} = \sum_{i=1}^{N} s_i - \left(\bigcup_{i=1}^{N} s_i \right) \tag{5.3}$$

where: N = total number of stencils; and s_i = area of stencil i.

The objective function of the dredge cut nesting problem is expressed mathematically as follows:

$$\text{minimize } Z = \alpha_{esc} \left(A_{escape} \right)^{\beta_{esc}} + \alpha_{npl} \left(A_{non-placement} \right)^{\beta_{npl}} + \alpha_{ovl} \left(A_{overlap} \right)^{\beta_{ovl}} \tag{5.4}$$

where: Z = total cost; α_{esc} = escape penalty factor; β_{esc} = escape penalty exponent; α_{npl} = non-placement penalty factor; β_{npl} = non-placement penalty exponent; α_{ovl} = overlap penalty factor; and β_{ovl} = overlap penalty exponent.

Subject to the constraints:

$$A_{non-placement} = 0 \tag{5.5}$$

The dredge cut nesting model uses standard C-sharp classes and methods for the purpose of polygon comparison in order to calculate escape, non-placement and overlap. The model of the dredger routing problem is presented next.

5.3 Dredger Routing Model

The dredger routing problem is modelled as a modified travelling salesperson problem of variable asymmetry, where planar coordinates of centroids of the square stencil components representing dredge cuts are used to represent nodes in Euclidean space. The coordinates of centroids are extracted from final nest solutions of the associated dredge cut nesting problem. The main modification consists of how problem decision variables are treated for dredger routing problems in comparison to how they are treated in a conventional travelling salesperson problem. The dredger routing model used here takes into account tour lengths as well as turning angles measured between tour edges whereas conventional travelling salesperson problems traditionally only concern themselves with tour lengths.

The parameter space of the dredger routing problem is considered to be dimensionless since the positions of nodes are fixed and the edge exchange mechanism used for modifying tours is also fixed. In the code of the dredger routing model, however, the parameter space dimension is set to unity to not divide by zero in Equations 3.32 and 3.33 (see Section 3.5.4). To modify dredger routes a fixed 2-opt edge exchange mechanism is used throughout the solution process. Figure 5.2 depicts the concept of a 2-opt edge exchange mechanism.

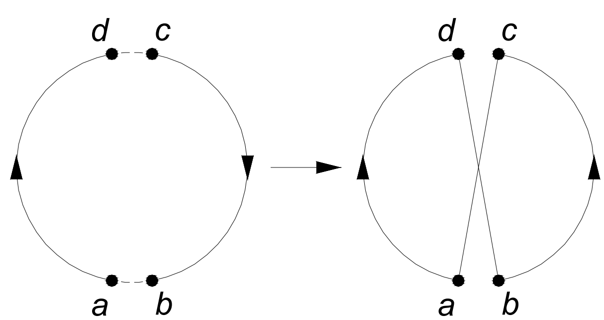


Figure 5.2 Two-opt edge exchange mechanism (Helsgaun, 2000).

In Figure 5.2 the 2-opt exchange mechanism converts the tour denoted by [*a*, *b*, *c*, *d*] into the tour denoted by [*a*, *c*, *b*, *d*] by replacing two edges. The replacement consists of: An exchange of the edge between nodes *a* and *b* with the edge between nodes *a* and *c*; and an exchange of the edge between nodes *c* and *d* with the edge between nodes *b* and *d*. Considering dredger routing problems as dimensionless renders the part of the adaptive simulated annealing algorithm where parameter temperatures are annealed, re-annealed and subjected to sensitivity analysis obsolete. However, annealing and re-annealing of the acceptance criterion is kept in place and therefore the optimization process can still be considered as one of adaptive simulated annealing. The dredger routing model requires the user to specify a node to define the location where dredging commences. This option is included because in practice it is not necessary to end dredging at the same node from where it started. In addition, although dredging areas are considered to have unrestricted access, the need to specify a start location for dredging can arise in practice when, for example, pre-dredging depths limit the number of locations where dredging can commence as a result of minimum water depth requirements imposed by the draft of the cutter suction dredger considered for use.

The dredger routing model considers the length of the edge between the last tour node and the start node equal to zero. Despite ignoring the length of the last tour edge the constraint that solutions must form a tour is kept in place to maintain the basic model structure of the travelling salesperson problem in the developed code. For the same reason as to why the length of the last tour edge is considered zero, the turning angles between the penultimate and the last, and the last and the first edge of tours are considered zero. The dredger routing model problem has two decision variables, which are defined as follows:

$$x_{ij} = \begin{cases} 1, \text{ if node } j \text{ is reached from node } i \\ 0, \text{ otherwise} \end{cases} \tag{5.6}$$

$$y_{ijk} = \begin{cases} 1, \text{ if node } k \text{ is reached from node } j \text{ and node } j \text{ is reached from node } i \\ 0, \text{ otherwise} \end{cases} \tag{5.7}$$

The model of the dredger routing problem quantifies tour edge lengths and sums of turning angles. The tour length is defined as follows:

$$L_{TOUR} = \sum_{i=1}^{N} \sum_{j=1}^{N} F_j d_{ij} x_{ij} \tag{5.8}$$

$d_{ij} = \infty$ for $i = j$ $\wedge$ $d_{ij} = 0$ for i = last tour node and j = first tour node

where; N = total number of nodes; F_j = edge length reduction factor; d_{ij} = distance from node i to node j.

The sum of turning angles in a tour is defined as follows:

$$A_{TOUR} = \left(\sum_{j=1}^{N} \Delta\gamma_j \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} y_{ijk} \right) \times \frac{L_{TOUR}}{180N} \tag{5.9}$$

$\Delta\gamma_j$ = 0 for *j* = last and *j* = first tour node

where; *N* = total number of nodes; $\Delta\gamma_j$ is the plane angle difference between incoming and outgoing tour edges at node *j*.

The objective function of the dredger routing problem is the following:

$$minimize\ Z = \alpha_{length} \left(L_{TOUR}\right)^{\beta_{length}} + \alpha_{angle} \left(A_{TOUR}\right)^{\beta_{angle}} \tag{5.10}$$

where; *Z* = total cost; α_{length} = tour length penalty factor; β_{length} = tour length penalty exponent; α_{angle} = turning angle sum penalty factor; and β_{angle} = turning angle sum penalty exponent.

Subject to the constraints:

$$\sum_{j=1}^{N} x_{ij} = 1 \qquad i = 1, 2, 3, \ldots, N \tag{5.11}$$

$$\sum_{i=1}^{N} x_{ij} = 1 \qquad j = 1, 2, 3, \ldots, N \tag{5.12}$$

$$\sum_{i=1}^{N} y_{ijk} = 1 \qquad j, k = 1, 2, 3, \ldots, N \tag{5.13}$$

$$\sum_{j=1}^{N} y_{ijk} = 1 \qquad i, k = 1, 2, 3, \ldots, N \tag{5.14}$$

$$\sum_{k=1}^{N} y_{ijk} = 1 \qquad i, j = 1, 2, 3, \ldots, N \tag{5.15}$$

$$x_{ij} = (0, 1) \qquad \text{for all } i \text{ and } j \tag{5.16}$$

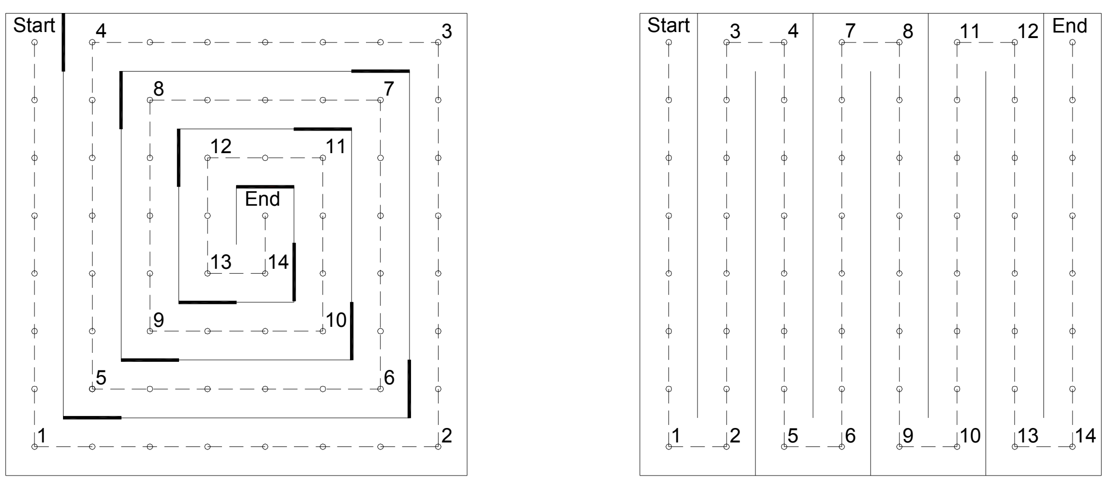
$$y_{ijk} = (0, 1) \qquad \text{for all } i, j \text{ and } k \tag{5.17}$$

Solution forms a tour. (5.18)

In Equation 5.9 the sum of turning angles is divided by 180 (the maximum turning angle) and multiplied by the average tour edge length to make the sum of turning angles independent of problem size. Otherwise identical solutions to very similar dredger routing problems of different scale will have different ratios of average turning angle to average tour edge length.

In Equation 5.8 the lengths of edges between nodes are subjected to a variable reduction factor when the turning angle between the tour edge under consideration and the preceding tour edge is less than a specified maximum. The application of this reduction factor is what gives the dredger routing problem a variable asymmetry. The value of the reduction factor decreases for each additional tour edge that exhibits a turning angle with its precursor which is less than a specified maximum. The maximum allowable turning angle which defines consecutive in-line edges should be kept small to encourage the dredger route to exhibit the greatest number of longest ‘straight’ lines possible. Parts of the dredger route made up of such lines are referred to here as links. It is important to note that single edges which are not aligned with either their precursor or the edge which follows are not considered links.

The reason why it is preferable to have a dredger route with the maximum number of maximum length links is a practical one and is mainly related to minimizing the number of occasions of having to dredge head on into previously dredged areas. Figure 5.3 illustrates this practical issue with two solutions to a continuous two-dimensional 64 node square grid routing problem which are both considered optimal according to graphical definitions given in Collins (2003) .



Solution A

Solution B

Figure 5.3 Optimal routing problem solutions (Collins, 2003).

Collins (2003) defines both routes in Figure 5.3 as having optimal total route lengths and optimal sums of turning angles. However, Figure 5.3 has to be looked at more closely to assess the suitability of each route for cutter suction dredgers. In solution A the dredger will dredge head on into previously dredged areas for a total number of 12 times, marked by bold line sections, therefore swinging across a total equivalent length of 12 times the grid spacing. Whereas in solution B the dredger is not made to dredge head on into previously dredged areas. In solution B the total distance dredged swinging sideways into previously dredged cuts is equivalent to a length of 49 times the grid spacing; whereas in Solution A the total distance dredged swinging sideways into previously dredged cuts is equal to a length of 35 times the grid spacing. In general, dredging into previously dredged areas results in greater losses of dredging production when experienced ahead of a cutter suction dredger than when experienced sideways. Therefore, for a cutter suction dredging, solution B in Figure 5.3 should take preference over solution A. A second practical reason for choosing solution B over A is that the spiral route of solution A will require repeated disconnection and reconnection of a floating pipeline, if used.

To quantify optimal routes for cutter suction dredgers, such as the route of Solution B in Figure 5.3, use is made of four equations. These four equations quantify attributes of optimal dredger routes which visit all nodes on continuous square grids of square or rectangular shape with widths and lengths of 2 nodes or more. Equations 5.19 and 5.20, respectively, give the lower bounds of total route length, L_{MIN}, and sum of turning angles of optimal dredger routes, A_{MIN}, for such grids.

$$L_{MIN} = d\left(n_W \cdot \left(n_L - 1\right) + \left(n_W - 1\right)\right) \tag{5.19}$$

where; n_L = number of nodes along length of rectangular grid; n_W = number of nodes across width of rectangular grid; and d = square grid spacing.

$$A_{MIN} = 180\left(n_W - 1\right) \tag{5.20}$$

where; n_W = number of nodes across width of rectangular grid.

In addition, Equations 5.21 and 5.22, respectively, define the upper bounds of link length, M_{MAX}, and of the number of maximum length links, N_{MAX}, in optimal dredger routes.

$$M_{MAX} = d\left(n_L - 1\right) \tag{5.21}$$

where; n_L = number of nodes along length of rectangular grid; and d = square grid spacing.

$$N_{MAX} = n_W \tag{5.22}$$

where; n_W = number of nodes along width of rectangular grid.

For a grid spacing of 1,Table 5.2 lists values which quantify the two routes depicted in Figure 5.3 using Equations 5.19 to 5.22 inclusive. In addition, Table 5.2 gives the actual average link length for each route, which is equal to the sum of link lengths divided by the total number of links.

Table 5.2 Optimal route attributes – 64 Node Square Grid

Item	Route Attribute	Solution A	Solution B
1	Length	63	63
2	Sum Angles	1260	1260
3	Maximum Link Length	7	7
4	Number of Maximum Length Links	3	8
5	Sum Link Lengths	61	56
6	Number of Links	13	8
7	Average Link Length	4.69	7

Item 4 in Table 5.2 shows that, with respect to Equation 5.22, the route of Solution B in Figure 5.3 is an optimal dredger route and Solution A is not. This is also reflected by the average link length, item 7, of both routes. Following this, it is said that optimal routes for cutter suction dredgers in continuous square grid routing problems of square or rectangular shape can be identified by the number of maximum length links they exhibit.

However, the determination of the upper bound for the number of maximum length links in a route for continuous *irregular* grids of *irregular* shape can be problematic and depends on the maximum allowable angle between consecutive route edges. In addition, assessing an entire dredger route in an irregular grid on the basis of the number of maximum length links can be of little worth if only a small number of such links exist: Nothing would be said about links of (slightly) lesser lengths. On the other hand, determining the average link length of a dredger route on any grid is straightforward and therefore it is considered meaningful for evaluating dredger routes.

Having said that, average link length, as defined here, should not be used as the only criterion for defining the optimality of dredger routes: If, for example, a solution to the routing problem in Figure 5.3 were to have only one link of maximum length (a greater number being possible) and all remaining route edges were not links, but unaligned route edges, then the average link length would still be considered ‘optimal’, when in reality the corresponding dredger route itself is not: It will be longer and have a greater sum of turning angles than Solution B in Table 5.2. Despite the shortcomings of assessing dredger routes on the basis of their average link lengths, these lengths are calculated and given in Section 6, *Results and Discussion*, to gain additional insight, in particular for the engineering application of the dredger routing model, which concerns an irregular routing problem.

The application of an edge reduction factor to route edge lengths is thought to be adequate for encouraging the formation of as many maximum length links as are possible in a given dredger routing problem. The edge length reduction factor in Equation 5.8 is calculated for each route edge as follows:

$$F_j = 1 - \frac{w(n-1)^2}{n_{max}} \tag{5.23}$$

where; w = reduction constant; n = number of edges in a link before node j; and n_{max} = maximum expected number of edges in a link.

Figure 5.4 gives values of the edge length reduction factor for a hypothetical dredger routing problem. In Figure 5.4 the reduction constant is 0.05 (selected on the basis of providing for practically useful edge reduction factors) and the maximum expected number of edges in a link is 15, a value that is estimated by visual inspection of valid route nodes extracted from final nests generated by the dredge cut nesting model.

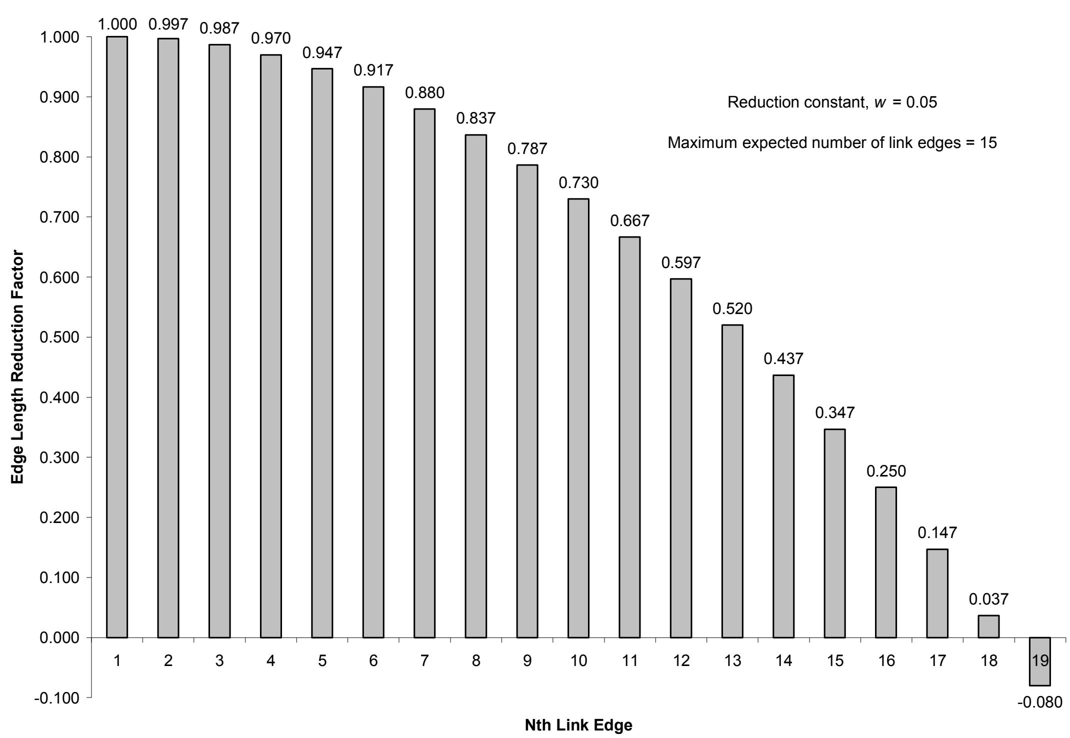


Figure 5.4 Edge length reduction factors.

Figure 5.4 shows that the actual maximum number of link edges can be 20% higher than 15 without edge length reduction factors taking on negative values. Negative edge lengths will not cause the objective function of the dredger routing model to function differently, but are avoided because they can complicate the comparison of total factored route costs. The experimental design is presented next.

5.4 Experimental Design

The experimentation undertaken here to test the research hypothesis consists of three main groups: Validation of the dredge cut nesting and dredger routing models against relevant problems taken from literature; further testing on hypothetical problems of varying complexity; and an engineering application of both models. Success with which the dredge cut nesting model optimizes nesting problems presented here is measured by comparing areas of escape (if applicable), overlap and non-placement of final nest layouts. Areas of escape, overlap and non-placement are generally expressed as a percentage of the total (inner) sheet area used. Success with which the dredger routing model optimizes routing problems presented here is measured by comparing final route lengths, sums turning angles and average link lengths of final routes. In addition, a selection of final nests and final routes are presented graphically for further analysis.

5.4.1 Model Validation

The dredge cut nesting and dredger routing models are validated against relevant problems related to an irregular nesting problem taken from literature.

5.4.1.1 Validation of Dredge Cut Nesting Model

The dredge cut nesting model is validated against an irregular nesting problem solved with an adaptive simulated annealing-based solution approach in Yuping *et al*. (2005). Figure 5.5 depicts the sheet and stencils of this irregular nesting problem.

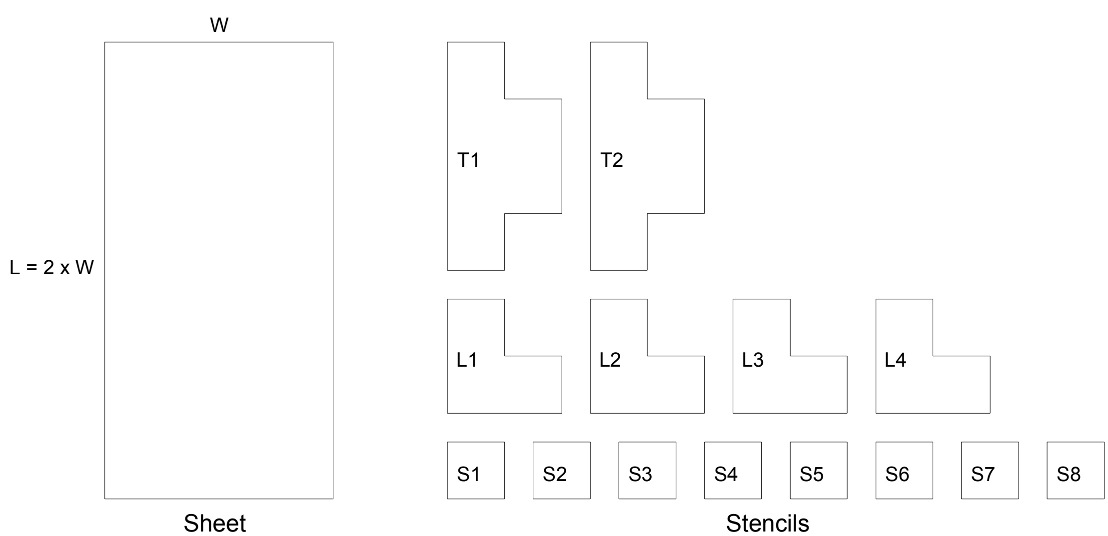


Figure 5.5 Nesting validation problem (Yuping *et al*., 2005).

The width of the sheet depicted in Figure 5.5 is exactly half its length. The set of stencils depicted in Figure 5.5 is irregular and consists of: Two large T-shaped stencils; four medium L-shaped stencils; and eight small square stencils. In Figure 5.5, the length of the sides of the square stencils, S1 to S8, are equal to a quarter of the width of the sheet. Each T-shaped stencil, T1 and T2, is made up of six square stencils, and each L-shaped stencil. L1 to L4, is made up of three square stencils. The area sum of all stencils is equal to the sheet area. It should be noted that, further to the stencil set depicted in Figure 5.5, the dredge cut nesting model was developed to use a maximum of three different types of stencil. In accordance with the solution approach adopted in Yuping *et al*. (2005) the experiments solving the nesting problem of Figure 5.5 do not allow for any escape of stencils beyond sheet boundaries. Table 5.3 gives the model settings used for validating the dredge cut nesting model.

Table 5.3 Dredge Cut Nesting model settings – Validation

Item	Description	Setting	Source
1	Sampled States	1,000	-
2	Annealing Limit Accepted States	1,000[1)]	Ingber (1989)
3	Annealing Limit Generated States	1,000[2)]	Ingber (2006)
4	Re-annealing Limit Accepted States	100	*(ibid.)*
5	Re-annealing Limit Generated States	1,000	-
6	Stop Limit Accepted States	10,000	*(ibid.)*
7	Stop Limit Generated States	99,999	*(ibid.)*
8	Small Change Sensitivity Analysis	0.001	*(ibid.)*
9	Initial Parameter Temperature	1.0	*(ibid.)*
10	Initial Cost Temperature Factor	1.0	-
11	Initial Placement	Random	Yuping *et al*. (2005)
12	Parameter Space Dimensions	42	*(ibid.)*
13	Parameter Control Tuning Factor	3, 4, 5, 6, 7, 8	Chen *et al*. (1999)
14	Cost Control Tuning Factor	3, 4, 5, 6, 7, 8, 9, 10, 11	*(ibid.)*
15	Stencil Modification Selection Mode	Sequential	Yuping *et al*. (2005)
16	Total stencil area / sheet area	1.0	*(ibid.)*
17	Square Unit Cut Side Length	100	-
18	Sheet dimensions	800×400	-

Notes: 1) Cost Temperature annealed only. 2) Parameter Temperatures annealed only.

For the validation of the dredge cut nesting model all penalty factors and exponents for overlap and non-placement are set to unity (see Equation 5.4). In Table 5.3, items 3, 4, 6, 7, 8 and 9 are set to default values taken from the Adaptive Simulated Annealing Code Manual version 26.22 (Ingber, 2006). The default value for item 1 is 5 (*ibid*.), but it is set at 1,000 to find a more representative initial cost temperature. The default value for item 2 is 0 (*ibid*.), which, if set as such, causes annealing of the cost and parameter temperatures at identical intervals based on the number of generated states. However, Ingber (1989) states that it has proven fruitful to anneal the cost temperature using the number of accepted states instead and therefore item 2 is set to a non-zero value equal to that of item 3, namely 1,000.

In Table 5.3, item 3 is set to 1,000 to have an expected 99 parameter temperature annealing events over the total number of 99,999 generated states as specified for item 7. According to Ingber (2006) default values for item 5 are 1,000,000 or 10,000, but it is set to 1,000 to reduce the risk of getting stuck in local minima in case the number of acceptances no longer increases and therefore can no longer be relied upon for re-annealing. The values for items 13 and 14 in Table 5.3 are chosen further to Chen *et al*. (1999), which states that values for the cost and parameter control tuning factors used in Equation 3.32 (see Section 3.5.4) often range between 1 and 10. To validate the dredge cut nesting model, a total of $6 \times 9 = 54$ different nesting experiments are carried out, one for each unique combination of control tuning factors. Each experiment of 99,999 iterations is repeated 20 times, totalling approximately 108 million iterations.

5.4.1.2 Validation of Dredger Routing Model

Inspired by square grid routing problems used in Henderson *et al*. (2003), the dredger routing model is validated against a square grid routing problem extracted from a global optimum nest layout of the irregular nesting problem solved in Yuping *et al*. (2005). Figure 5.6 depicts the nodes of this routing problem, with an associated global optimum nest of stencils marked in dashed lines.

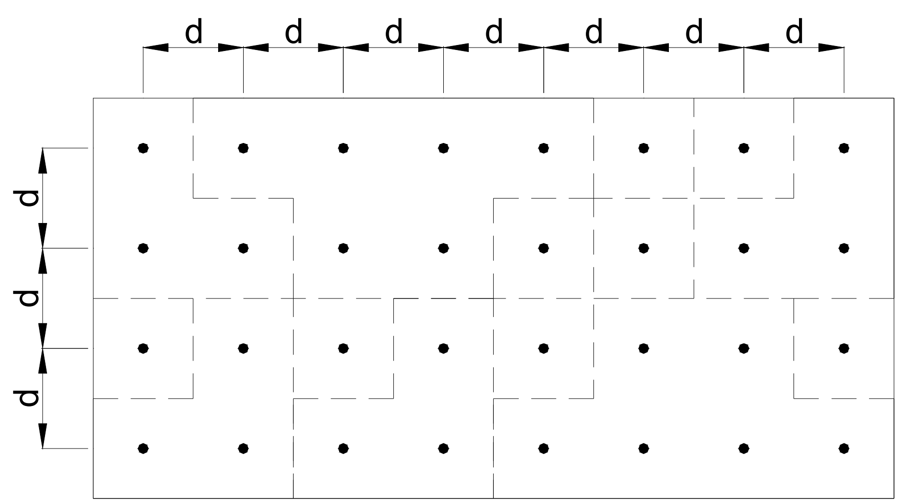


Figure 5.6 Routing validation problem (Yuping *et al*., 2005).

In Figure 5.6, the 32 nodes derived from the centroids of squares which make up the stencils define the planar square grid problem solved in this part of the experimentation. Table 5.4 gives the model settings used for validating the dredger routing model.

Table 5.4 Dredger Routing model settings – Validation

Item	Description	Setting	Source
1	Sampled States	1,000	-
2	Annealing Limit Accepted States	0	Ingber (2006)
3	Annealing Limit Generated States	1,000[1)]	*(ibid.)*
4	Re-annealing Limit Accepted States	100	*(ibid.)*
5	Re-annealing Limit Generated States	10,000	*(ibid.)*
6	Stop Limit Accepted States	10,000	*(ibid.)*
7	Stop Limit Generated States	99,999	*(ibid.)*
8	Initial Parameter Temperature	1.0	*(ibid.)*
9	Initial Cost Temperature Factor	0.0161	Johnson (1990)
10	Initial tour	Random	Henderson *et al*. (2003)
11	Parameter Space Dimensions	1[2)]	-
12	Parameter Control Tuning Factor	0.047[3)]	Ingber (2006)
13	Cost Control Tuning Factor	0.047[3)]	*(ibid.)*
14	Edge Exchange Mode	2-opt	Koulamas *et al*. (1994)
15	Edge Exchange Selection Mode	Random	*(ibid.)*
16	Local Search	1, 8	*(ibid.)*
17	Maximum Expected In-Line Edges	7	-
18	Edge Length Reduction Constant	0.05	-
19	Link Edge Angle Range	[-1°, 1°]	-
21	Start Position (x, y)	(25, 25)	-
22	Horizontal and Vertical Grid Spacing	50	-

Notes: 1) Cost and Parameter Temperatures annealed. 2) For code only. 3) Not rounded in code.

For the validation of the dredge cut nesting model all penalty factors and exponents for tour length and sum of turning angles are set to unity (see Equation 5.10). In Table 5.4, the values for items 2 to 8 inclusive are set to default values taken from the Adaptive Simulated Annealing Code Manual version 26.22 (Ingber, 2006). The value for item 9 is derived from Johnson (1990), which states that initial acceptance temperatures can be used which are "*roughly*" equal to half the average edge length of the initial random tour. In contrast to exploring combinations of fixed values for parameter and cost control tuning factors, as done for dredge cut nesting problems, the values of items 12 and 13 in Table 5.4, are calculated according to guidelines given in Ingber (2006). These guidelines recommend using the following values for the control coefficients of Equation 3.33 (see Section 3.5.4):

$$m_i = -ln(0.01) \qquad (5.24)$$

$$n_i = ln(99) \qquad (5.25)$$

In Table 5.4. values for item 16, the local search, are set to 1 and 8, the latter value being an upper limit for local search recommended in Koulamas *et al*. (1994). To validate the dredger routing model, a total of 2 different routing experiments are carried out, one with a local search of 1 and the other with local search of 8. Each experiment of 99,999 base iterations was repeated 20 times, totalling approximately 18 million iterations.

5.4.2 Variation of Nesting Problem Variables

After validation, the performance of the dredge cut nesting model is further tested on five sub-groups of additional nesting problems, which includes an engineering application. Table 5.5 gives the main settings of the dredge cut nesting model used for *all* additional nesting problems.

Table 5.5 Main Dredge Cut Nesting model settings for all additional nesting problems

Item	Description	Value(s)/Setting	Source
1	Sampled States	1,000	-
2	Annealing Limit Accepted States	1,000[1)]	Ingber (1989)
3	Annealing Limit Generated States	1,000[2)]	Ingber (2006)
4	Re-annealing Limit Accepted States	100	*(ibid.)*
5	Re-annealing Limit Generated States	1,000	-
6	Stop Limit Accepted States	10,000	*(ibid.)*
7	Stop Limit Generated States	99,999	*(ibid.)*
8	Initial Parameter Temperature	1.0	*(ibid.)*
9	Initial Cost Temperature Factor	1.0	-
10	Initial Placement	Random	Yuping *et al*. (2005)
11	Stencil Modification Selection Mode	Sequential	*(ibid.)*

Notes: 1) Cost Temperature annealed only. 2) Parameter Temperatures annealed only.

The following sections detail the additional groups of nesting problems and the nesting problem solved in the engineering application of the dredge cut nesting model.

5.4.2.1 Dredge Cut Nesting – Relaxed Sheet Boundary Conditions

The first stage of evaluating if the dredge cut nesting model can be used for engineering applications consists of investigating what effect increasing the number of relaxed sheet boundary conditions has on final nests for the irregular nesting problem depicted in Figure 5.5. The relaxation of sheet boundary conditions consists of a step-wise increase in the number of sheet boundaries across which stencils can escape.

It is thought the issue of having to provide for sets with many stencils of different size and shape, so that all inner sheet dimensions can be matched, can be overcome by providing selective escape regions for stencils. Figure 5.7 depicts the three modified sheet arrangements used to investigate the effect of sheet boundary relaxation on final nests.

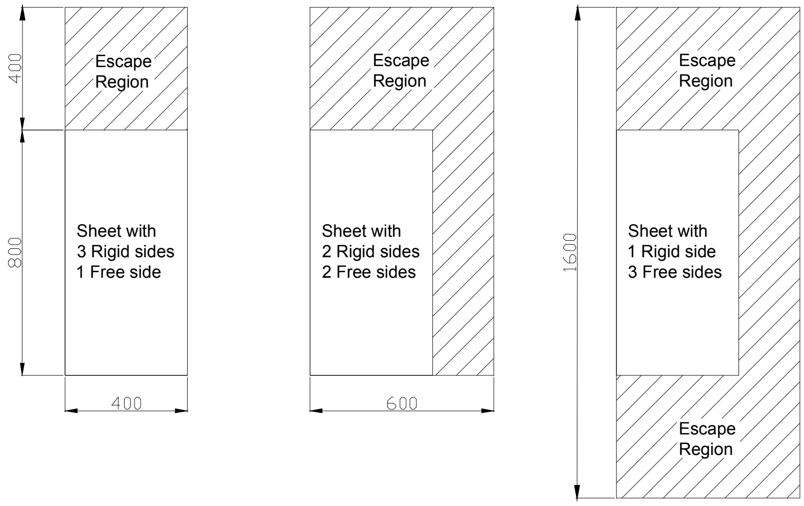


Figure 5.7 Sheet arrangements with relaxed boundaries.

The same set of stencils as depicted in Figure 5.5, with a square unit cut side length of 100, is used for nesting in the modified sheet arrangements shown in Figure 5.7, therefore the total inner sheet area to total stencil area ratio remains 1:1. Table 5.6 gives the specific settings of the dredge cut nesting model used for this part of the experimentation.

Table 5.6 Specific model settings – Irregular nesting – Relaxed sheet boundaries

Item	Description	Value(s)/Setting	Source
1	Parameter Space Dimensions	42	Yuping *et al*. (2005)
2	Square Unit Cut Side Length	100	-
3	Parameter Control Tuning Factor	3, 4, 5, 6, 7, 8	Chen *et al*. (1999)
4	Cost Control Tuning Factor	3, 4, 5, 6, 7, 8, 9, 10, 11	*(ibid.)*
5	Total stencil area / inner sheet area	1.0	-
6	Sheet dimensions	800×400	-

To investigate the effect of relaxing sheet boundary conditions on final nests all penalty factors and exponents for escape, overlap and non-placement are set to unity in the dredge cut nesting model. For this part of the experimentation a total of 3 × 6 × 9 = 162 different nesting experiments are carried out, one for each unique combination of sheet arrangement and control tuning factors. Each experiment of 99,999 iterations is repeated 20 times, giving a total of approximately 324 million iterations.

5.4.2.2 Dredge Cut Nesting – Reduced Sheet Areas

The second stage of evaluating if the dredge cut nesting model can be used for engineering applications consists of investigating the effect of changing the ratio of total inner sheet area to total stencil area on final nests. From now on this ratio is referred to as the sheet to stencil area ratio. The effect of changing this ratio is investigated for variants of the irregular nesting problem taken from Yuping *et al*. (2005) with two relaxed inner sheet boundary conditions. Changes in sheet to stencil area ratios consist of step-wise reductions of inner sheet dimensions whilst maintaining the total stencil area constant.

For a given sheet, Yuping *et al*. (2005) states that the number of stencils to be used in a solving irregular stock cutting problems is that which results in the total stencil area being "*roughly*" equal to the sheet area. Since feasible solutions to dredge cut nesting problems are permitted to exhibit non-zero escape it is reasonable to expect that a surplus of stencil area is required if the condition of zero non-placement is to be met. Figure 5.8 depicts the four different sheet arrangements, with two relaxed boundary conditions each, used in this part of the experimentation.

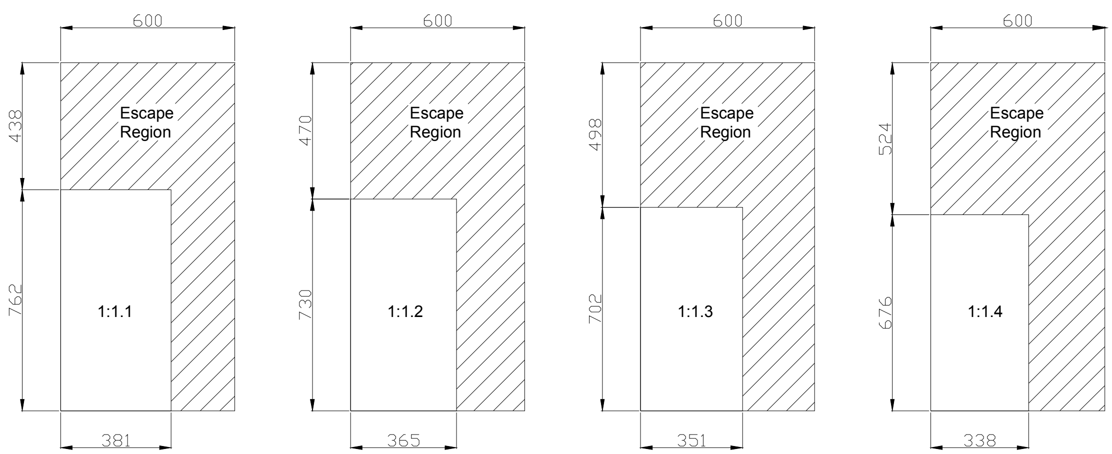


Figure 5.8 Reduced inner sheet area arrangements.

The same set of stencils as depicted in Figure 5.5, with a square unit cut side length of 100, is used for nesting in the reduced inner sheet areas depicted in Figure 5.8, thereby giving, from left to right, sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4. Table 5.7 gives specific settings of the dredge cut nesting model used for investigating the effect of decreased inner sheet areas on final nest layouts of irregular nesting problems.

Table 5.7 Specific model settings – Irregular nesting – Reduced inner sheets

Item	Description	Value(s)/Setting	Source
1	Parameter Space Dimensions	42	Yuping *et al*. (2005)
2	Square Unit Cut Side Length	100	-
3	Parameter Control Tuning Factor	3, 4, 5, 6, 7, 8	Chen *et al*. (1999)
4	Cost Control Tuning Factor	3, 4, 5, 6, 7, 8, 9, 10, 11	*(ibid.)*
5	Total stencil area / inner sheet area	1.1, 1.2, 1.3, 1.4	-
6	Sheet dimensions	762×381, 730×365, 702×351, 676×338	-

To investigate the effect of decreasing inner sheet area for constant total stencil area, all penalty factors and exponents for overlap and non-placement in the dredge cut nesting model are set to unity. The penalty factor for escape, however, is set to zero to eliminate its influence on nesting solutions. For this part of the experimentation a total of 4 × 6 × 9 = 216 different nesting experiments are carried out, one for each unique combination of reduced sheet size and control tuning factors. Each experiment of 99,999 iterations is repeated 20 times, giving a total of approximately 432 million iterations.

5.4.2.3 Dredge Cut Nesting – Reduced Sheet Areas for Square Stencils

The third stage of evaluating if the dredge cut nesting model can be used for engineering applications consists of investigating the effect of using square stencils only for nesting problems with decreased inner sheets and two relaxes sheet boundaries. This is done for the nesting problems with varying sheet to stencil area ratios described in Section 5.4.2.2. The third sub-group of experiments therefore is almost identical to the second sub-group with the exception that the third sub-group uses 32 square stencils giving 96 parameter space dimensions instead of 42 for the original set of 14 stencils depicted in Figure 5.5.

Apart from the change in parameter space dimensions the specific settings used for solving the third sub-group of additional nesting problems are the same as those given in Table 5.7. To investigate using square stencils only, effectively regularizing the nesting problems solved, a total of 4 × 6 × 9 = 216 nesting experiments are carried out, one for each unique combination of reduced sheet size and control tuning factors. Each experiment of 99,999 iterations is repeated 20 times, giving a total of approximately 432 million iterations.

5.4.2.4 Dredge Cut Nesting – Cost Penalty Increase for Square Stencils

The fourth stage of evaluating if the dredge cut nesting model can be used for engineering application consists of investigating the effect of increasing penalty factors and exponents for overlap and non-placement cost. This is investigated for the nesting of 32 identical square stencils in the sheet arrangements depicted in Figure 5.8. Table 5.8 gives specific settings for the dredge cut nesting model used for the fourth sub-group of additional nesting problems.

Table 5.8 Specific model settings – Regular nesting – Increased cost penalties

Item	Description	Value(s)/Setting	Source
1	Parameter Space Dimensions	96	-
2	Square Unit Cut Side Length	100	-
3	Parameter Control Tuning Factor	10 / 9 / 11 / 9	-
4	Cost Control Tuning Factor	6 / 6 / 6 / 5	-
5	Total stencil area / inner sheet area	1.1 / 1.2 / 1.3 / 1.4	-
6	Sheet dimensions	762×381 / 730×365 / 702×351 / 676×338	-
7	Escape penalty factor	0	-
8	Escape penalty exponent	0	-
9	Overlap penalty factor	4[1)]	Yuping *et al.* (2005)
10	Overlap penalty exponent	1[1)]	*(ibid.)*
11	Non-placement penalty factor	50[1)]	*(ibid.)*
12	Non-placement penalty exponent	2[1)]	*(ibid.)*

Note: 1) Penalty values taken from referenced source but applied differently to decision variables.

To investigate the effect of higher cost penalties using square stencils only, a total of 4 different nesting experiments of 99,999 iterations are carried out, each of which is repeated 20 times, giving a total of approximately 8 million iterations.

5.4.3 Variation of Routing Problem Variables

After validation, the performance of the dredger routing model is tested on three additional routing problems, which includes an engineering application. Table 5.9 gives the main settings of the dredger routing model used for *all* additional routing problems.

Table 5.9 Main Dredger Routing model settings for all additional routing problems

Item	Parameter Description	Value/Setting	Source
1	Annealing Limit Accepted States	0	Ingber (2006)
2	Annealing Limit Generated States	1,000[1)]	*(ibid.)*
3	Re-annealing Limit Accepted States	100	*(ibid.)*
4	Re-annealing Limit Generated States	10,000	*(ibid.)*
5	Stop Limit Accepted States	10,000	*(ibid.)*
6	Stop Limit Generated States	99,999	*(ibid.)*
7	Initial tour	Random	Henderson *et al*. (2003)
8	Parameter Space Dimensions	1[2)]	-
9	Parameter Control Tuning Factor	0.047	Ingber (2006)
10	Cost Control Tuning Factor	0.047	*(ibid.)*
11	Edge Exchange Mode	2-opt	Koulamas *et al*. (1994)
12	Edge Exchange Selection Mode	Random	*(ibid.)*

Notes: 1) Cost and Parameter Temperatures annealed. 2) For code only.

The following sections detail the additional routing problems, including the problem solved in the engineering application of the dredger routing model.

5.4.3.1 Dredger Routing – 64 Node Square Grid Problem

The first additional routing problem solved with the dredger routing model consists of a planar square grid problem with 64 nodes representing the centroids of square dredge cut stencils in an optimum solution of a hypothetical dredge cut nesting problem. Figure 5.9 depicts the 64 node routing problem and Table 5.10 gives the specific model settings used for solving this problem.

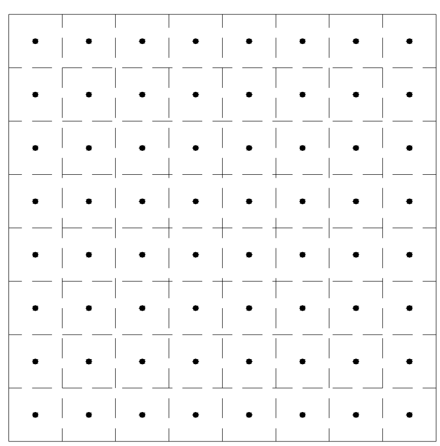


Figure 5.9 Square grid routing problem with 64 nodes.

Table 5.10 Specific model settings – Regular routing – 64 Nodes

Item	Description	Value/Setting	Source
1	Sampled States	1,000	-
2	Initial Cost Temperature Factor	0.0079	Johnson (1990)
3	Local Search	1, 8	Koulamas *et al*. (1994)
4	Link Edge Angle Range	[-1°, 1°]	-
5	Horizontal and Vertical Grid Spacing	50	-
6	Start Position (x, y)	(25, 25)	-
7	Maximum Expected In-Line Edges	7	-
8	Edge Length Reduction Constant	0.05	-

To investigate the performance of the dredger routing model on the 64 node routing problem a total of 2 routing experiments are carried out, one with local search of 1 and the other with local search of 8. Each experiment of 99,999 base iterations is repeated 20 times, giving a total of approximately 18 million iterations.

5.4.3.2 Dredger Routing – 256 Node Square Grid Problem

The second additional routing problem solved with the dredger routing model is a planar square grid problem with 256 nodes representing the centroids of squares of an optimum solution to a hypothetical dredge cut nesting problem. Figure 5.10 depicts the 256 node routing problem and Table 5.11 gives the specific model settings used for optimization.

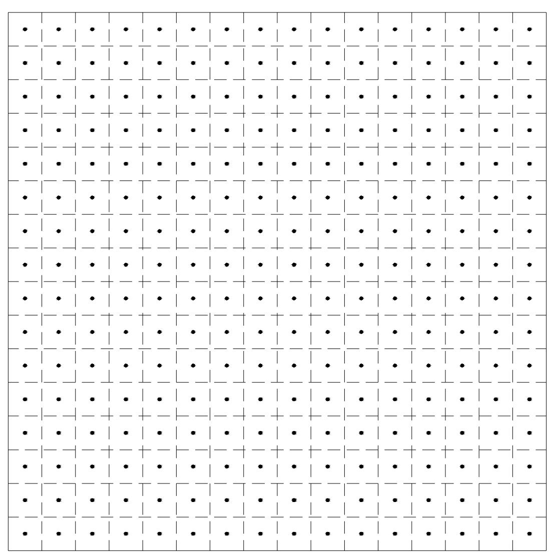


Figure 5.10 Square grid routing problem with 256 nodes.

Table 5.11 Specific model settings – Regular routing – 256 Nodes

Item	Parameter Description	Value/Setting	References
1	Sampled States	1,000	-
2	Initial Cost Temperature Factor	0.0020	Johnson (1990)
3	Local Search	1, 8, 16, 32, 64, 128	-
4	Link Edge Angle Range	[-1°, 1°]	-
5	Horizontal and Vertical Grid Spacing	50	-
6	Start Position (x, y)	(25, 25)	-
7	Maximum Expected In-Line Edges	15	-
8	Edge Length Reduction Constant	0.05	-

To investigate the performance of the dredger routing model on the 256 node routing problem a total of 6 routing experiments are carried out, with local search equal to 1, 8, 16, 32, 64 and 128. Each experiment of 99,999 base iterations is repeated 20 times, giving a total of approximately 498 million iterations.

5.4.4 Engineering Application

The performance of the dredge cut nesting and dredger routing models for engineering applications is evaluated by solving a two-dimensional cut planning problem for cutter suction dredgers derived from the Laem Chabang Port Project Phase 2 Stage 1 in Thailand. Figure 5.11 depicts the project in plan view. The hatched area seen in Figure 5.11 was dredged between 1998 and 1999 with the cutter suction dredger “Cyrus”, at the time operated by Dragomar S.p.A. of Italy.

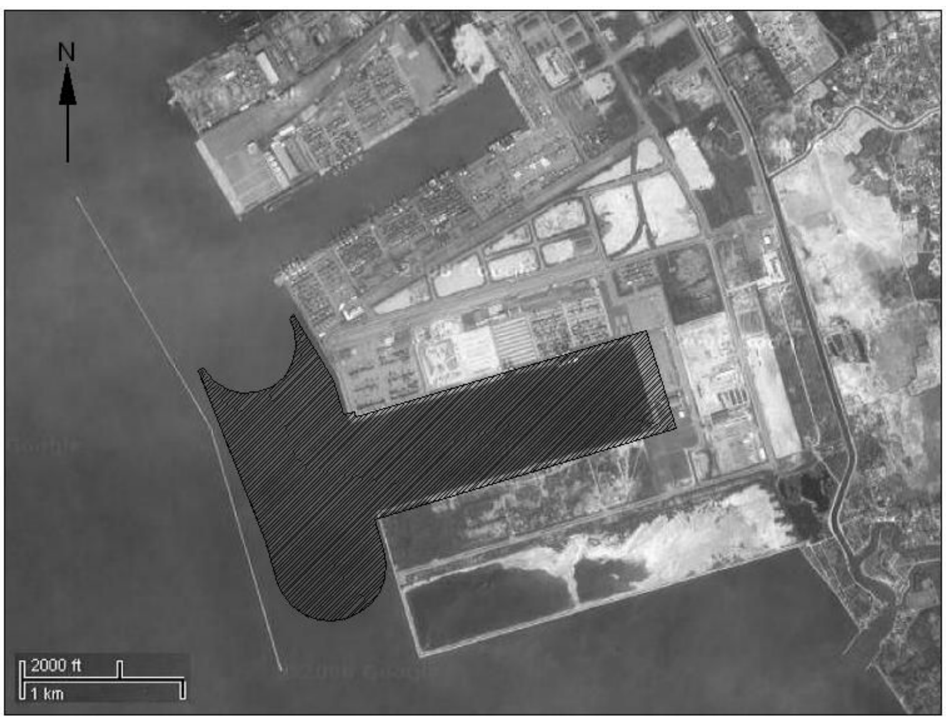


Figure 5.11 Engineering application – Dredging area.

Appendix B gives a daily dredger report showing that cutter suction dredger “Cyrus” achieved dredge cut widths of up to 112 metres wide on the Laem Chabang Port Project. For the dredger’s main characteristics Appendix C is referred to.

5.4.4.1 Dredge Cut Nesting – Engineering Application

Whilst awaiting results of the experimentation on nesting problems, over 15 different preliminary dredge cut nesting experiments were carried out for the real-world dredging area depicted in Figure 5.11. The model settings used in these preliminary experiments were not those which performed best in other nesting experiments, since results of these experiments were not yet available. Some results of this preliminary experimentation on the real-world dredge cut planning problem are of interest to the research presented here because they influenced the final shapes of escape regions and stencils used in the engineering application of the dredge cut nesting model. However, the model settings used for the preliminary experiments vary considerably and it is felt that clarity would be lost if they are included now. Therefore, appendices with relevant model settings, will be referred to in Section 6, *Results and Discussion*, if and when preliminary results of the engineering application of the dredge cut nesting model are mentioned. Figure 5.12 depicts the inner sheet, escape regions and a sample of a square unit dredge cut used in the engineering application of the dredge cut nesting model.

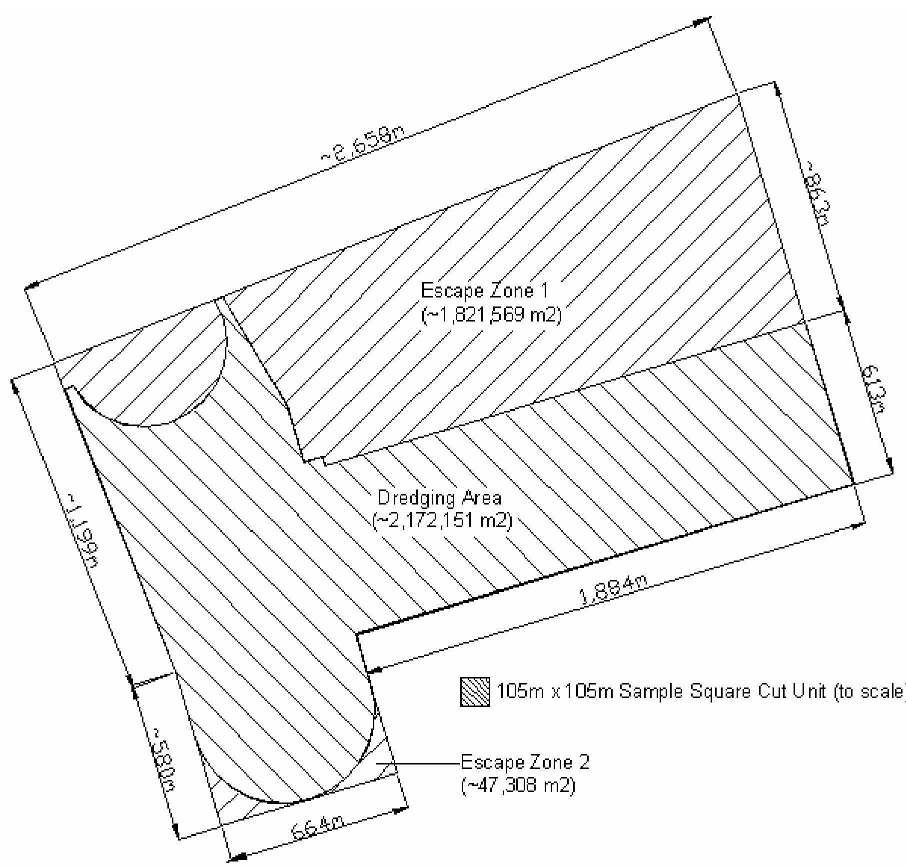


Figure 5.12 Engineering application – Inner and outer sheets.

Appendix D gives the coordinate pairs defining the inner and outer sheets depicted in Figure 5.12. Figure 5.13 depicts the stencil set used in the engineering application of the dredge cut nesting model.

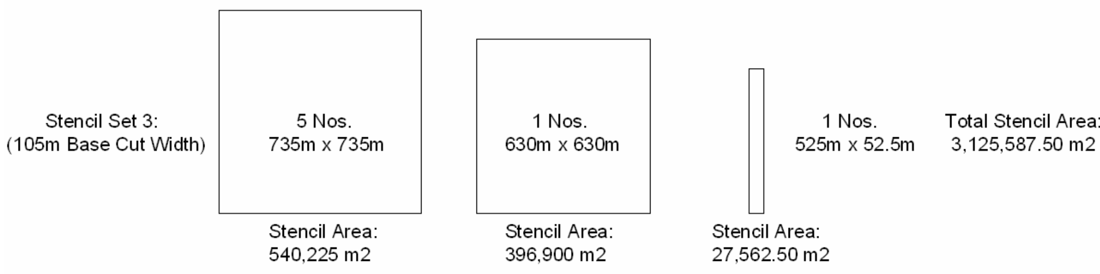


Figure 5.13 Engineering application – Stencil set.

The stencil set in Figure 5.13 gives a sheet to stencil area ratio is 1:1.439. Table 5.12 gives the specific settings used in the engineering application of the dredge cut nesting model.

Table 5.12 Specific model settings – Irregular nesting – Engineering application

Item	Description	Value(s)/Setting	Source
1	Parameter Space Dimensions	21	-
2	Square Unit Cut Side Length	105	-
3	Parameter Control Tuning Factor	7	-
4	Cost Control Tuning Factor	5	-
5	Total stencil area / sheet area	1.439	-
6	Escape penalty factor	0	-
7	Escape penalty exponent	0	-
8	Overlap penalty factor	4[1)]	Yuping *et al.* (2005)
9	Overlap penalty exponent	1[1)]	*(ibid.)*
10	Non-placement penalty factor	50[1)]	*(ibid.)*
11	Non-placement penalty exponent	2[1)]	*(ibid.)*

Note: 1) Penalty values taken from referenced source but applied differently to decision variables.

The value of 105 for item 2 in Table 5.12 is taken from the daily dredger reports given in Appendix B. For the engineering application of the dredge cut nesting model 20 replications of 99,999 iterations are carried out.

5.4.4.2 Dredger Routing – Engineering Application

The model of the real-world dredger routing problem is derived from a final solution of the engineering application of the dredge cut nesting model. Route nodes are made up of centroids of the square unit dredge cuts which are wholly inside or intersect a boundary of the inner sheet representing the dredging area. Figure 5.14 illustrates how centroids of square unit cuts which intersect a dredging area boundary are selected for inclusion in the engineering application of the dredger routing model.

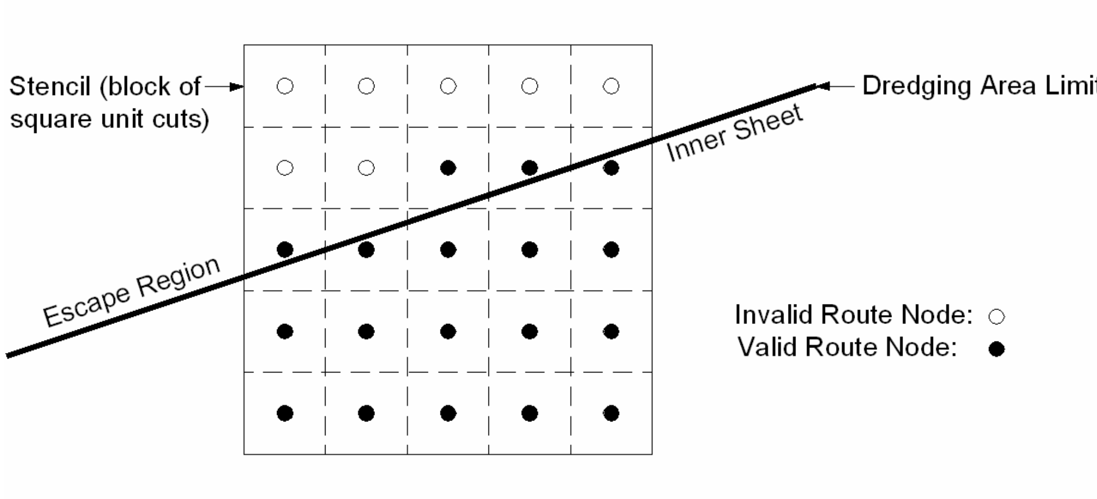


Figure 5.14 Engineering application – Route node selection.

Where dredge cut stencils overlap to the extent that square unit cuts of one stencil lie entirely within another stencil the following guidelines are used for the elimination of unnecessary route nodes:

1) Route nodes of stencils of bigger size take priority over those of smaller ones.
2) When stencils of equal size overlap, route nodes of the stencil which overlaps with the highest number of other equally sized stencils take priority.
3) When two stencils of equal size overlap and each one overlaps with an equal number of other equally sized stencils, route nodes of the stencil which has the least amount of escape take priority.

The selection of valid route nodes extracted from a final solution of the engineering application of the dredge cut nesting model is given in Section 6 where results are presented and discussed. Table 5.13 gives specific model settings used in the engineering application of the dredger routing model.

Table 5.13 Specific model settings – Irregular routing – Engineering application

Item	Description	Value/Setting	Source
1	Initial Cost Temperature Factor	½·(Total nodes)$^{-1}$	Johnson (1990)
2	Local Search	128	-
3	Link Edge Angle Range	[-1°, 1°]	-
4	Start Position (x, y)	204, 1625	-
5	Horizontal and Vertical Grid Spacing	Variable	-

For the engineering application of the dredger routing model 10 replications of 99,999 base iterations are carried out. For clarity, Table 5.14 lists all the experiments with a short description and their main objectives, after which results of all experiments are presented and discussed.

Table 5.14 Experimentation summary with objectives

No.	Experiment	Objective	Section
1	Irregular nesting – taken from Yuping *et al*. (2005)	Validation of Dredge Cut Nesting Model	5.4.1.1
2	Regular routing – 32 nodes derived from Yuping *et al*. (2005)	Validation of Dredger Routing Model for local search of 1 and 8	5.4.1.2
3	Irregular nesting – with increasing number of relaxed sheet boundaries and sheet to stencil area ratios of 1:1	To conclude if allowing escape of stencils leads to better final dredge cut nesting solutions	5.4.2.1
4	Irregular nesting – with 2 of relaxed sheet boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4	To conclude if providing an excess of stencil area leads to better final dredge cut nesting solutions	5.4.2.2
5	Regular nesting – with 2 of relaxed sheet boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4	To conclude if irregular or regular stencils lead to better final dredge cut nesting solutions	5.4.2.3
6	Regular nesting – with 2 of relaxed sheet boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4 with revised cost penalties	To conclude if revised cost penalties lead to better final dredge cut nesting solutions	5.4.2.4
7	Regular routing – 64 nodes	To conclude if for local search of 1 and 8 optimal dredger routes can be found	5.4.3.1
8	Regular routing – 256 nodes	To conclude if for local search of 1, 8, 16, 32, 64 and 128 optimal dredger routes can be found	5.4.3.2
9	Irregular nesting – engineering application	To conclude if the Dredge Cut Nesting Model can optimize a real-world problem	5.4.4.1
10	Irregular routing – engineering application 228 nodes	To conclude if the Dredger Routing Model can optimize a real-world problem	5.4.4.2

6 Results and Discussion

This section presents and discusses results of experiments in the same order as the previous section described the sub-groups of nesting and routing experiments.

6.1 Validation of Dredge Cut Nesting Model

On average each validation nesting experiment of 99,999 iterations required 22.83 minutes to complete. The average of 22.83 minutes observed here is much higher than the solution time of 1.22 minutes reported in Yuping *et al*. (2005) for the same nesting problem, especially since Yuping *et al*. (2005) reports having used a computer with a processor speed of 333 megahertz and 128 megabytes of random access memory. It is not known if the experiments of Yuping *et al*. (2005) were run for the same number of iterations as done here. Although not explicitly mentioned, it is possible that Yuping *et al*. (2005) used stopping criteria for early termination of nesting optimization processes. Also, Yuping *et al*. (2005) does not explicitly state how stencils were rotated during the nesting process, but in the original solution approach of Jain *et al*. (1998) stencils were rotated in steps of 90 degrees only. The model used here allows for near-continuous rotation of stencils, and rotation angles expressed to the nearest 1×10^{-9} of a degree were observed. With early stopping criteria and by limiting stencil rotation it is thought solution times can be reduced, but a limitation on the rotation of stencils can reduce the effectiveness of the dredge cut nesting model to cope with nesting in irregular sheets with boundaries which are not parallel to coordinate axes. Figure 6.1 depicts an example of a random initial placement of stencils used as an initial solution at the start of the nesting optimization process, which has an overlap area equal to 47.71% and a non-placement area equal to 47.70% of the sheet area.

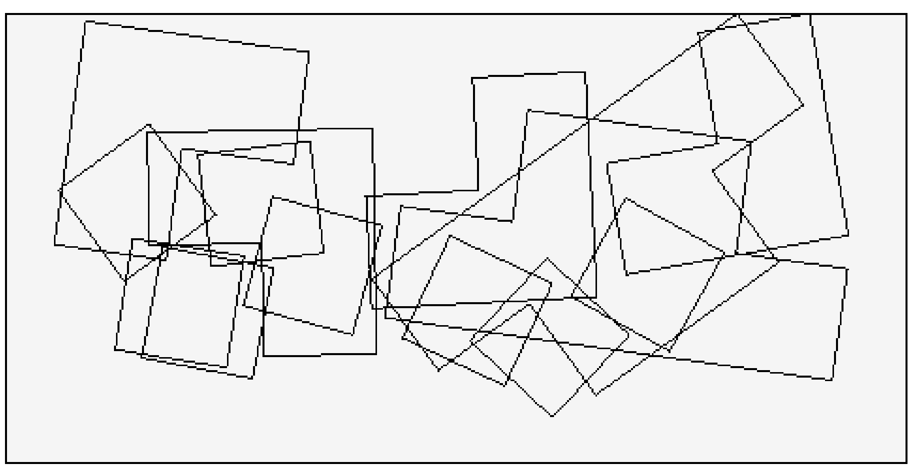


Figure 6.1 Random initial nest – Irregular nesting – Validation.

The difference in overlap and non-placement calculated for the nest in Figure 6.1 is explained after Table 6.1. Table 6.1 presents the results of the validation experiment for the dredge cut nesting model. Values of overlap and non-placement given are for 20 replications carried out, and are expressed as a percentage of total sheet area.

Table 6.1 Irregular nesting – Sheet to stencil area ratio 1:1

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	36.14	4.91	27.62	43.32	36.16	4.92	27.63	43.35
3 / 4	9.77	2.59	6.88	16.57	9.77	2.60	6.89	16.58
3 / 5	5.17	1.26	3.38	8.35	5.17	1.26	3.37	8.34
3 / 6	7.00	1.93	4.26	10.96	7.00	1.93	4.26	10.96
3 / 7	7.23	2.61	3.73	12.82	7.23	2.62	3.74	12.83
3 / 8	6.67	2.05	3.02	11.16	6.67	2.05	3.03	11.16
4 / 3	36.84	4.10	27.29	45.64	36.85	4.10	27.31	45.66
4 / 4	8.32	2.24	5.24	15.18	8.33	2.24	5.25	15.19
4 / 5	4.38	1.76	2.44	7.88	4.39	1.76	2.45	7.88
4 / 6	6.42	1.85	3.19	10.92	6.43	1.85	3.19	10.93
4 / 7	6.29	2.46	2.35	12.57	6.29	2.46	2.36	12.58
4 / 8	5.08	2.14	1.75	9.74	5.08	2.14	1.75	9.75
5 / 3	37.28	3.87	29.50	44.32	37.30	3.87	29.51	44.34
5 / 4	5.68	2.22	2.65	12.68	5.70	2.22	2.67	12.68
5 / 5	3.43	1.90	1.78	9.35	3.44	1.90	1.79	9.37
5 / 6	4.26	2.68	1.28	9.65	4.26	2.68	1.27	9.66
5 / 7	4.96	2.73	1.15	10.29	4.96	2.73	1.15	10.30
5 / 8	5.37	3.07	0.97	11.65	5.37	3.07	0.98	11.65
6 / 3	36.65	4.48	25.50	43.77	36.67	4.48	25.52	43.79
6 / 4	4.62	1.70	1.89	9.09	4.63	1.70	1.89	9.09
6 / 5	1.36	1.36	0.50	4.82	1.36	1.36	0.51	4.82
6 / 6	3.05	2.30	0.35	8.52	3.05	2.30	0.35	8.52
6 / 7	4.33	2.88	0.93	9.54	4.34	2.88	0.93	9.54
6 / 8	4.81	2.79	0.38	8.87	4.81	2.79	0.38	8.83
7 / 3	36.65	2.37	32.36	40.63	36.66	2.38	32.37	40.64
7 / 4	3.01	0.78	1.84	4.82	3.02	0.78	1.84	4.84
7 / 5	1.70	2.32	0.20	7.52	1.71	2.32	0.21	7.54
7 / 6	3.08	3.08	0.12	10.06	3.08	3.08	0.12	10.07
7 / 7	4.22	2.94	0.07	9.34	4.22	2.94	0.08	9.35
7 / 8	5.56	2.65	0.28	8.74	5.56	2.65	0.26	8.75
8 / 3	35.76	4.94	25.72	48.95	35.78	4.94	25.74	48.96
8 / 4	2.06	1.12	0.32	5.45	2.07	1.13	0.33	5.46
8 / 5	2.40	2.33	0.02	6.76	2.40	2.33	0.02	6.78
8 / 6	3.81	1.59	0	6.63	3.81	1.59	0	6.64
8 / 7	3.11	2.12	0	8.42	3.11	2.11	0	8.43
8 / 8	4.10	2.72	0	8.88	4.09	2.72	0	8.83
9 / 3	36.68	4.74	29.99	45.10	36.70	4.74	30.01	45.11
9 / 4	1.24	0.83	0.54	4.20	1.25	0.83	0.54	4.22
9 / 5	2.38	2.43	0	5.77	2.38	2.44	0	5.83
9 / 6	4.11	2.36	0	7.31	4.11	2.36	0	7.31
9 / 7	3.70	2.61	0	8.79	3.69	2.61	0	8.77
9 / 8	4.18	2.43	0	7.62	4.16	2.42	0	7.60
10 / 3	35.41	3.36	28.04	41.44	35.42	3.36	28.08	41.45
10 / 4	**0.83**	0.67	0.27	2.99	**0.84**	0.67	0.28	3.01
10 / 5	2.33	2.76	0	7.33	2.34	2.77	0	7.33
10 / 6	3.14	2.81	0	8.53	3.14	2.81	0	8.55
10 / 7	4.39	2.80	0	8.89	4.38	2.80	0	8.90
10 / 8	4.55	3.72	0	11.99	4.53	3.71	0	11.99
11 / 3	35.08	4.91	27.32	45.77	35.10	4.91	27.35	45.78
11 / 4	1.14	1.59	0.08	7.16	1.15	1.59	0.08	7.18
11 / 5	1.42	2.39	0	8.05	1.43	2.40	0	8.07
11 / 6	3.47	2.75	0	8.55	3.47	2.76	0	8.56
11 / 7	3.54	3.25	0	11.50	3.53	3.25	0	11.53
11 / 8	5.60	3.18	0	10.57	5.58	3.18	0	10.54

Table 6.1 shows that on average the best final nests were obtained with a parameter tuning factor of 10 and a cost tuning factor of 4. The relevant row in Table 6.1 is grey scaled and minimum averages are given in bold text. As mentioned for the nest in Figure 6.1, Table 6.1 shows small differences in average values for overlap and non-placement for each pair of control tuning factors. When escape is not allowed and the total stencil area is equal to the sheet area, Equations 5.2 and 5.3 (see Section 5.2) are the same,

and therefore areas of overlap and non-placement should be equal for every nest found. In Table 6.1 this is not reflected. The differences in values for overlap and non-placement are thought to be the result of the standard C-sharp methods used in the code of the dredge cut nesting model to calculate areas of polygons. Yuping *et al*. (2005) reported having used the Weiler algorithm (Weiler, 1980) for polygon comparison. To calculate areas of polygons in the code of the dredge cut nesting model, corresponding regions were filled with a limited number of non-overlapping rectangles with sides parallel to the coordinate axes. Next, the area sum of these rectangles was calculated and taken as the area of the polygon considered. Polygon edges, as shown in Figure 6.1, were not always parallel to the coordinate axes and because the number of filling rectangles is limited, polygons were not always completely covered. The incomplete coverage of polygon regions is what must have caused errors in calculating overlap and non-placement areas, in the order of the differences seen in Table 6.1. The average difference between percentage values of overlap and non-placement observed was 0.008%, with a maximum of 0.025% of the total sheet area. Figure 6.2 depicts an overview of average final costs of overlap and non-placement expressed as percentages of total sheet area.

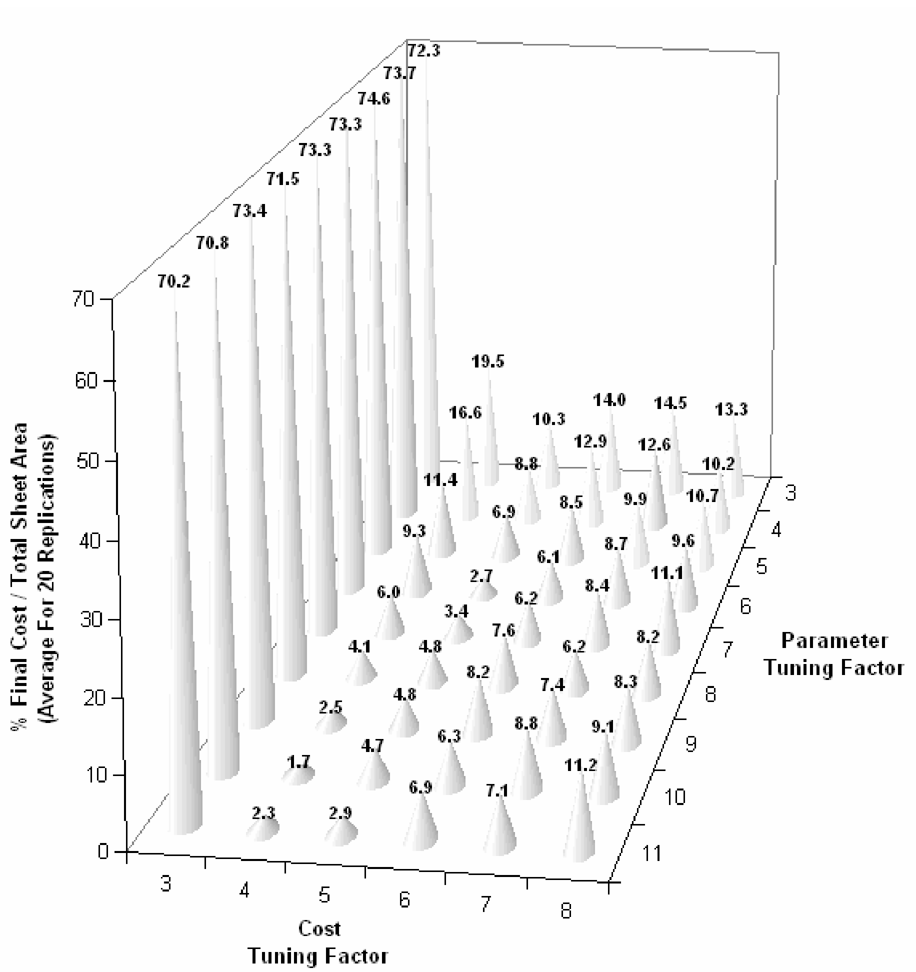


Figure 6.2 Overview irregular nesting results – Validation.

Figure 6.2 shows that final costs are more sensitive to the value of the cost tuning factor than to that of the parameter tuning factor. It also shows that using a cost tuning factor of 3 consistently resulted in the poorest final nest layouts, indicating a lack in annealing of the acceptance function to below the threshold required for finding minima. Figure 6.2 shows that for a cost tuning factor of 4 a clear trend of improved final nest layouts was observed up to a parameter tuning factor of 10, after which a slight increase in final costs was observed for a parameter tuning factor of 11. None of the other cost tuning factors used showed such a clear trend for different values of parameter tuning factors. The lack of clearer trends can be the result of the limited number of replications (20 for each experiment) that were carried out. Table 6.2 compares the best performing pair of tuning factor values identified here with values used/recommended in literature (see Equations 3.33, 5.24 and 5.25).

Table 6.2 Adaptive Simulated Annealing settings for different applications

Variable	Dredge Cut Nesting Validation	Leather Nesting (Yuping *et al*., 2005)	Signal Processing (Chen *et al*., 1999)	A.S.A. Theory (Ingber, 2006)
Parameter Space Dimensions	42	45	2 and 4	42
Control Coefficient m_i	-	$-\ln(1\times10^{-30})$	-	$-\ln(\geq1\times10^{-5})$
Control Coefficient n_i	-	ln(200)	-	ln(100)
Parameter Tuning Factor	10	61.40	[1,10]	≤10.32
Cost Tuning Factor	4	61.40	[1,10]	≤10.32

Table 6.2 shows that values of parameter and cost tuning factors for which, on average, the best nesting results were obtained here, fall within the theoretical ranges recommended in Ingber (2006). If the values of the parameter and cost tuning factors would have exceeded the values recommended in Ingber (2006) then the statistical guarantee of finding global optima would have been lost because simulated quenching would have been carried out instead. This could have been the case in the leather nesting experiments carried out in Yuping *et al*. (2005) for which, as shown in Table 6.2, it is thought a much higher value of 61.40 was used for the parameter and cost tuning factor.

Yuping *et al*. (2005) actually states having used a value of $\ln(1\times10^{-30}) = -69.08$ for the control coefficient m_i (see Equation 5.24). However, a negative value for the control coefficient m_i would lead to negative values of tuning factors (see Equation 3.33), in the case of Yuping *et al*. (2005) minus 61.40, which in turn would give negative values of rescaled annealing times for uneven parameter space dimensions (see Equation 3.35). Negative annealing times will in turn cause an increase in annealing temperatures as the solution process progresses (see Equation 3.32), therefore heating instead of annealing the system. Since the leather nesting problem solved in Yuping *et al*. (2005) has 45 parameter space dimensions, an uneven number, it is thought that a value of plus 61.40

was used for tuning factors instead, otherwise the nesting processes could not have converged to global optima as reported in Yuping *et al*. (2005). If so, then it can be said that in Yuping *et al*. (2005) the leather nesting problem was indeed solved by adaptive simulated quenching instead of adaptive simulated annealing.

The parameter and cost tuning factor values of 10 and 4 which performed best here also fall within the bounds recommended in Chen *et al*. (1999). When the default values for control coefficients in Table 6.1 are used for parameter space dimensions of 2 and 4, values for tuning factors of 1.15 and 3.64 are arrived at, respectively. Considering Figure 6.2, which shows poor results for a cost tuning factor of 3, it is thought the use of a cost tuning factor of 1.15 for the nesting problem discussed here would have given equally poor or worse results.

The difference in parameter space dimensions between problems solved here and in Chen *et al*. (1999) was initially overlooked at the stage of experimental design: Originally, only the use of parameter tuning factor values between 3 and 8 was foreseen, which covered the majority of the range of 1 to 10 recommended in Chen *et al*. (1999). Further to the perceived shortage of good quality final nests, as confirmed by visual inspection of all final nests, additional experiments were carried out for parameter tuning factors 9, 10 and 11.

All 1,080 final nest layouts were inspected visually for nearness to global optima. The criterion used was subjective, and was satisfied if it was thought that by further simulated annealing or quenching the final nest observed was likely to converge to a nest with zero overlap and zero non-placement. With the results of visual inspections it can be argued that for parameter and cost tuning factor combinations of 6 / 4, 7 / 4, 8 / 4, 9 / 4 and 10 / 4, 20 out of 20 final nest satisfy this subjective criterion. These tuning factor values are all within the tuning factor value ranges required for finding global optima as recommended in Ingber (2006). Figure 6.3 depicts the final nest with the highest final cost which was still considered near-optimal, and which was found for a parameter and cost tuning factor combination of 6 and 4.

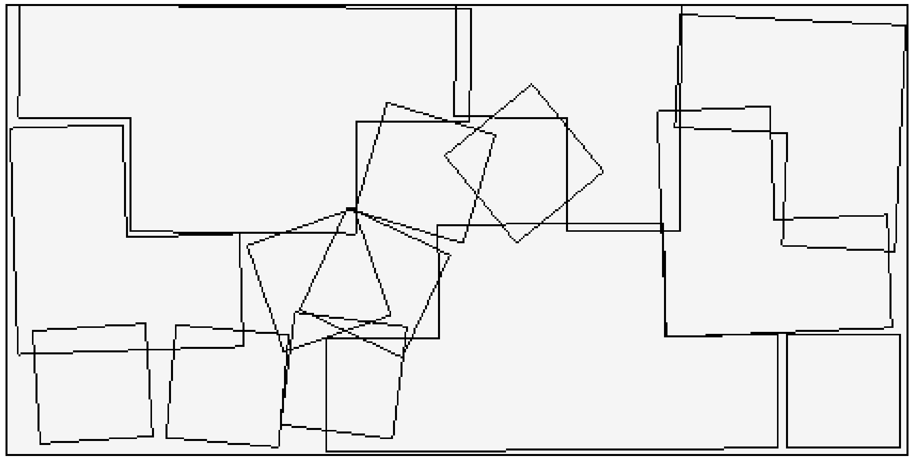


Figure 6.3 Sub-optimal nest – Irregular nesting – Validation.

The nest shown in Figure 6.3 has an overlap area and a non-placement area each equal to 9.09% of the sheet area. Figure 6.4 depicts the best local minimum final nest found, which has an overlap area equal to 2.63%, and a non-placement area equal to 2.57% of the sheet area. Figures 6.3 and 6.4 illustrate the usefulness of visually inspecting final nests: The final cost of the nest depicted in Figure 6.3 is approximately 4 times higher than that of the nest depicted in Figure 6.4. However, the nest in Figure 6.4 was considered a local minimum which is unlikely to converge to a nest with zero overlap and zero non-placement after further simulated annealing or quenching.

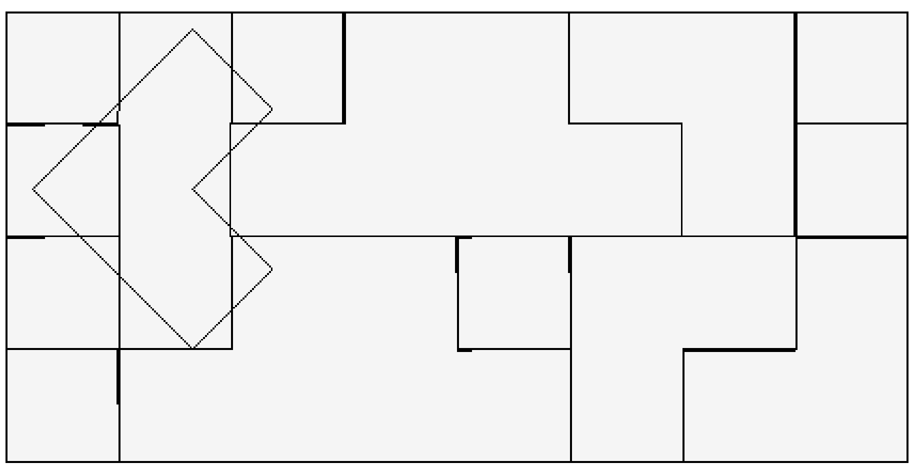


Figure 6.4 Local minimum nest – Irregular nesting – Validation.

Table 6.1 also shows that the dredge cut nesting model is capable of arriving at final nests which exhibit zero overlap and zero non-placement. Figure 6.5 shows the percentages of 20 final nests which exhibited zero overlap and zero non-placement and gives the parameter and cost tuning factors with which they were arrived at.

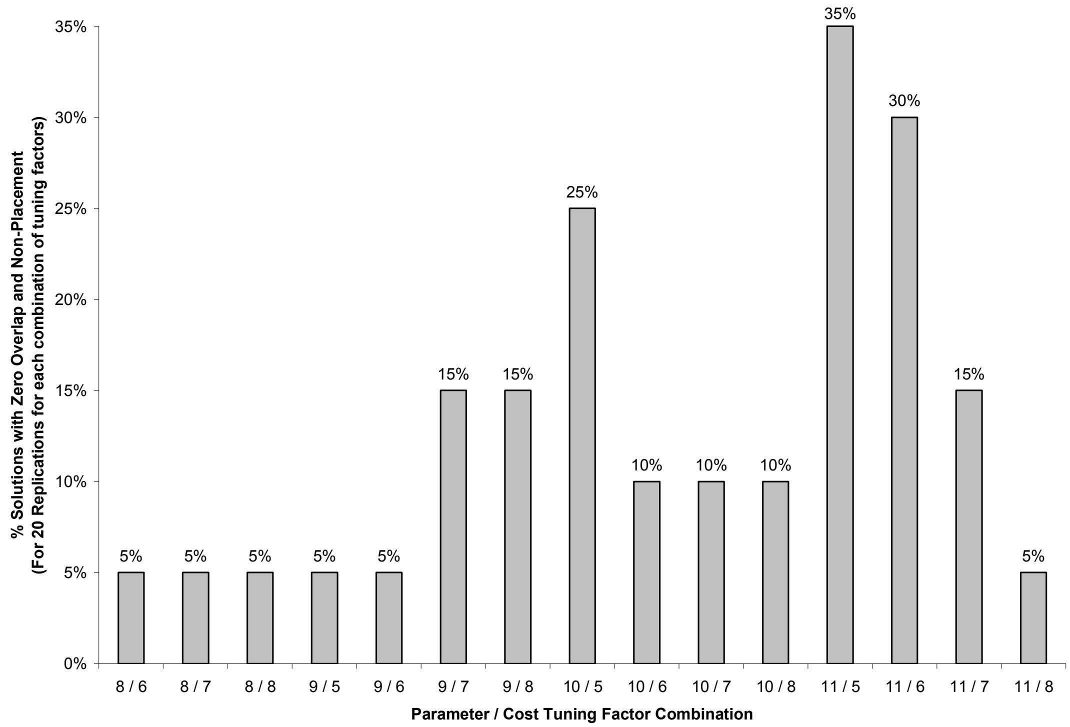


Figure 6.5 Percentages global minima – Validation.

Figure 6.5 shows that, out of a total of 1,080 nesting experiments carried out, only 39 of the final nests found, exhibited zero overlap and non-placement, which equates to a success rate of 3.61%. The highest percentage of 35% (7 out of 20) in Figure 6.5 was observed for a parameter tuning factor of 11 and a cost tuning factor of 5. A tuning factor value of 11 is greater than the maximum of 10.32 recommended in Ingber (2006), and therefore the use of a tuning factor of 11 can be said to have compromised the ergodicity of the adaptive simulated annealing-based solution process. Figure 6.6 depicts the first final nest found with zero overlap and zero non placement, which was obtained with a parameter tuning factor of 11 and a cost tuning factor of 5.

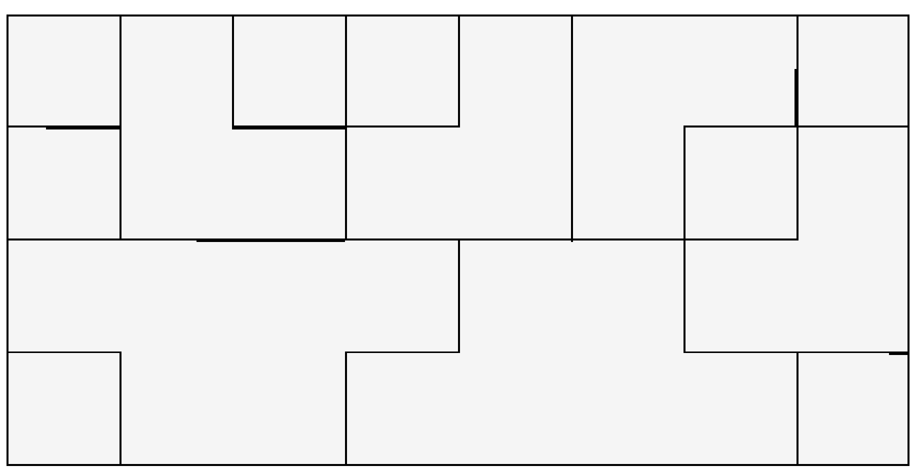


Figure 6.6 Global minimum nest – Irregular nesting – Validation.

For the nesting problem discussed here, if the statistical guarantee of finding a global optimum is to be maintained then values of tuning factors should not exceed those recommended in Ingber (2006). However, results obtained here with values of tuning factors which guarantee finding a global optimum and which satisfy the zero non-placement constraint of the dredge cut nesting model (see Constraint 5.5), gave at most 5 satisfactory final nests out of 20 replications for the best performing pair of tuning factors, or 22 satisfactory final nests out of a total of 980 experiments carried out. This equates to a success rate of 2.24%. This success rate is lower than the 3.61% achieved when simulated quenching of parameter temperatures is accepted.

It is thought that invalidating the guarantee of finding a global optimum by using a parameter and cost tuning factor combination of 11 and 5 can be accepted for the nesting problem solved here if finding a greater number of final nests which exhibit zero non-placement is seen as important, in this case an increase of 10% (up from 5 to 7 out of 20). Another reason for accepting the use of a value which goes against theory can be that it was used as a tuning factor which influences parameter temperatures for stencil motion, which, as seen in Figure 6.2, does not affect final costs as much as the value of the cost tuning factor. For the nesting problem discussed here, more caution should be taken when using cost tuning factor values which are outside the theoretical range.

Despite achieving low overall success rates, the global minima in Figure 6.5 validate the dredge cut nesting model for the irregular nesting problem with 14 stencils taken from Yuping *et al*. (2005). The fact that 7 of the global minima in Figure 6.5 were obtained with a parameter tuning factor value outside the recommended upper bound highlights a criticism often levelled at simulated annealing-based solution approaches. This criticism is that in some instances simulated annealing can be a very poor algorithm to search for global optima, which leads to simulated quenching-based solution approaches being adopted instead (Ingber, 1993).

The low success rates of finding final nests with zero overlap and zero non-placement obtained here suggest that the adaptive simulated annealing algorithm is not well suited to searching for global optima of the irregular nesting problem discussed here. However, as noted in Ingber (1993), to some extent the low success rates can be considered to have been offset by the relative ease with which the dredge cut nesting problem was approached and coded. In contrast to the 4,000 lines of code developed here to optimize nesting problems, Heistermann *et al*. (1995) reports that the implementation of a heuristic greedy algorithm to solve leather nesting problems for industrial use required 115,000 lines of code. Validation results for the dredger routing model are presented and discussed next.

6.2 Validation of Dredger Routing Model

Using Equations 5.19 and 5.20 for the 32 node routing problem, with a square grid spacing of 50, the minimum route length is 1,550 and the minimum sum of turning angles is 540 degrees. Figure 6.7 depicts a random route used as an initial solution at the start of the routing optimization process. The route length and the sum of turning angles of the route depicted in Figure 6.7 are 5,120.48 and 3,610.08 degrees, respectively.

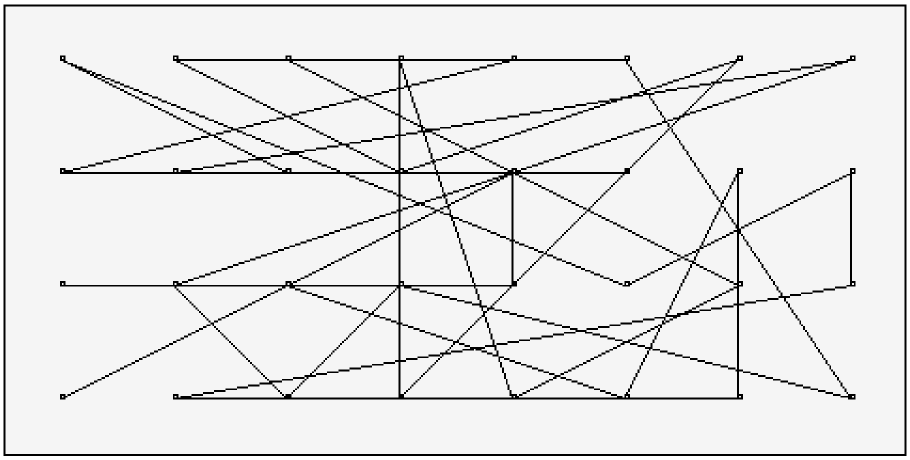


Figure 6.7 Random initial route – Regular routing – Validation.

The initial route depicted in Figure 6.7 does not have any links, while for the 32 node routing problem, with a square grid spacing of 50, the maximum link length is 350 and the optimum number of maximum length links is 4 (see Equations 5.21 an 5.22). Table 6.3 reports the mean, μ, standard deviation, σ, and minimum and maximum of route attributes for 20 final routes arrived at for local search, LS, values of 1 and 8 for the validation of the dredger routing model on the 32 node square grid routing problem solved here.

Table 6.3 Regular Routing – 32 Nodes

	LS	μ	σ	Min	Max
Route Length (m)	1	1,555.61	17.90	1,550.00	1,620.71
	8	1,552.07	9.26	1,550.00	1,591.42
Sum Turning Angles	1	549.00º	27.70º	540.00º	630.00º
	8	567.00º	120.75º	540.00º	1,080.00º
Average Link Length (m)	1	334.21	35.86	214.29	350.00
	8	335.67	40.31	183.33	350.00

For a local search of 1 each 32 node routing experiment of 99,999 base iterations on average took 13.35 minutes to complete, whereas each experiment with 99,999 base iterations and a local search of 8 on average took 17.23 minutes to complete. For a local search of 1 the dredger routing model found final routes with optimum route lengths and sums of turning angles 17 out of 20 times and for a local search of 8 it did so 19 out of 20 times. These results correspond with Koulamas *et al*. (1994) which, for a simulated annealing-based solution approach, reported finding improved final tours of symmetric travelling salesperson problems when local search was increased from 1 to 8.

Of the 17 final routes found with a local search of 1 which had optimal route lengths and sums of turning angles, 16 also had 4 maximum length links, giving them an optimal average link length of 350. Therefore, 16 out of 20 routes found with a local search of 1 were optimal dredger routes. Figure 6.8 depicts the first optimal dredger route arrived at with a local search of 1 and the depicted route is identical to all other optimal dredger routes found. It should be noted that they were identical because of the fixed starting position.

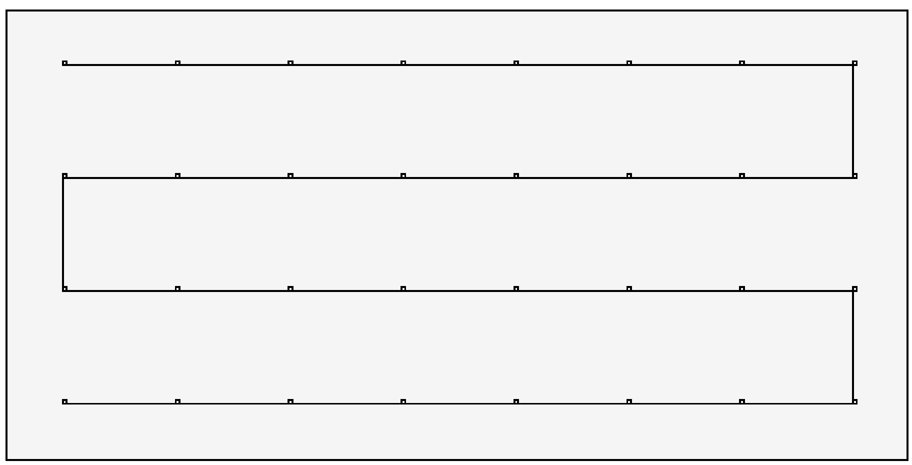


Figure 6.8 Optimal route – Regular routing – Validation.

Figure 6.9 on the next page depicts the one final route found with a local search of 1 which had an optimal route length and sum of turning angles, but which only had 3 maximum link lengths. The optimal route in Figure 6.8 has an average link length of 350 whereas the near-optimal route in Figure 6.9 has an average link length of 290.

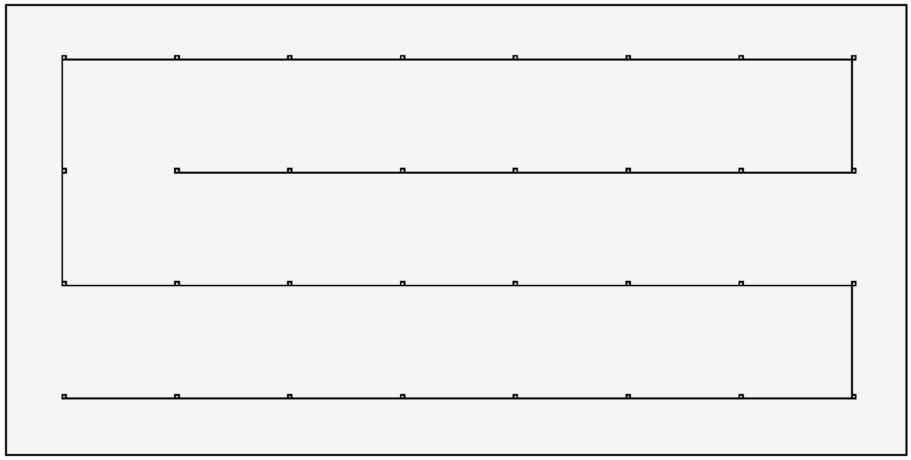


Figure 6.9 Near-optimal route – Regular routing – Validation – Local search 1.

Figures 6.8 and 6.9 highlight the intended effect which the edge length reduction factor (see Equation 5.23) has on the average link length of final routes. Without an edge reduction factor the dredger routing model, given enough replications, would find final routes like those depicted in Figures 6.8 and 6.9 in equal measure, but as an optimal dredger route only routes like the one in Figure 6.8 should be accepted.

For a local search of 8, all of the 17 out of 20 routes found with optimal route lengths and sums of turning angles were also found to have 4 maximum length links. For a local search of 8 one exceptional final route was found, which had a sum of turning angles of 1,080.00 degrees, exactly double the optimum, and had an average link length of 183.33, approximately half the optimum. Figure 6.10 depicts the relevant final route.

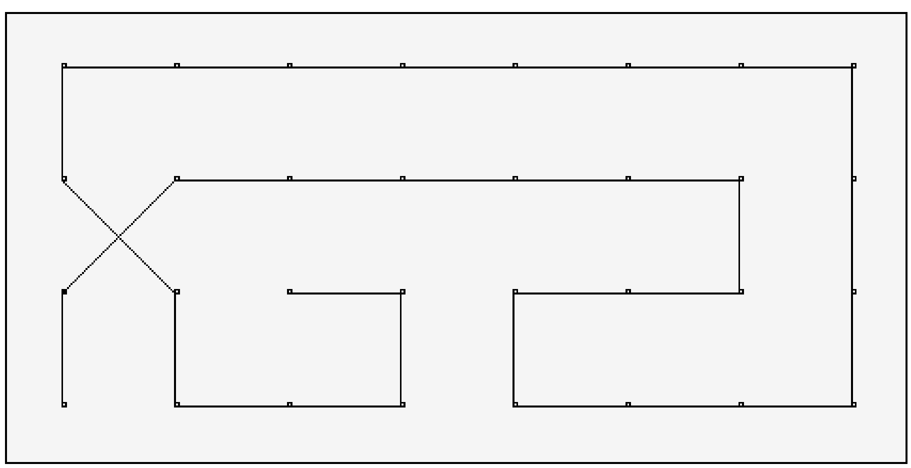


Figure 6.10 Sub-optimal route – Regular routing – Validation – Local search 8.

The final record of the optimization process leading to the final route depicted in Figure 6.10 shows that the best ever recorded route length and sum of turning angles were both optimal at some point. However, no record of the corresponding average link length exists. It is reasonable to assume that increasing local search not only increases the probability of finding better routes but also increases the probability of finding worse routes with smaller cost differences from current routes. Therefore, as increasing the local search from 1 to 8 increases probabilities of accepting worse routes, it can also increase the probability of escaping from global minima.

In general, it is thought the final route depicted in Figure 6.10 resulted from having escaped a global minimum with an optimum average link length. In particular, it is thought a local search of 8 could have adversely affected the benefits normally experienced from re-annealing events in the adaptive simulated annealing algorithm and have encouraged the undesired escape from a global minimum. It has already been stated that increasing local search increases probabilities of accepting better and worse states. When re-annealing occurs the current solution cost is stored and used at the next annealing event to rescale the acceptance temperature. Increased numbers of acceptances of better states lead to more frequent re-annealing, especially in the early stages of the optimization process when mostly better states are found, even more so with a local search of 8. This in turn, after annealing, leads to reductions in the acceptance temperature and therefore has a positive effect on the optimization process.

For this part of the experimentation re-annealing was carried out every 100 acceptances of better states and after every 10,000 consecutive iterations during which no better state was accepted. Annealing was carried out every 1,000 generated states, irrespective of the number of better states accepted. Re-annealing based on generated states (caused by a lack of acceptances of better states) does not tend to occur in the early stages of the optimization process. Despite that no continuous records of annealing progress were made for this part of the experimentation, it is thought that a re-annealing event triggered at a very late stage of the solution process, which subsequently was not followed by enough annealing events, played a significant part in arriving at the sub-optimal final route depicted in Figure 6.10. The use of a local search of 8 made finding such a final route more likely than when a local search of 1 was used.

Overall the results presented in this section validate the dredger routing model for 32 node continuous square grid routing problems of rectangular shape. The majority of final routes found were optimal dredger routes with optimal average link lengths. With a local search of 1 a success rate of 80% was achieved and with a local search of 8 this rate increased to 85%. Results of experiments with nesting problems of increased complexity are presented next, starting with results obtained for nesting problems with relaxed inner sheet boundaries.

6.3 Dredge Cut Nesting – Relaxed Sheet Boundaries

Experimental results obtained from the application of the dredge cut nesting model to irregular nesting problems with increasing numbers of relaxed sheet boundaries are presented in Tables 6.4, 6.5 and 6.6, which report the mean, μ, standard deviation, σ, and minimum and maximum of values of escape, overlap and non-placement for 20 replications carried out for each pair of tuning factors. Escape, overlap and non-placement are expressed as a percentage of the total inner sheet area used.

Table 6.4 Irregular nesting – 1 Relaxed sheet boundary

Tuning Factor Parameter / Cost	Escape (% of Sheet Area)				Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	8.79	4.75	1.28	17.25	29.84	5.85	19.39	40.64	38.66	3.73	31.73	45.38
3 / 4	1.21	0.87	0.09	3.38	10.31	2.44	6.82	15.94	11.53	2.43	7.82	18.04
3 / 5	0.99	0.95	0.02	3.58	6.70	1.71	4.08	10.44	7.69	1.60	5.39	10.86
3 / 6	1.29	1.05	0.08	3.39	7.37	1.82	4.52	10.99	8.66	1.66	5.81	12.06
3 / 7	0.78	0.74	0.03	3.07	7.80	2.15	3.28	11.23	8.58	2.07	4.52	11.54
3 / 8	1.59	1.10	0.01	3.93	7.76	1.62	4.11	10.17	9.36	1.48	6.48	12.10
4 / 3	7.89	3.63	1.16	15.59	29.21	5.89	17.74	43.21	37.12	6.05	27.80	56.48
4 / 4	0.87	0.77	0.05	3.15	9.09	2.40	4.96	14.05	9.96	2.63	5.97	15.07
4 / 5	0.81	0.65	0.12	2.92	4.98	1.92	2.43	9.55	5.80	2.04	2.74	11.16
4 / 6	0.80	1.18	0.01	4.37	6.66	1.93	3.45	10.09	7.47	2.34	3.60	13.07
4 / 7	1.58	1.13	0.02	3.91	5.38	1.66	2.63	8.94	6.96	1.97	3.25	10.14
4 / 8	1.49	1.20	0.01	4.61	6.26	2.40	2.40	10.30	7.76	2.20	3.31	10.81
5 / 3	7.58	5.53	0.41	20.91	30.56	6.51	20.42	38.24	38.15	4.73	27.75	44.04
5 / 4	1.31	1.40	0.03	5.01	6.98	2.08	3.03	11.75	8.30	2.42	3.70	12.26
5 / 5	0.48	0.90	0	3.77	3.60	1.33	2.09	6.46	4.08	1.40	2.16	6.46
5 / 6	0.76	1.03	0.06	3.13	4.95	2.25	1.28	9.42	5.72	2.41	1.88	9.65
5 / 7	1.50	1.29	0.02	3.27	4.58	2.00	1.24	9.59	6.08	1.81	2.29	9.62
5 / 8	1.67	1.28	0	4.87	5.54	1.96	1.52	8.32	7.22	2.37	1.74	10.47
6 / 3	8.79	5.07	0.22	17.83	30.79	5.61	19.33	40.40	39.60	3.99	28.82	48.26
6 / 4	0.55	0.54	0	2.11	6.44	2.71	3.39	12.35	7.00	3.03	3.58	14.47
6 / 5	0.59	0.80	0.03	3.15	2.85	2.13	0.73	9.10	3.45	2.46	0.77	10.15
6 / 6	1.06	1.13	0.02	3.52	3.25	2.57	0.42	7.26	4.31	2.77	0.64	9.03
6 / 7	1.38	1.24	0	3.34	5.56	2.26	1.07	9.11	6.95	2.05	1.08	9.75
6 / 8	0.92	1.01	0.04	3.08	5.10	2.87	0.43	9.11	6.02	3.11	0.50	11.03
7 / 3	9.14	5.07	3.22	19.55	30.54	4.35	19.42	38.34	39.70	4.31	31.30	47.01
7 / 4	0.30	0.20	0.03	0.66	3.68	2.03	1.75	11.28	3.99	2.06	1.93	11.75
7 / 5	0.26	0.53	0	2.29	1.85	1.98	0.23	6.73	2.11	2.13	0.35	7.42
7 / 6	0.64	0.97	0	3.14	2.85	2.29	0.10	7.22	3.49	2.66	0.23	7.50
7 / 7	0.99	1.03	0	3.09	4.16	2.64	0.12	8.02	5.16	2.70	0.30	8.98
7 / 8	1.47	1.27	0	3.22	3.92	2.69	0.17	9.74	5.38	2.37	0.17	10.19
8 / 3	9.34	5.47	1.37	20.98	29.21	4.58	22.46	36.88	38.57	3.98	31.54	47.77
8 / 4	0.33	0.59	0.03	2.77	2.68	1.68	1.33	9.15	3.03	2.23	1.37	11.95
8 / 5	0.54	0.96	0	3.25	2.32	2.37	0.14	6.66	2.87	2.49	0.18	7.22
8 / 6	0.93	1.32	0	3.15	2.40	2.25	0.01	7.45	3.33	2.05	0.01	7.52
8 / 7	1.35	1.67	0	5.87	5.21	2.75	0.06	10.85	6.56	2.71	0.06	10.85
8 / 8	0.96	1.18	0	3.47	4.88	2.57	0.05	8.68	5.83	2.45	0.08	9.06
9 / 3	7.78	3.90	1.66	15.16	29.40	4.99	21.76	38.76	37.20	3.39	31.43	44.36
9 / 4	0.20	0.25	0	1.09	2.13	2.12	0.38	8.83	2.35	2.31	0.41	9.32
9 / 5	0.77	1.17	0	3.10	2.24	1.89	0.03	5.95	3.01	2.03	0.03	7.28
9 / 6	0.48	0.82	0	3.11	3.72	2.35	0.12	7.97	4.21	2.49	0.12	8.04
9 / 7	0.22	0.49	0	1.91	4.08	3.17	0	9.78	4.30	3.35	0	9.90
9 / 8	1.09	1.21	0	3.45	4.72	2.09	0.03	8.15	5.79	2.26	0.03	9.04
10 / 3	10.60	6.48	2.71	25.68	27.99	5.48	16.14	38.26	38.61	5.58	23.91	47.15
10 / 4	**0.19**	0.28	0	0.94	2.78	3.43	0.73	10.28	2.99	3.68	0.75	11.24
10 / 5	0.56	0.87	0	3.16	3.12	3.11	0	8.64	3.69	3.43	0	10.08
10 / 6	0.97	1.42	0	5.08	4.56	2.02	0.29	9.60	5.53	2.29	3.11	10.71
10 / 7	0.64	1.15	0	3.14	4.78	2.83	0	10.07	5.41	2.65	0.01	10.16
10 / 8	0.64	0.93	0	3.12	3.55	2.61	0	7.74	4.16	2.68	0	7.98
11 / 3	9.45	5.77	0.46	23.74	29.79	5.58	18.58	39.91	39.26	5.19	31.21	50.47
11 / 4	0.27	0.62	0	2.62	**1.73**	2.47	0.16	7.83	**2.02**	2.90	0.20	8.32
11 / 5	0.90	1.28	0	3.54	3.20	2.74	0	9.60	4.11	2.87	0	10.49
11 / 6	1.13	1.32	0	3.13	2.28	2.24	0	6.10	3.42	2.35	0	6.60
11 / 7	1.13	1.28	0	3.80	5.02	2.02	2.30	9.62	6.13	2.21	2.88	9.87
11 / 8	1.43	1.27	0	3.13	4.15	2.43	0.03	8.86	5.55	2.03	3.10	9.28

Table 6.4 shows that on average, for one relaxed sheet boundary, the best final nests with minimum non-placement – the most important decision variable for dredging – were obtained with a parameter tuning factor of 11 and a cost tuning factor of 4. The relevant row in Table 6.4 is grey scaled and minimum averages for all three decision variables are given in bold text. Before summarizing results, results of nesting experiments with two relaxed inner sheet boundaries are given in Table 6.5.

Table 6.5 Irregular nesting – 2 Relaxed sheet boundaries

Tuning Factor Parameter / Cost	Escape (% of Sheet Area)				Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	16.01	3.93	10.81	23.97	22.08	5.02	14.04	30.19	38.11	4.42	30.76	46.10
3 / 4	3.54	1.66	2.31	9.75	10.33	2.49	5.11	14.99	13.88	2.60	7.71	18.87
3 / 5	2.59	1.11	1.18	4.59	5.85	1.14	3.64	8.41	8.45	1.57	5.40	10.57
3 / 6	2.65	1.34	0.57	5.78	6.20	1.31	3.72	8.20	8.85	1.77	5.42	11.86
3 / 7	3.28	1.18	0.98	5.57	6.64	0.84	4.29	8.09	9.92	1.33	7.34	12.86
3 / 8	3.18	0.96	1.48	5.55	6.74	0.90	5.04	8.53	9.91	1.13	7.56	11.60
4 / 3	16.01	5.60	6.40	25.49	21.56	5.20	12.64	30.18	37.59	4.61	28.79	47.57
4 / 4	2.82	1.48	0.97	6.76	7.68	2.61	3.52	13.23	10.51	3.24	4.88	15.74
4 / 5	1.82	0.81	0.58	3.78	4.66	1.52	2.55	7.33	6.49	1.99	3.49	9.21
4 / 6	2.80	0.98	1.19	4.25	5.45	1.96	3.05	9.37	8.25	2.41	4.53	13.41
4 / 7	2.44	0.90	0.78	3.38	5.40	1.51	2.46	8.24	7.85	1.68	4.57	9.90
4 / 8	3.00	1.49	0.69	6.38	4.64	1.91	1.25	7.68	7.64	2.69	2.91	11.93
5 / 3	15.64	5.28	5.27	25.48	22.33	5.48	12.70	31.40	37.99	4.27	29.99	45.73
5 / 4	3.41	1.39	0.73	6.34	7.24	2.31	3.54	11.10	10.66	3.09	5.83	16.13
5 / 5	1.56	1.04	0.38	3.77	3.74	1.82	1.18	7.05	5.30	2.40	1.63	10.82
5 / 6	1.83	1.23	0.50	4.50	3.90	1.98	1.02	8.29	5.74	1.77	1.99	8.78
5 / 7	1.69	1.23	0.14	4.00	3.58	1.72	1.26	6.90	5.28	2.31	2.18	8.93
5 / 8	2.29	1.66	0.18	5.62	4.11	1.60	1.12	7.78	6.40	2.57	1.65	9.82
6 / 3	13.68	4.47	5.82	22.24	23.24	6.08	13.69	41.49	36.94	5.65	29.32	50.83
6 / 4	1.62	0.86	0.29	3.14	5.28	2.23	1.99	10.28	6.92	2.60	3.44	13.09
6 / 5	1.64	1.61	0.10	4.97	3.05	1.61	0.92	6.74	4.69	2.34	1.09	8.39
6 / 6	1.32	1.07	0.12	3.59	3.59	2.04	0.50	7.20	4.91	2.72	0.77	9.06
6 / 7	1.75	1.26	0.13	3.66	3.16	1.53	0.54	5.92	4.91	1.82	1.05	7.29
6 / 8	1.85	1.14	0.18	3.57	4.20	2.32	0.51	9.09	6.05	2.83	0.80	10.64
7 / 3	17.12	6.31	6.19	32.46	20.48	5.26	8.13	30.87	37.63	5.08	25.47	44.55
7 / 4	1.12	0.51	0.32	1.97	3.45	1.35	1.45	7.28	4.59	1.52	2.70	8.84
7 / 5	0.88	0.72	0.05	2.20	2.80	2.07	0.32	6.53	3.69	2.43	0.63	7.26
7 / 6	1.83	1.27	0.07	3.90	3.16	1.91	0.47	6.16	4.99	2.31	0.59	7.47
7 / 7	1.83	1.48	0.06	4.42	3.91	2.53	0.28	8.04	5.74	3.24	0.36	10.14
7 / 8	1.67	1.42	0.02	5.28	3.38	1.59	0.59	6.47	5.06	1.89	0.73	9.03
8 / 3	14.02	4.84	0.97	22.49	23.24	4.87	14.45	32.93	37.28	4.63	30.12	47.99
8 / 4	1.42	0.98	0.29	3.39	3.69	2.28	1.30	10.42	5.13	3.06	1.96	13.80
8 / 5	0.78	1.02	0.02	4.26	**2.08**	1.67	0.18	4.79	**2.87**	2.32	0.28	8.79
8 / 6	1.21	1.14	0	3.46	2.87	1.69	0.09	6.19	4.08	2.18	0.11	7.55
8 / 7	1.37	1.33	0	3.91	3.22	1.61	0.03	5.94	4.59	2.03	0.09	8.33
8 / 8	1.47	1.12	0.04	3.41	3.59	1.93	0.06	8.83	5.05	1.90	2.89	9.31
9 / 3	16.19	5.64	5.63	24.56	21.56	5.42	13.40	35.17	37.77	2.76	32.54	43.67
9 / 4	1.30	1.11	0.27	4.38	2.91	2.03	0.53	8.97	4.22	2.78	1.22	10.62
9 / 5	1.03	1.18	0	4.55	2.72	2.16	0.02	6.80	3.76	2.95	0.07	8.31
9 / 6	1.35	1.34	0	4.21	3.01	1.94	0.04	6.20	4.37	2.30	0.06	7.30
9 / 7	2.43	1.72	0	6.59	4.33	2.01	0	6.43	6.76	2.87	0	10.40
9 / 8	2.58	1.51	0	4.77	3.02	2.02	0	6.99	5.58	2.71	0	10.01
10 / 3	14.22	5.52	5.09	22.80	22.23	5.52	14.96	32.10	36.47	3.42	25.64	41.28
10 / 4	1.53	1.75	0	5.63	11.89	2.72	7.36	18.42	13.43	3.17	7.42	20.31
10 / 5	**0.58**	0.86	0	3.12	7.60	1.27	5.88	10.45	8.20	1.52	6.14	11.29
10 / 6	0.58	0.61	0	1.73	8.34	1.78	5.03	12.81	8.93	1.70	6.21	12.82
10 / 7	1.53	1.21	0	3.87	8.37	1.30	6.12	10.59	9.91	1.67	7.19	12.77
10 / 8	0.96	0.94	0	3.07	8.91	2.39	4.87	13.84	9.86	2.16	5.88	13.82
11 / 3	15.53	6.57	5.75	29.03	22.21	6.39	12.84	31.91	37.76	6.61	28.17	52.83
11 / 4	2.03	2.01	0.06	6.51	10.17	1.81	7.92	14.31	12.22	2.37	8.86	16.45
11 / 5	0.89	1.15	0	3.67	7.89	1.44	4.84	9.91	8.80	1.56	5.51	11.01
11 / 6	0.99	1.20	0	3.20	7.77	1.95	4.47	12.07	8.77	1.98	5.11	12.08
11 / 7	1.30	1.09	0	4.02	8.44	2.10	6.03	15.04	9.73	1.95	6.21	15.61
11 / 8	1.63	1.76	0	5.55	8.38	1.91	5.14	11.92	10.01	1.75	6.88	13.69

Table 6.5 shows that on average, for two relaxed sheet boundaries, the best final nests with minimum non-placement were obtained with a parameter tuning factor of 8 and a cost

tuning factor of 5. Table 6.6 gives the results of nesting experiments with three relaxed inner sheet boundaries.

Table 6.6 Irregular nesting – 3 Relaxed sheet boundaries

Tuning Factor Parameter / Cost	Escape (% of Sheet Area)				Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	19.61	6.15	12.40	35.59	18.59	5.02	9.83	28.07	38.22	6.21	25.14	48.04
3 / 4	4.70	2.17	1.79	9.38	9.56	2.32	5.49	14.70	14.27	3.17	8.18	21.73
3 / 5	3.31	1.02	1.05	5.67	5.62	1.76	2.92	9.58	8.93	2.03	5.79	12.64
3 / 6	3.59	1.27	1.24	6.17	6.54	1.32	4.67	9.81	10.13	1.21	7.35	12.06
3 / 7	3.77	1.16	1.78	5.71	6.08	1.16	2.72	7.56	9.85	1.62	5.28	12.44
3 / 8	4.56	1.40	1.45	7.27	5.84	1.32	3.39	9.32	10.40	1.65	5.94	13.41
4 / 3	17.87	6.74	7.90	41.95	18.16	4.50	9.70	26.61	36.05	5.53	29.16	54.52
4 / 4	4.13	1.68	1.83	8.74	8.47	2.01	3.85	12.56	12.61	1.90	9.27	15.87
4 / 5	2.07	0.74	0.73	3.43	4.52	1.18	2.52	6.27	6.60	1.74	3.70	9.06
4 / 6	3.19	1.42	1.26	6.63	5.14	1.41	2.77	8.54	8.33	1.87	4.31	11.36
4 / 7	3.16	1.62	0.80	6.42	4.85	1.69	2.21	7.67	8.01	2.05	3.44	10.39
4 / 8	2.89	1.25	0.91	5.11	5.43	1.49	2.23	8.32	8.33	1.92	4.36	11.92
5 / 3	20.88	5.98	11.30	34.52	16.68	5.07	10.87	30.19	37.58	4.23	30.75	48.11
5 / 4	4.24	2.35	1.45	11.01	7.43	1.78	3.75	11.34	11.67	3.33	6.10	18.76
5 / 5	1.98	1.10	0.27	4.04	3.57	1.30	1.72	6.09	5.55	2.02	2.53	9.17
5 / 6	2.32	1.37	0.24	5.93	3.03	1.29	1.06	5.25	5.36	2.16	2.19	9.60
5 / 7	2.32	1.28	0.65	4.81	4.23	1.41	1.24	7.00	6.55	2.03	2.46	9.40
5 / 8	3.28	1.76	0.90	7.48	4.48	1.95	0.81	9.12	7.77	2.18	2.08	11.40
6 / 3	18.97	6.15	10.82	36.21	19.52	4.57	9.48	26.28	38.51	3.68	32.61	45.71
6 / 4	2.93	1.14	0.87	5.78	5.82	1.58	3.39	9.48	8.77	2.29	4.71	13.20
6 / 5	1.42	0.91	0.16	3.39	3.27	1.97	0.78	6.86	4.69	2.79	1.21	9.33
6 / 6	2.01	1.39	0.31	5.11	2.97	1.44	0.70	4.70	4.98	2.27	1.01	7.76
6 / 7	2.35	1.73	0.31	5.91	3.48	1.94	0.80	6.39	5.83	3.07	1.33	9.95
6 / 8	3.10	1.74	0.09	6.41	4.44	1.46	0.73	7.60	7.54	2.12	1.07	10.28
7 / 3	18.63	3.67	13.44	24.64	17.53	4.15	10.57	27.68	36.18	4.75	28.79	46.63
7 / 4	2.42	1.07	0.74	4.37	4.80	1.89	1.79	8.42	7.24	2.71	2.83	12.31
7 / 5	1.30	1.04	0.17	3.43	**2.61**	1.87	0.43	7.58	3.92	2.37	0.71	9.12
7 / 6	1.90	1.40	0.14	4.07	2.95	1.93	0.25	5.90	4.86	2.74	0.43	8.12
7 / 7	1.83	1.58	0.06	6.78	3.66	2.32	0.29	7.86	5.50	2.99	0.40	9.98
7 / 8	1.58	1.07	0.06	3.53	4.47	1.72	0.65	7.29	6.05	1.84	3.11	9.72
8 / 3	19.23	6.81	10.11	33.64	18.03	4.45	9.65	25.71	37.28	5.54	29.57	48.17
8 / 4	2.06	1.39	0.57	6.26	3.93	2.18	1.30	9.08	6.01	3.48	2.36	15.37
8 / 5	1.45	1.18	0.06	4.53	3.04	1.81	0.20	5.97	4.49	2.45	0.40	7.40
8 / 6	1.45	1.15	0.05	3.77	3.06	1.81	0.28	6.48	4.52	2.38	0.42	9.19
8 / 7	2.48	1.70	0	6.64	3.04	2.13	0.07	7.15	5.51	2.56	0.11	8.94
8 / 8	2.35	1.24	0.28	4.48	3.94	1.10	2.21	6.00	6.28	1.59	3.43	8.42
9 / 3	21.21	6.74	9.63	40.52	15.09	4.45	4.53	23.06	36.32	4.37	28.55	45.05
9 / 4	1.53	1.03	0.48	3.66	3.64	2.28	1.35	8.31	5.19	3.12	2.04	11.20
9 / 5	1.39	1.21	0.02	4.34	2.87	1.99	0.02	5.87	4.27	2.85	0.05	8.60
9 / 6	1.70	1.24	0.04	3.32	3.38	2.24	0.15	6.66	5.09	2.70	0.19	9.55
9 / 7	2.03	1.58	0	4.52	3.83	1.95	0.01	7.53	5.85	2.66	0.08	9.47
9 / 8	1.69	1.14	0.02	3.79	3.83	1.96	0.11	8.92	5.51	2.27	0.10	11.34
10 / 3	19.84	6.31	10.11	33.14	17.35	3.53	11.12	24.45	37.21	5.15	28.25	47.72
10 / 4	1.65	1.63	0.18	5.51	3.09	2.24	0.68	8.80	4.76	3.73	1.03	14.33
10 / 5	1.14	1.20	0	3.90	2.92	2.28	0.02	6.48	4.07	3.15	0.02	9.03
10 / 6	2.38	1.40	0	4.33	2.81	1.84	0	6.06	5.19	2.47	0	8.52
10 / 7	2.29	1.58	0	4.94	3.45	2.40	0.04	7.58	5.73	2.59	0.04	9.78
10 / 8	2.49	1.43	0	4.46	3.50	1.54	0.80	6.87	5.97	2.36	2.89	10.47
11 / 3	19.80	6.69	12.42	41.67	17.69	4.73	12.22	28.85	37.51	6.35	28.12	55.27
11 / 4	**1.11**	1.19	0.09	4.09	2.63	2.26	0.47	7.93	**3.75**	3.16	0.61	12.04
11 / 5	1.56	1.04	0	3.34	3.50	2.09	0.02	6.91	5.08	2.11	0.02	8.82
11 / 6	1.83	1.13	0	4.24	3.95	1.59	0	5.94	5.78	2.04	0	8.68
11 / 7	2.74	1.40	0.24	5.06	3.70	1.87	0.01	6.98	6.43	2.10	2.64	10.18
11 / 8	2.70	1.58	0.02	6.26	4.31	1.77	0.05	7.27	6.98	2.23	0.06	9.38

Table 6.6 shows that on average, for a sheet with three relaxed boundaries, the best final nests with minimum non-placement were obtained with a parameter tuning factor of 11 and a cost tuning factor of 4. Overall average solution quality in Tables 6.4 and 6.5 is worse than in Table 6.1. Figure 6.11 summarizes average final costs consisting of average final overlap and non-placement costs expressed as percentages of the total inner sheet area used.

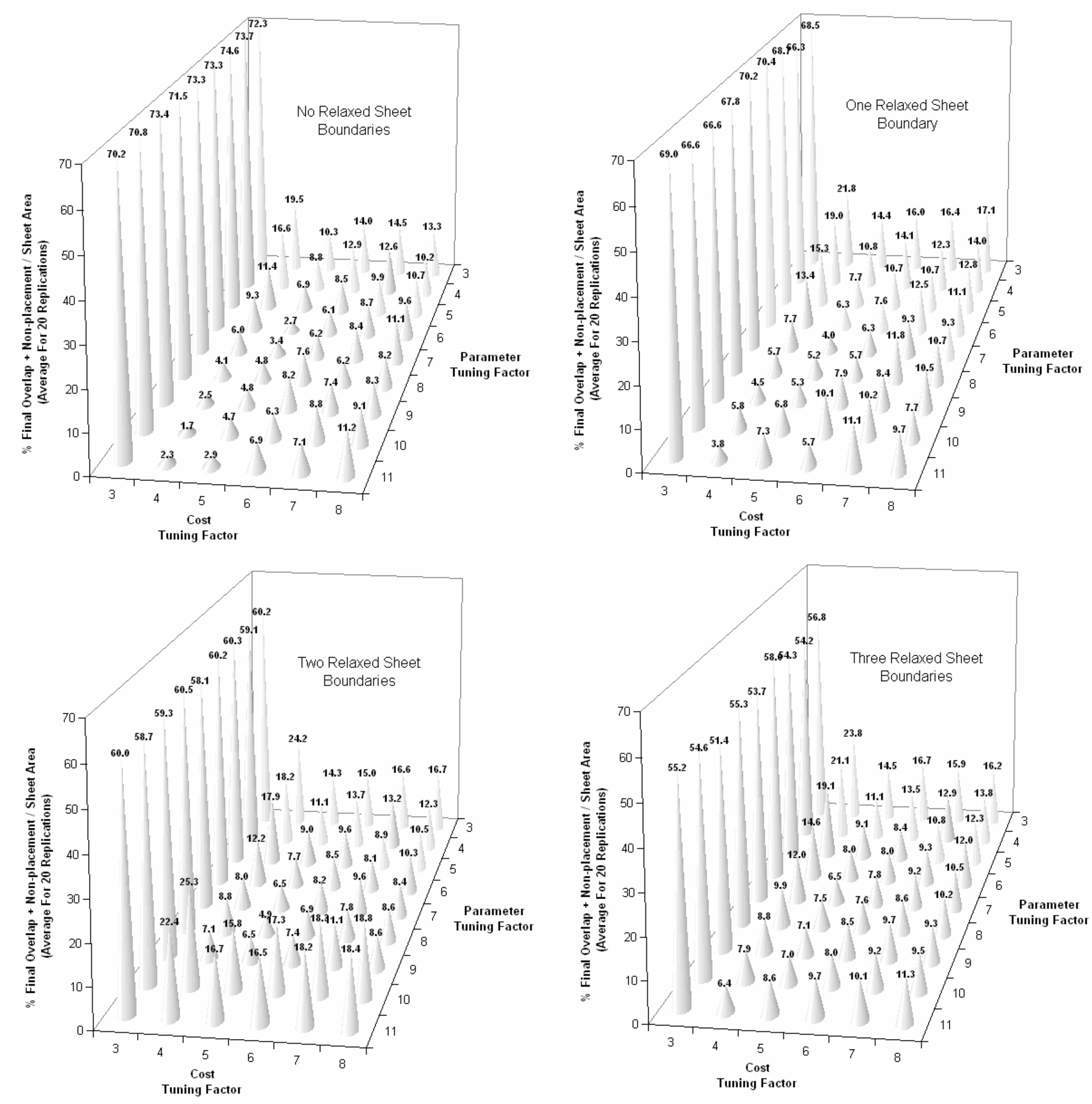


Figure 6.11 Overview irregular nesting results – Relaxed sheet boundaries.

Figure 6.11 shows that the worst final nests for all sheet arrangements were found for a cost tuning factor of 3, and that (for all tuning factor values used) the overall average solution quality for 0, 1, 2, and 3 relaxed sheet boundaries, respectively, was 19.03, 19.79, 20.38, and 18.25. Therefore it can be said that increasing the number of relaxed sheet boundaries up to 2 reduced the overall solution quality of final nests in comparison to a sheet with fixed boundaries where no escape was allowed. The minimum overall average solution quality was found for three relaxed sheet boundaries. However, the minimum average final cost of 1.7 was found for a sheet with fixed boundaries. It should be noted that cost penalties for escape, overlap and non-placement were all set to unity for this part of the experimentation. Table 6.7 gives minimum averages of overlap and non-placement obtained for the irregular nesting problem with varying numbers of relaxed sheet boundaries.

Table 6.7 Minimum averages irregular nesting – Relaxed sheet boundaries

Relaxed Sheet Boundaries	Minimum Average Overlap (% of Sheet Area)	Minimum Average Non-placement (% of Sheet Area)
0	0.83	0.84
1	1.73	2.02
2	2.08	2.87
3	2.61[1)]	3.75[1)]

Note: 1) Not obtained with same combination of parameter and cost control tuning factors.

Table 6.7 shows that an increase in the number of relaxed sheet boundaries increased average minima of overlap and non-placement for the irregular nesting problems solved, all of which have sheet to stencil area ratio of 1:1. Figure 6.12 shows percentages of 20 final nests which exhibited zero overlap and zero non-placement and the parameter and cost tuning factors with which they were arrived at. Note that Figure 6.12 shows that no such final nests were found for two relaxed sheet boundaries.

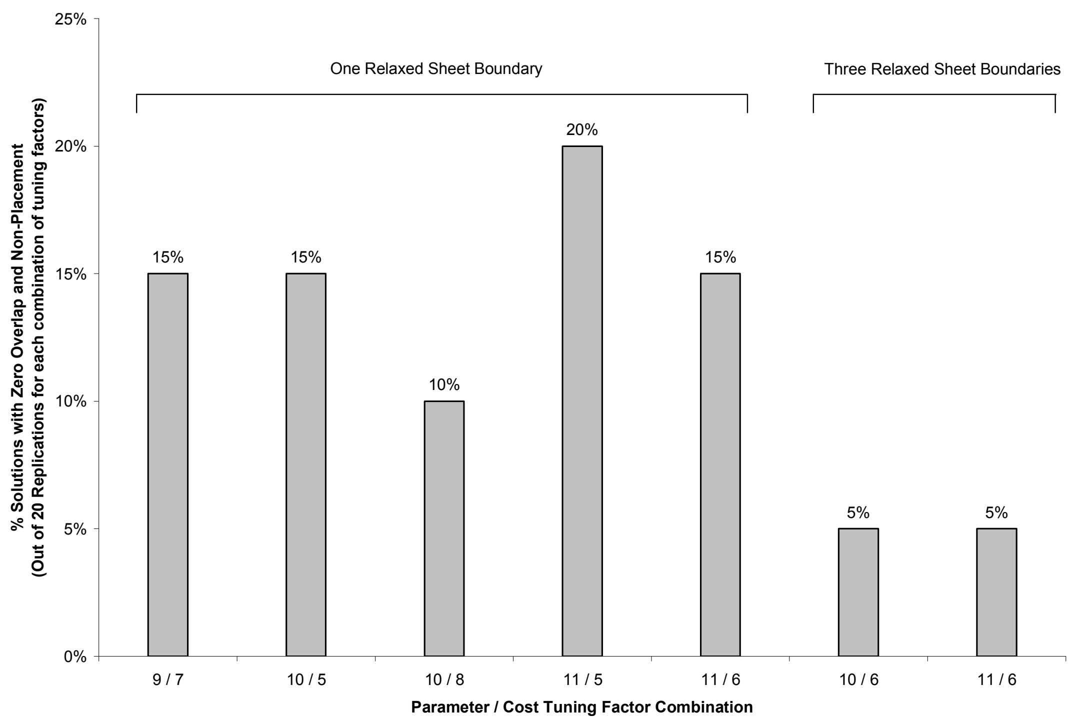


Figure 6.12 Percentages global minima – Relaxed sheet boundaries.

A comparison of Figures 6.5 and 6.12 further confirms that relaxing sheet boundaries increased the difficulty of finding global optima with zero overlap and zero non-placement with the dredge cut nesting model for irregular nesting problems with a sheet to stencil area ratio of 1:1 and decision variable penalties all set to unity. Figure 6.12 shows that out of a total of 3,240 nesting experiments carried out only 17 of the final nests found, exhibited zero overlap and non-placement, which equates to an average overall success rate of 0.52%, down from 3.61% for the 39 global optima found out of 1,080 experiments with a sheet with fixed boundaries. The highest percentage of 20% (4 out of 20) in Figure 6.12 was observed for a parameter tuning factor of 11 and a cost tuning factor of 5. A tuning factor value of 11 is greater than the maximum of 10.32 recommended in Ingber (2006), and therefore it can be said again that the use of a tuning factor of 11 may have compromised the ergodicity of the adaptive simulated annealing-based solution process. Most notably, no global optimum final nests were found for a sheet with two relaxed boundaries. Figure 6.13 depicts the best final nest for two relaxed sheet boundaries which was found with parameter and cost tuning factors 9 and 7, respectively.

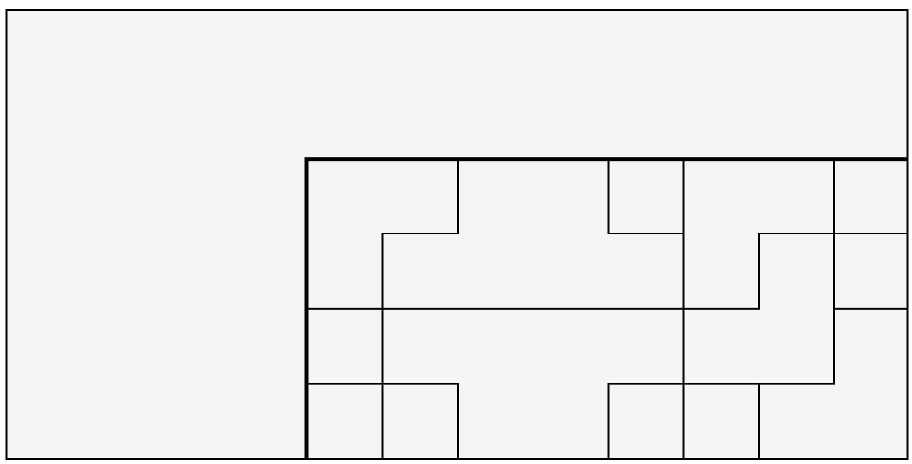


Figure 6.13 Best nest – 2 Relaxed sheet boundaries.

The final nest in Figure 6.13 has an overlap area and a non-placement area each equal to 0.000031% of the inner sheet area. The graphical representation of the final nest shown in Figure 6.13 appears optimal. However, since the dredge cut nesting model found a multiple of final nests with zero overlap and zero non-placement for other sheet arrangements the minimal values of overlap and non-placement themselves are not thought to be the result of calculation error, and therefore the nest in Figure 6.13 has to be considered near-optimal. Results of nesting experiments with two relaxed sheet boundaries and reduced sheet areas are presented and discussed next.

6.4 Dredge Cut Nesting – Reduced Sheet Areas

Experimental results obtained from the application of the dredge cut nesting model to irregular nesting problems with two relaxed sheet boundaries and reduced sheet areas are presented in Tables 6.8, 6.9, 6.10 and 6.11, which report the mean, μ, standard deviation, σ, and minimum and maximum of values of overlap and non-placement for 20 replications carried out for each pair of tuning factors. Overlap and non-placement are expressed as a percentage of the total inner sheet area used. Escape penalties were zero for this part of the experimentation and thus escape is excluded from these tables.

Table 6.8 Irregular nesting – Sheet to stencil area ratio 1:1.1

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	13.05	4.06	4.92	20.72	41.14	5.28	28.29	47.62
3 / 4	3.39	1.11	2.00	6.38	16.18	3.62	11.01	23.85
3 / 5	2.98	1.02	1.02	4.89	10.52	1.66	6.60	13.34
3 / 6	2.41	0.95	1.07	4.52	13.20	3.12	8.20	18.47
3 / 7	2.58	0.83	1.28	3.97	14.34	2.32	10.73	19.96
3 / 8	2.29	0.91	0.90	4.34	14.66	2.97	6.37	18.43
4 / 3	12.09	4.36	4.56	21.44	39.54	5.16	32.47	54.70
4 / 4	3.05	1.52	0.86	6.82	13.06	4.38	7.51	23.10
4 / 5	2.37	1.11	0.77	4.29	10.16	1.52	7.76	13.10
4 / 6	2.14	1.03	0.76	4.41	11.67	2.35	7.87	16.12
4 / 7	2.04	0.74	0.36	3.55	12.31	2.04	7.04	15.87
4 / 8	1.71	0.75	0.73	3.28	13.98	2.34	9.89	19.27
5 / 3	10.69	4.05	5.23	21.98	39.85	5.90	28.75	58.46
5 / 4	2.81	1.24	0.81	5.20	12.97	4.93	5.58	23.58
5 / 5	2.02	1.03	0.35	3.91	8.24	2.39	4.22	13.65
5 / 6	1.87	0.52	1.01	3.06	8.93	2.44	5.09	13.95
5 / 7	1.91	1.10	0.77	5.63	10.53	2.63	5.88	15.56
5 / 8	1.65	0.72	0.59	3.04	12.31	2.62	8.51	16.55
6 / 3	12.31	4.27	6.01	19.09	38.07	4.61	30.10	44.99
6 / 4	2.60	1.21	0.73	4.79	11.42	3.85	5.64	18.33
6 / 5	1.67	1.16	0.12	3.84	7.76	2.32	4.88	13.80
6 / 6	1.41	0.65	0.39	3.06	9.12	2.60	4.28	14.98
6 / 7	1.55	0.65	0.59	3.00	9.42	2.50	4.81	14.28
6 / 8	1.60	0.72	0.39	2.74	11.30	2.37	7.31	16.34
7 / 3	12.77	5.46	6.49	30.17	40.89	5.35	30.41	48.60
7 / 4	2.55	1.13	0.67	4.88	11.73	4.34	4.88	20.41
7 / 5	1.67	0.95	0.29	3.63	7.43	3.02	0.86	15.54
7 / 6	1.63	0.87	0.23	3.04	8.64	2.69	3.79	13.89
7 / 7	**0.95**	0.55	0.11	2.23	9.29	2.06	6.48	14.26
7 / 8	1.41	0.83	0.28	3.23	9.02	2.63	3.61	14.06
8 / 3	12.98	5.07	4.33	24.19	41.63	4.92	34.83	50.16
8 / 4	2.26	0.94	0.82	4.06	10.65	4.34	3.86	22.61
8 / 5	1.43	0.82	0.17	2.79	6.12	2.84	0.33	11.71
8 / 6	1.53	0.75	0.26	2.60	9.16	2.44	4.73	15.48
8 / 7	1.61	0.80	0.14	3.52	9.17	2.99	4.84	16.05
8 / 8	1.33	1.04	0.01	4.43	8.88	3.54	2.41	15.88
9 / 3	11.48	3.90	5.43	20.59	37.00	6.91	23.94	52.68
9 / 4	2.38	1.08	0.45	4.22	9.50	4.82	1.35	18.79
9 / 5	1.96	1.38	0.07	3.97	5.34	3.12	0.26	11.20
9 / 6	1.45	0.82	0.10	3.75	8.32	2.80	2.18	12.40
9 / 7	1.28	0.77	0.02	2.55	10.57	2.56	6.89	16.54
9 / 8	1.34	0.63	0.36	2.91	10.63	2.49	5.89	15.51
10 / 3	12.61	4.38	5.42	21.34	38.03	6.06	29.13	50.50
10 / 4	2.42	1.45	1.04	6.65	9.29	4.04	4.02	19.56
10 / 5	1.46	1.13	0	3.53	**5.39**	3.01	0.25	10.34
10 / 6	1.68	1.18	0.01	4.18	6.93	2.63	0.07	11.88
10 / 7	1.52	0.70	0.20	2.67	9.50	2.83	4.10	15.82
10 / 8	1.33	0.78	0.10	2.55	9.81	2.40	3.58	13.68
11 / 3	12.39	3.87	6.17	21.74	39.64	4.41	29.86	47.69
11 / 4	2.65	0.93	1.33	4.50	11.06	4.52	2.94	19.01
11 / 5	1.64	0.95	0.08	3.22	7.31	3.23	2.02	14.42
11 / 6	1.24	0.92	0.01	2.64	8.24	3.33	2.81	14.15
11 / 7	1.75	1.00	0	3.54	8.74	2.69	2.79	12.96
11 / 8	1.63	1.01	0.32	4.61	10.63	2.82	4.74	15.21

Table 6.8 shows that on average, for a sheet to stencil area ratio of 1:1.1, the best final nests with minimum non-placement – the most important decision variable for dredging – were obtained with a parameter tuning factor of 10 and a cost tuning factor of 5. The relevant row in Table 6.8 is grey scaled and minimum averages for overlap and non-placement are given in bold text. Before summarizing results, results of nesting experiments with a sheet to stencil area ratio of 1:1.2 are given in Table 6.9.

Table 6.9 Irregular nesting – Sheet to stencil area ratio 1:1.2

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	10.63	4.70	2.98	20.67	35.57	5.31	25.48	46.49
3 / 4	3.41	1.21	1.33	5.81	14.63	4.42	7.83	25.05
3 / 5	2.89	0.96	1.24	4.54	11.79	1.86	9.36	15.36
3 / 6	3.08	1.19	1.68	6.11	12.43	2.53	7.54	15.96
3 / 7	2.69	1.28	0.58	4.63	12.40	1.94	8.66	15.06
3 / 8	2.70	1.38	0.57	6.30	13.18	2.92	5.46	17.63
4 / 3	11.74	4.29	6.51	20.35	35.33	6.57	23.93	46.94
4 / 4	3.19	1.54	1.43	6.56	11.32	4.42	6.75	21.72
4 / 5	2.36	0.99	0.60	4.20	9.60	1.99	6.47	13.29
4 / 6	2.04	0.82	0.53	3.83	11.09	3.33	5.67	18.55
4 / 7	2.16	0.96	1.05	4.78	11.75	3.12	7.34	17.94
4 / 8	2.25	1.15	0.39	4.22	12.10	2.48	8.62	16.39
5 / 3	11.85	4.82	6.47	20.96	41.01	7.48	28.28	52.93
5 / 4	2.74	1.44	0.55	5.88	10.02	3.29	5.63	16.49
5 / 5	1.97	0.80	0.66	3.34	9.13	2.16	4.50	12.50
5 / 6	1.56	0.59	0.54	2.33	8.19	2.07	4.36	11.68
5 / 7	2.20	0.67	0.83	3.34	10.73	2.53	5.60	16.01
5 / 8	2.05	0.82	0.90	3.60	11.28	2.93	6.92	19.22
6 / 3	11.75	5.37	5.08	24.00	34.18	5.07	25.86	42.38
6 / 4	2.23	1.19	0.96	5.70	9.52	5.18	2.67	20.84
6 / 5	1.63	1.10	0.12	3.66	6.59	2.79	2.85	12.51
6 / 6	1.80	0.81	0.56	3.51	8.25	2.03	4.58	11.98
6 / 7	1.49	0.86	0.24	3.50	8.66	2.89	5.29	15.55
6 / 8	1.75	0.97	0.53	4.53	10.51	3.30	3.50	15.20
7 / 3	12.74	4.42	5.53	21.44	36.76	5.24	25.34	46.01
7 / 4	1.82	1.20	0.56	4.43	7.97	4.54	2.88	19.49
7 / 5	2.02	1.21	0.18	4.31	7.15	2.73	1.92	11.73
7 / 6	1.50	0.91	0.09	2.89	7.60	2.70	2.22	11.96
7 / 7	1.66	0.87	0.31	3.21	9.21	2.06	6.49	12.94
7 / 8	1.88	0.66	0.95	3.09	9.83	2.34	5.38	13.74
8 / 3	12.38	4.76	2.08	21.94	35.42	6.22	25.30	49.03
8 / 4	1.99	0.87	0.73	4.43	8.24	3.97	3.77	17.36
8 / 5	1.93	1.27	0.19	5.06	7.07	3.15	3.66	13.52
8 / 6	1.61	0.99	0.01	3.16	7.67	2.66	3.26	11.44
8 / 7	1.92	0.78	0.25	3.12	8.70	3.08	3.05	15.75
8 / 8	1.56	1.16	0	4.43	8.10	3.39	3.59	14.20
9 / 3	12.84	3.67	6.69	18.33	38.51	5.06	29.24	50.22
9 / 4	2.49	1.06	0.57	4.26	9.48	3.82	3.98	16.73
9 / 5	1.54	1.01	0.11	3.38	6.40	2.72	1.69	11.58
9 / 6	2.04	1.35	0.03	4.75	7.57	2.93	2.23	12.28
9 / 7	1.50	0.99	0.01	3.66	8.69	3.32	3.15	15.33
9 / 8	1.88	0.75	0.40	3.05	10.30	2.72	4.21	14.86
10 / 3	10.76	3.41	5.63	16.35	38.78	6.19	27.47	54.79
10 / 4	1.84	1.33	0.32	4.58	8.66	4.25	3.05	16.19
10 / 5	1.68	1.26	0.07	3.52	6.84	3.81	2.20	13.90
10 / 6	1.93	0.95	0.09	3.24	7.68	2.07	4.19	11.05
10 / 7	1.40	0.75	0.05	2.92	9.66	2.59	4.10	14.33
10 / 8	**1.37**	0.65	0.05	2.34	9.68	3.13	4.20	13.89
11 / 3	9.74	4.08	4.52	20.33	36.77	6.14	25.87	50.90
11 / 4	2.30	1.48	0.49	5.80	9.64	5.11	1.94	19.24
11 / 5	1.96	0.97	0	3.47	**6.29**	2.59	0.86	10.98
11 / 6	1.86	1.10	0.10	4.12	7.75	2.04	4.14	10.76
11 / 7	1.91	0.89	0.52	3.85	8.13	2.94	4.34	16.04
11 / 8	1.82	1.06	0.16	3.97	9.19	2.97	4.61	16.98

Table 6.9 shows that on average, for a sheet to stencil area ratio of 1:1.2, the best final nests with minimum non-placement were obtained with a parameter tuning factor of 11 and a cost tuning factor of 5. Table 6.10 gives the results of nesting experiments with sheet to stencil area ratio of 1:1.3.

Table 6.10 Irregular nesting – Sheet to stencil area ratio 1:1.3

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	12.32	3.18	4.98	18.25	35.91	6.18	22.61	49.07
3 / 4	3.44	1.38	0.91	6.25	14.04	3.08	6.46	19.02
3 / 5	3.03	1.12	1.62	4.61	12.40	2.06	8.75	16.10
3 / 6	2.78	1.05	0.58	5.18	12.23	2.05	8.62	16.20
3 / 7	2.81	1.24	0.78	5.42	12.87	2.29	8.98	16.39
3 / 8	2.65	1.37	0.57	6.07	13.72	2.38	10.03	18.25
4 / 3	11.29	4.38	4.71	22.70	35.54	6.70	25.55	50.84
4 / 4	2.93	1.17	0.96	5.42	11.42	4.23	5.90	20.19
4 / 5	2.05	0.98	0.74	3.78	8.98	3.15	3.52	15.76
4 / 6	2.41	0.98	0.71	3.84	10.31	2.20	6.24	15.30
4 / 7	2.12	0.83	0.84	3.70	10.65	3.07	4.98	15.38
4 / 8	2.30	1.08	0.46	4.98	11.84	2.06	7.51	15.91
5 / 3	13.89	3.59	8.29	20.83	35.67	7.54	25.05	49.23
5 / 4	1.95	0.84	0.47	3.45	7.83	3.84	2.87	15.84
5 / 5	1.76	1.20	0.28	3.95	7.92	3.50	1.93	15.80
5 / 6	1.82	0.68	0.61	3.12	9.32	2.31	4.87	12.71
5 / 7	1.77	0.87	0.41	3.78	8.49	2.57	3.96	13.30
5 / 8	1.70	0.62	0.50	3.07	10.53	3.73	2.94	16.59
6 / 3	11.44	2.46	7.33	15.75	35.02	5.39	22.71	47.60
6 / 4	1.79	1.06	0.41	4.31	7.45	4.65	2.17	15.53
6 / 5	2.09	1.35	0.17	4.72	7.43	3.86	0.88	14.41
6 / 6	1.70	0.89	0.09	2.99	6.92	2.60	2.59	11.76
6 / 7	1.92	1.33	0.06	4.65	8.08	2.95	4.16	13.79
6 / 8	2.04	0.77	0.45	3.45	9.18	2.63	4.79	15.35
7 / 3	12.60	3.72	6.53	18.35	34.08	5.83	22.26	42.69
7 / 4	1.73	1.27	0.33	4.80	5.86	4.66	1.12	13.66
7 / 5	1.91	1.16	0.09	3.81	6.49	3.91	0.46	13.84
7 / 6	1.61	1.16	0.05	3.48	6.84	3.41	0.76	12.97
7 / 7	2.11	0.95	0.12	3.68	8.34	2.82	1.69	14.09
7 / 8	1.65	0.68	0.05	3.25	9.25	2.43	3.94	13.89
8 / 3	12.18	4.39	5.45	22.15	33.28	4.38	21.69	43.27
8 / 4	1.90	1.72	0.16	6.65	6.62	4.22	0.95	17.14
8 / 5	1.85	1.00	0.03	4.06	5.96	2.82	0.40	10.98
8 / 6	2.05	1.46	0	4.25	5.90	2.78	0.40	10.51
8 / 7	1.77	1.18	0	4.76	8.09	2.51	2.55	11.57
8 / 8	2.04	1.09	0.56	4.77	8.02	2.28	4.02	12.42
9 / 3	12.73	5.09	2.95	28.90	34.07	7.22	24.28	54.38
9 / 4	1.81	1.36	0.18	4.68	6.64	5.11	0.96	15.17
9 / 5	1.87	1.47	0	4.46	**5.24**	2.93	0.27	9.96
9 / 6	2.21	0.91	0	3.61	6.28	2.61	0.46	11.22
9 / 7	1.84	1.26	0.09	4.23	6.43	2.74	0.64	10.28
9 / 8	1.56	0.78	0	2.73	9.41	2.38	6.06	12.98
10 / 3	13.93	4.62	7.25	22.50	32.91	5.27	24.63	41.10
10 / 4	1.92	1.00	0.16	3.37	7.57	5.41	0.85	19.63
10 / 5	1.92	1.54	0	4.54	5.77	3.51	0.29	11.72
10 / 6	1.91	1.16	0	4.55	7.25	1.94	3.69	9.81
10 / 7	2.05	0.95	0.42	3.81	8.06	2.31	1.95	11.08
10 / 8	1.84	0.76	0.22	3.24	9.86	2.18	6.56	15.11
11 / 3	12.53	5.08	4.78	21.99	33.10	5.37	24.28	45.75
11 / 4	1.92	1.40	0.03	4.70	7.16	5.27	0.47	17.89
11 / 5	**1.51**	0.89	0.02	2.85	5.34	2.73	0.16	10.10
11 / 6	1.71	1.20	0	4.44	6.19	2.73	0.58	10.37
11 / 7	1.51	0.98	0	3.49	6.80	3.18	0.48	11.33
11 / 8	1.98	1.09	0.42	5.09	9.03	2.56	5.48	14.82

Table 6.10 shows that on average, for a sheet to stencil area ratio of 1:1.3, the best final nests with minimum non-placement were obtained with a parameter tuning factor of 11 and a cost tuning factor of 5. Table 6.11 gives the results of nesting experiments with a sheet to stencil area ratio of 1:1.4.

Table 6.11 Irregular nesting – Sheet to stencil area ratio 1:1.4

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area) μ	σ	Min	Max	Non-placement (% of Sheet Area) μ	σ	Min	Max
3 / 3	12.93	3.97	6.74	21.65	34.01	6.90	20.72	43.92
3 / 4	2.92	1.76	0.35	8.19	14.14	2.96	9.28	20.94
3 / 5	2.78	0.88	1.17	4.58	11.87	2.19	8.72	17.06
3 / 6	2.71	1.23	0.68	5.36	11.40	2.38	6.65	17.78
3 / 7	2.47	0.91	0.53	3.82	12.14	1.73	9.74	16.64
3 / 8	2.76	1.19	1.31	6.02	12.58	2.36	8.58	17.60
4 / 3	13.98	6.78	5.58	25.93	32.82	5.94	19.75	42.42
4 / 4	2.93	1.22	0.63	5.88	12.44	3.25	7.72	18.76
4 / 5	2.19	1.42	0.37	6.30	9.73	2.61	5.84	14.39
4 / 6	2.52	1.10	0.57	4.79	9.57	2.91	2.93	14.01
4 / 7	1.86	0.87	0.26	3.38	9.95	2.17	6.69	14.67
4 / 8	2.40	0.92	0.90	4.12	11.06	1.70	6.90	13.51
5 / 3	11.58	4.95	4.93	26.44	32.43	7.34	18.55	45.02
5 / 4	2.12	1.00	0.49	4.34	10.30	3.94	4.73	19.18
5 / 5	1.94	1.18	0.48	4.50	7.10	3.27	2.53	14.20
5 / 6	2.13	0.84	0.63	3.71	9.19	2.77	4.77	13.74
5 / 7	2.08	1.28	0.75	6.31	7.45	2.10	4.10	11.48
5 / 8	1.91	0.97	0.61	4.56	9.37	2.20	4.34	13.23
6 / 3	13.97	5.63	1.93	27.38	35.14	7.51	24.29	55.94
6 / 4	1.90	1.34	0.52	5.44	7.33	3.79	2.79	17.09
6 / 5	2.08	1.28	0.38	4.31	6.61	2.29	1.32	9.39
6 / 6	1.76	1.34	0.09	6.54	7.55	2.02	1.57	9.72
6 / 7	1.91	0.93	0.18	3.81	7.50	2.70	2.26	12.85
6 / 8	1.94	1.25	0.43	4.55	8.93	2.74	4.03	14.73
7 / 3	14.60	4.82	7.94	23.84	33.03	4.98	22.31	41.05
7 / 4	2.83	1.35	0.32	5.62	9.09	4.50	1.21	19.18
7 / 5	1.73	1.41	0.15	4.25	**4.51**	3.30	0.36	10.84
7 / 6	1.96	1.07	0.17	4.14	7.59	1.76	3.98	10.54
7 / 7	**1.64**	1.21	0	4.04	6.21	2.94	1.15	13.77
7 / 8	2.23	0.95	0.14	3.83	7.89	2.68	2.32	12.06
8 / 3	13.04	3.97	5.06	18.72	33.07	7.10	21.18	46.70
8 / 4	2.20	1.14	0.12	4.32	7.89	3.53	0.69	12.67
8 / 5	1.86	1.29	0.05	3.96	6.28	3.25	1.83	14.44
8 / 6	1.65	1.24	0.01	3.31	5.52	3.08	1.18	11.81
8 / 7	1.94	0.84	0.66	4.10	8.24	2.68	2.26	13.10
8 / 8	2.04	0.93	0.54	3.91	7.29	2.30	2.45	11.09
9 / 3	12.94	4.50	5.52	23.69	30.65	5.80	18.36	44.27
9 / 4	2.09	1.40	0.12	4.13	7.62	5.16	0.48	19.86
9 / 5	2.24	1.27	0.04	4.49	6.10	3.10	0.78	9.84
9 / 6	2.06	1.00	0.15	3.93	5.93	2.04	2.65	9.65
9 / 7	1.88	1.04	0.07	4.93	7.40	2.73	3.66	13.94
9 / 8	2.17	1.04	0.72	4.10	7.82	3.34	1.83	11.65
10 / 3	12.68	4.95	4.64	20.92	32.58	5.23	23.08	42.66
10 / 4	2.33	1.51	0.23	5.41	7.96	5.03	0.48	16.05
10 / 5	2.23	1.12	0.24	4.19	5.71	2.87	1.58	11.43
10 / 6	2.07	1.29	0.01	4.40	7.70	3.49	0.61	13.52
10 / 7	2.13	1.22	0	4.51	7.75	2.56	2.50	11.69
10 / 8	1.69	0.84	0.13	3.04	7.96	3.39	0.57	12.18
11 / 3	14.81	5.24	6.10	25.63	35.37	6.14	22.18	48.02
11 / 4	1.86	1.33	0.04	5.08	6.28	5.08	0.35	22.43
11 / 5	2.34	1.15	0	4.47	6.42	3.77	0.18	12.17
11 / 6	1.90	1.50	0.03	4.75	6.14	2.49	2.38	9.75
11 / 7	2.23	1.08	0.06	4.52	7.76	2.91	3.83	14.40
11 / 8	1.98	1.00	0.03	4.30	7.50	3.34	1.12	13.41

Table 6.11 shows that on average, for a sheet to stencil area ratio of 1:1.4, the best final nests with minimum non-placement were obtained with a parameter tuning factor of 7 and a cost tuning factor of 5. Overall average solution quality in Tables 6.8 to 6.11 is better than that achieved for the nesting problem with two relaxed sheet boundaries with a sheet to stencil area ratio of 1:1, results of which are given in Table 6.5. It should be noted however, that results in Table 6.5 were arrived at with escape penalties set to unity whilst the results in Tables 6.8 to 6.11 were obtained with escape penalties of zero. Figure 6.14 summarizes average final costs for increasingly smaller sheet sizes.

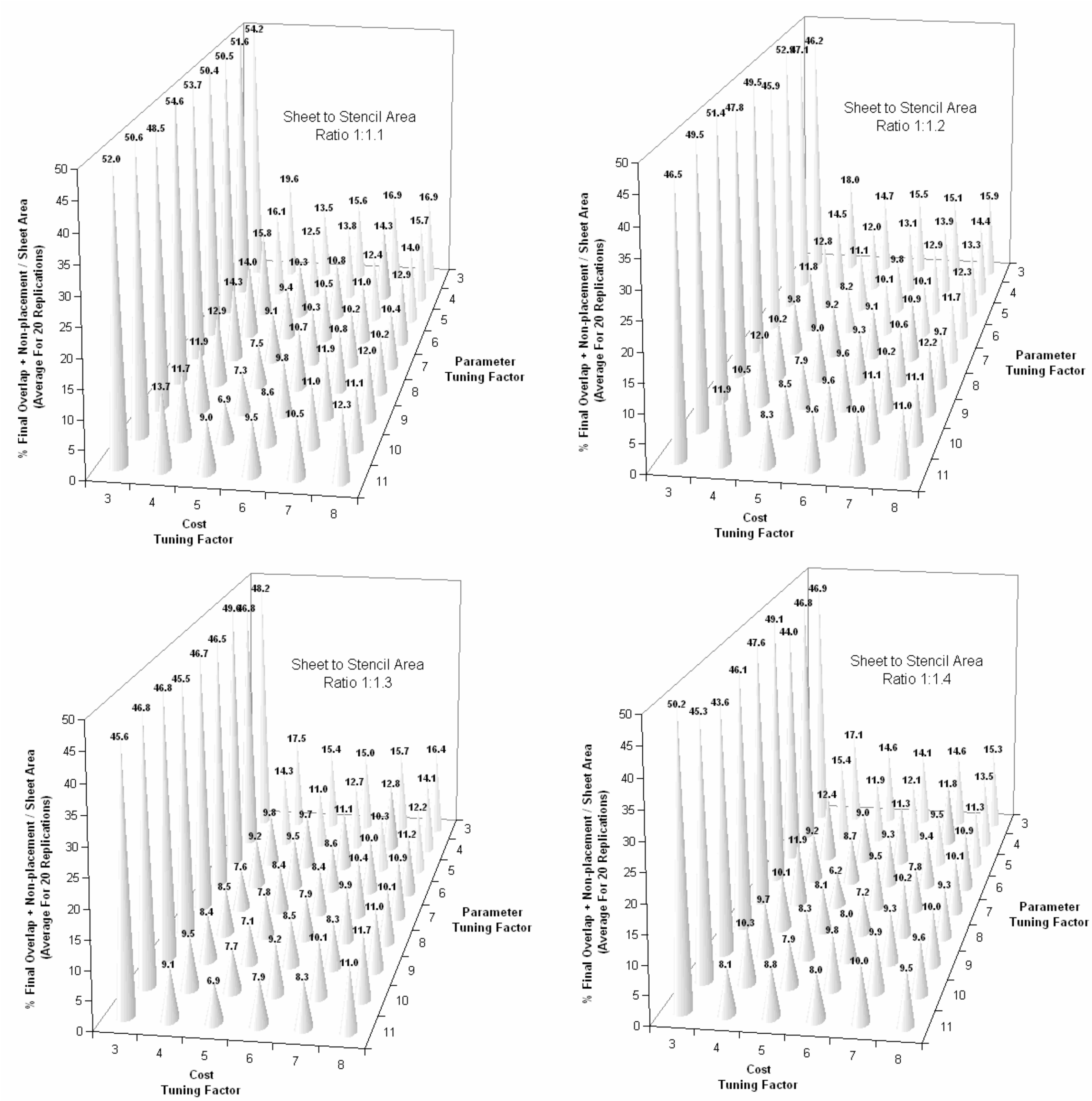


Figure 6.14 Overview irregular nesting results – Reduced sheets.

Figure 6.14 shows that the worst final nests for all sheet arrangements were found for a cost tuning factor of 3, and that (for all tuning factor values used) the overall average solution quality for sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3, and 1:1.4, respectively, was 18.63%, 17.58%, 16.55%, and 16.46%. Therefore it can be said that reducing the sheet areas improved the overall solution quality of final nests. The minimum overall average solution quality was found for a sheet to stencil area ratio of 1:1.4. In addition, the minimum average final cost of 6.2 was also found for a ratio of 1:1.4. It should be noted that the overall average solution quality for a sheet to stencil area ratio of 1:1 was 20.38%, but then escape penalties were unity instead of zero. Table 6.12 gives minimum averages of overlap and non-placement, expressed as percentages of inner sheet area, obtained for each irregular nesting problem of varying sheet to stencil area ratio.

Table 6.12 Minimum averages irregular nesting – Variable sheet areas

Sheet to Stencil Area Ratio	Minimum Average Overlap (% of Sheet Area)	Minimum Average Non-placement (% of Sheet Area)
1:1[1)]	2.08	2.87
1:1.1[2)]	0.95	5.39
1:1.2[2)]	1.37	6.29
1:1.3[2)]	1.51	5.25
1:1.4[2)]	1.64	4.51

Notes: 1) Escape penalties unity 2) Escape penalties zero.

Table 6.12 shows that when escape penalties are zeroed, minimum averages for non-placement approximately doubled for all non-equal sheet to stencil area ratios, while minimum averages for overlap at first more than halved and then steadily increased as inner sheets got smaller.

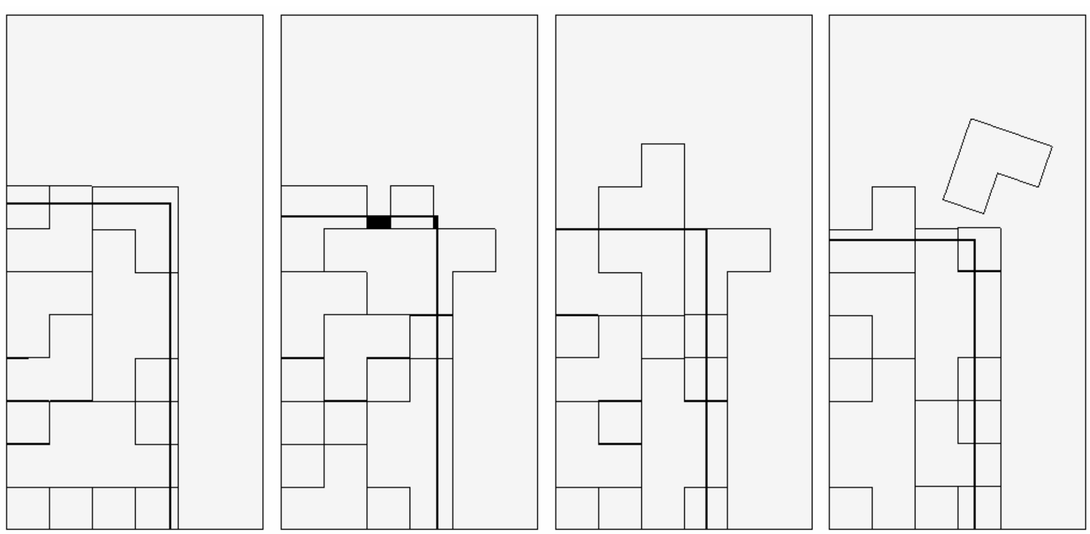
Perhaps contrary to expectation, none of the 4,320 final nests generated for irregular nesting problems where the total stencil area exceeded the inner sheet area were found to have zero non-placement. It is thought this poor result was mainly caused by having set escape penalties to zero: Stencils were no longer ‘drawn’ into inner sheets by both escape and non-placement cost, but the optimization process now solely depended on non-placement cost to ‘draw’ stencils into the inner sheet. This suggested that non-equal penalties for overlap and non-placement could produce better final nests for problems where total stencil area exceeds inner sheet areas. For the total of 5,400 final nests generated for each sheet size, Table 6.13 gives a summary of the number of final nests found with zero stencil overlap and gives the parameter and cost tuning factors with which these nests were obtained.

Table 6.13 Zero overlap instances irregular nesting – Variable sheet areas

Sheet to Stencil Area Ratio	Instances of Zero Overlap	Parameter / Cost Tuning Factor
1:1[1)]	1	9 / 8
1:1.1[2)]	1	10 / 5
	1	11 / 7
1:1.2[2)]	-	-
1:1.3[2)]	3	9 / 5
	1	9 / 6
	1	9 / 8
	1	10 / 5
	1	10 / 6
	1	11 / 6
	1	11 / 7
1:1.4[2)]	-	-

Notes: 1) Escape penalties unity 2) Escape penalties zero.

Table 6.13 further illustrates the poor performance of the dredge cut nesting model in this last set of experiments: only 11 out of 4,320 final nest generated had zero stencil overlap. Results in Table 6.13 further strengthen the argument that, if achieving zero non-placement is to be prioritized over achieving zero stencil overlap, more weight should be given to non-placement cost in the solution process. Because no final nests were found with zero non-placement for this part of the experimentation, final nests with minimum non-placement were looked at. Figure 6.15 depicts the final nests found with minimum non-placement costs, expressed as percentages of inner sheet area, for sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4.



1 – 1 : 1.1
Overlap: 0.01%
Non-Placement: 0.07%

2 – 1 : 1.2
Overlap: 0.003%
Non-Placement: 0.86%

3 – 1 : 1.3
Overlap: 0.02%
Non-Placement: 0.16%

4 – 1 : 1.4
Overlap: 0.004%
Non-Placement: 0.18%

Figure 6.15 Minimum final cost nests – Irregular nesting – Reduced sheets.

The parameter and cost tuning factor combinations with which the final nests in Figure 6.15 were arrived at were 10 / 6 for a sheet to stencil area ratio of 1:1.1 and 11 / 5 for ratios of 1:1.2, 1:1.3, and 1:1.4. The final nests depicted in Figure 6.15 not only exhibited minimum non-placement costs for each sheet size, but also had minimum sums of overlap and non-placement costs. Despite that none of the final nests depicted in Figure 6.15 satisfy the zero non-placement constraint of the dredge cut nesting model, final nests 1, 3 and 4 could be considered for use in determining nodes for the dredger routing model. It is thought that the non-placement of these three layouts can be neglected and that coverage of the inner sheet can be considered adequate enough to ensure complete excavation of the inner sheet if it represented a dredging area. However, it must be noted that a total of 4,320 nesting experiments were carried out for this part of the experimentation, and then to only find 3 final nests suitable for further use in the dredger routing model reflects poorly on the adopted approach.

In Figure 6.15 final nest 4 displays a desired side-effect of the dredge cut nesting solution process: A stencil has been left completely outside of the inner sheet. For this to happen escape cost must be very small in comparison to overlap and non-placement costs, and this is why for nesting experiments with reduced inner sheet areas escape penalties were set to zero. As explained in Section 5.4.4.2, unit dredge cuts of stencils, which are entirely outside inner sheets can be left undredged by not including their centroids as nodes in the dredger routing model.

In Figure 6.15 final nest 2 has to be rejected for use in determining nodes for the dredger routing model as the non-placement area exhibited (marked in solid black) is considered too large to be neglected and therefore would result in areas not being dredged. The other 19 final nests found with a parameter and cost tuning factor combination of 11 and 5 (for which the minimum average non-placement was reported in Table 6.8) were inspected visually and were all considered unsuitable as well. As a final check, for a sheet to stencil area ratio of 1:1.2, the final nest with the second lowest final cost was looked at to see if it would be suitable for use in determining nodes for the dredger routing model. Figure 6.16 depicts the relevant final nest.

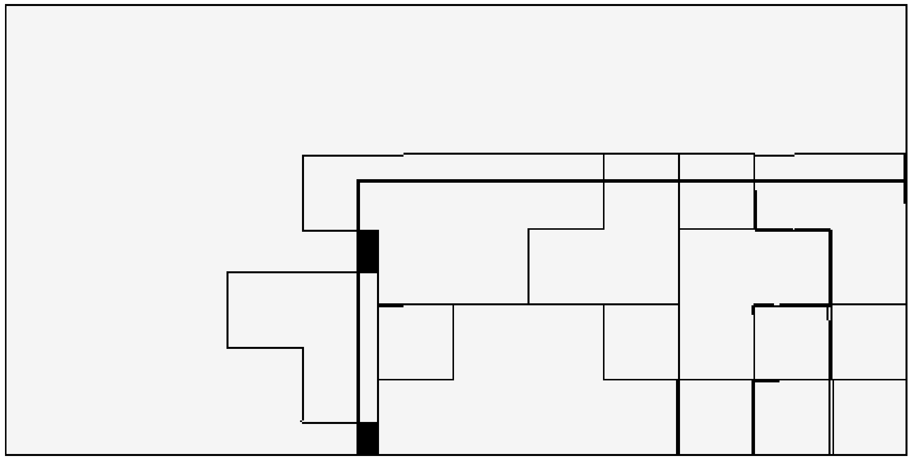


Figure 6.16 Second best nest – Irregular nesting – Sheet to stencil area ratio 1:1.2.

The final nest depicted in Figure 6.16 was arrived at with a parameter and cost tuning factor combination of 9 and 5, and has an overlap area equal to 0.11%, and a non-placement area equal to 1.69% of the inner sheet area. As is clear from Figure 6.16 the depicted final nest should also be considered unfit for determining nodes for the dredger routing model as the non-placement areas (marked in solid black) are too large to be neglected.

Gradual increase in the number of relaxed sheet boundaries and gradual reduction of inner sheet sizes, for two relaxed sheet boundaries and constant total stencil area, caused the solution quality of best final nests found to gradually deteriorate to the point that for the experimentation discussed here none of the 4,320 final nests generated satisfied the zero non-placement constraint of the dredge cut nesting model. It was thought that allowing for relaxed sheet boundaries and providing total stencil areas which exceeded inner sheet areas was key to solving irregular dredge cut nesting problems: Without these attributes it was considered difficult to achieve zero non-placement for irregularly shaped dredging areas.

The results presented in this section suggested adding more weight to non-placement cost in comparison to overlap cost to obtain better final nest layouts. Before this was done, the stencil set used for nesting was regularized. It was thought the nesting of square stencils, representing individual cuts with sides equal to an effective cut width of cutter suction dredgers could lead to increased numbers of final nests with zero non-placement. So, instead of the irregular stencil set shown in Figure 6.15 a set of 32 identical square stencils were nested in problems with sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4. The results of the regular nesting experiments carried out with a stencil set consisting of 32 identical squares are presented and discussed next.

6.5 Dredge Cut Nesting – Reduced Sheet Areas for Square Stencils

Figure 6.17 depicts an example of a random initial placement of 32 square stencils used as an initial solution at the start of a nesting optimization process for a nesting problem with a sheet to stencil area ratio of 1:1.1.

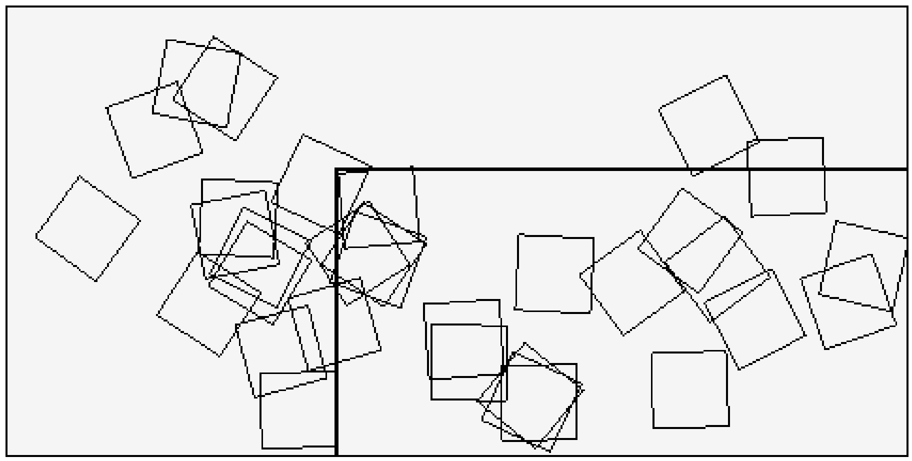


Figure 6.17 Random initial nest – Regular nesting.

The random initial nest in Figure 6.17 has an overlap area equal to 32.05%, and a non-placement area equal to 57.90% of the inner sheet area. Experimental results obtained from the application of the dredge cut nesting model to regular nesting problems with two relaxed sheet boundaries and reduced sheet areas are presented in Tables 6.14, 6.15, 6.16 and 6.17. These tables report the mean, μ, standard deviation, σ, and minimum and maximum of values of overlap and non-placement for 20 replications carried out for each pair of tuning factors. Overlap and non-placement are expressed as a percentage of the total inner sheet area used. Escape penalties were zero for this part of the experimentation and thus escape is excluded from these tables.

Table 6.14 Regular nesting – Sheet to stencil area ratio 1:1.1

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	17.35	3.62	10.99	23.75	49.20	4.28	43.30	57.24
3 / 4	9.70	1.79	5.71	12.33	35.27	4.70	29.29	46.75
3 / 5	4.15	0.67	3.01	5.47	15.16	1.83	11.97	19.73
3 / 6	3.01	0.98	1.58	5.68	12.26	1.28	9.42	14.43
3 / 7	2.68	0.64	1.68	3.68	12.30	1.26	10.38	15.04
3 / 8	3.02	0.86	1.58	4.35	12.16	1.71	9.44	15.69
4 / 3	16.99	2.92	12.28	23.20	48.63	4.88	39.88	61.16
4 / 4	9.22	2.33	5.73	13.77	36.26	4.32	30.42	45.66
4 / 5	4.24	0.89	2.40	6.62	13.08	1.88	9.60	16.56
4 / 6	2.16	0.50	1.24	3.00	11.47	1.44	8.54	14.73
4 / 7	2.00	0.44	0.90	2.51	11.20	1.48	8.08	14.63
4 / 8	2.39	0.95	1.14	4.87	11.17	1.10	8.26	12.60
5 / 3	16.58	2.76	12.04	23.09	50.48	4.37	42.62	57.69
5 / 4	10.61	2.80	6.21	18.42	36.02	3.23	29.38	43.28
5 / 5	3.20	0.80	1.72	4.96	11.24	1.35	8.01	13.56
5 / 6	1.73	0.49	1.00	2.87	9.81	1.79	5.30	12.15
5 / 7	1.62	0.63	0.75	2.99	9.62	1.58	6.54	13.15
5 / 8	1.69	0.51	0.89	2.92	10.48	1.97	6.79	13.26
6 / 3	17.05	4.58	9.11	25.49	50.26	4.06	40.54	57.37
6 / 4	10.08	2.22	6.07	15.17	37.43	3.17	30.76	42.18
6 / 5	2.64	0.54	1.94	3.85	10.51	0.97	9.07	12.67
6 / 6	1.46	0.47	0.71	2.49	8.26	1.40	5.74	10.64
6 / 7	1.32	0.50	0.54	2.19	8.53	1.57	5.99	11.51
6 / 8	1.20	0.47	0.47	2.22	9.37	1.95	5.83	12.22
7 / 3	17.17	4.30	8.82	25.51	50.02	4.70	43.27	58.66
7 / 4	10.62	2.56	6.28	16.01	35.30	3.45	29.07	43.35
7 / 5	2.40	0.70	1.33	4.08	9.94	1.62	7.68	14.19
7 / 6	1.31	0.54	0.74	2.69	6.58	1.27	5.12	9.59
7 / 7	1.13	0.37	0.54	1.84	7.79	1.76	5.03	11.66
7 / 8	1.37	0.47	0.25	2.34	8.07	1.76	3.65	10.56
8 / 3	17.06	3.11	9.13	22.94	49.54	4.32	43.19	58.31
8 / 4	9.14	1.60	6.45	11.82	37.84	4.09	30.73	44.67
8 / 5	2.24	0.66	1.23	3.50	8.43	1.69	5.72	12.83
8 / 6	1.24	0.38	0.60	1.90	6.78	1.40	3.81	9.81
8 / 7	1.14	0.41	0.32	2.06	7.22	1.64	4.94	11.22
8 / 8	1.18	0.42	0.60	1.92	7.84	1.54	3.52	10.26
9 / 3	15.72	2.86	11.28	20.90	48.45	3.60	40.33	54.80
9 / 4	10.01	2.76	5.25	18.40	35.37	3.99	27.77	42.39
9 / 5	2.01	0.65	1.03	3.45	8.02	1.71	5.19	11.87
9 / 6	1.13	0.44	0.47	2.12	6.68	1.35	3.59	9.05
9 / 7	1.16	0.36	0.48	1.99	7.02	1.09	4.71	9.21
9 / 8	**1.07**	0.47	0.33	2.22	8.58	2.15	5.33	12.22
10 / 3	15.53	3.05	8.38	22.65	48.91	5.60	36.63	58.99
10 / 4	9.41	1.79	5.09	12.24	35.37	3.50	27.45	41.35
10 / 5	2.08	0.72	0.88	3.61	8.66	2.20	3.47	12.44
10 / 6	1.37	0.74	0.22	3.20	**5.87**	1.59	3.13	10.01
10 / 7	1.20	0.35	0.60	1.99	7.12	2.05	4.15	11.46
10 / 8	1.37	0.61	0.62	2.54	7.75	1.53	4.77	11.04
11 / 3	16.89	4.10	10.32	25.01	51.58	4.70	41.82	59.37
11 / 4	10.67	3.19	6.13	17.66	36.30	3.43	28.31	41.21
11 / 5	1.84	0.69	0.88	3.44	8.36	1.82	4.77	11.08
11 / 6	1.16	0.49	0.27	2.00	6.50	1.44	3.83	9.33
11 / 7	1.25	0.38	0.77	2.38	7.37	1.64	4.60	11.04
11 / 8	1.16	0.33	0.53	1.71	7.72	1.63	4.24	10.69

Table 6.14 shows that on average, for a sheet to stencil area ratio of 1:1.1, the best final nests with minimum areas of non-placement – the most important decision variable for dredging – for square stencils were obtained with a parameter tuning factor of 10 and a cost tuning factor of 6. The relevant row in Table 6.14 is grey scaled and minimum averages for overlap and non-placement are given in bold text. Before summarizing results, results of regular nesting experiments with a sheet to stencil area ratio of 1:1.2 are given in Table 6.15.

Table 6.15 Regular nesting – Sheet to stencil area ratio 1:1.2

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	18.43	4.77	10.93	28.08	48.87	4.75	41.60	57.47
3 / 4	10.49	2.98	5.63	17.30	35.23	3.01	29.93	40.54
3 / 5	4.28	0.93	2.67	6.24	14.62	1.76	12.29	19.33
3 / 6	3.28	0.99	1.46	5.36	12.84	1.41	11.01	16.62
3 / 7	2.81	0.58	1.66	3.89	13.08	1.50	10.56	16.59
3 / 8	3.16	1.16	1.96	6.60	12.78	1.84	8.85	16.43
4 / 3	18.19	2.30	13.14	23.06	49.82	5.03	39.74	58.08
4 / 4	11.15	2.95	6.29	16.41	33.50	3.96	25.76	42.46
4 / 5	3.71	0.63	2.22	4.82	12.52	1.99	8.86	15.95
4 / 6	2.55	0.62	1.54	3.53	10.94	1.50	8.50	14.02
4 / 7	2.24	0.61	1.01	3.14	10.72	1.47	8.08	13.53
4 / 8	2.24	0.74	1.12	4.19	10.98	1.73	7.46	14.20
5 / 3	17.86	3.92	11.68	26.34	49.99	4.04	43.60	57.16
5 / 4	10.09	2.47	5.37	14.16	33.29	4.51	26.14	40.02
5 / 5	3.16	0.76	2.03	4.60	10.61	2.10	6.66	14.42
5 / 6	1.95	0.48	1.11	2.86	9.50	1.43	6.92	12.63
5 / 7	1.79	0.69	0.69	3.47	10.05	1.46	8.01	12.03
5 / 8	1.66	0.65	0.41	2.68	10.27	1.05	7.89	12.02
6 / 3	17.46	4.64	11.05	30.25	48.19	5.11	39.32	61.94
6 / 4	8.95	2.53	4.53	13.52	34.20	4.78	22.26	43.10
6 / 5	2.70	0.81	1.11	3.97	9.32	1.70	6.36	12.96
6 / 6	1.52	0.57	0.31	2.37	7.77	1.02	5.55	9.52
6 / 7	1.68	0.82	0.49	3.36	8.90	1.73	5.69	12.43
6 / 8	1.48	0.62	0.15	2.65	9.86	1.82	6.99	13.53
7 / 3	17.70	3.37	12.95	24.50	49.69	5.43	37.51	56.60
7 / 4	9.67	2.53	4.59	15.47	33.95	3.73	27.55	40.22
7 / 5	2.61	0.69	1.19	3.82	9.14	1.48	5.66	12.29
7 / 6	1.57	0.51	0.38	2.63	6.98	1.19	4.23	8.56
7 / 7	1.26	0.55	0.08	2.46	8.29	2.11	3.63	12.09
7 / 8	1.43	0.65	0.45	2.70	9.04	1.31	6.49	11.46
8 / 3	17.81	3.74	10.93	25.07	49.74	4.70	39.96	58.77
8 / 4	9.49	1.84	5.65	13.20	32.93	3.07	24.12	37.22
8 / 5	2.16	0.84	0.70	3.09	7.80	1.39	5.76	10.93
8 / 6	1.37	0.44	0.49	2.12	6.29	1.41	3.30	9.40
8 / 7	1.38	0.40	0.73	2.08	7.48	1.63	4.15	11.08
8 / 8	1.39	0.51	0.53	2.29	8.97	1.87	5.58	12.93
9 / 3	17.46	3.58	12.18	26.81	49.81	5.93	36.03	58.00
9 / 4	9.59	1.55	6.10	13.00	33.75	3.92	26.89	43.44
9 / 5	1.90	0.69	0.65	3.01	7.31	1.15	5.48	9.32
9 / 6	1.29	0.63	0.31	2.69	**6.10**	1.50	2.98	9.52
9 / 7	1.16	0.52	0.31	2.08	7.53	2.05	4.00	10.55
9 / 8	1.50	0.57	0.75	2.64	8.27	1.74	4.83	11.63
10 / 3	17.87	4.15	9.57	28.84	49.20	4.87	41.73	57.20
10 / 4	10.12	3.08	5.98	18.09	33.56	2.82	29.85	40.11
10 / 5	2.17	1.02	0.60	3.93	7.87	1.96	4.49	11.44
10 / 6	1.60	0.67	0.16	3.00	6.48	1.52	3.80	8.98
10 / 7	1.26	0.58	0.07	2.69	7.73	2.00	3.21	11.00
10 / 8	1.23	0.41	0.78	2.11	8.49	1.67	5.44	11.04
11 / 3	17.35	3.52	11.68	24.95	49.08	4.16	40.36	58.93
11 / 4	9.90	2.98	4.62	15.69	31.66	4.20	25.98	41.90
11 / 5	2.17	0.95	0.63	4.83	7.84	2.04	5.23	12.89
11 / 6	1.40	0.55	0.16	2.38	6.70	1.71	3.90	10.17
11 / 7	**1.14**	0.50	0.36	2.14	6.17	1.74	3.27	9.26
11 / 8	1.43	0.55	0.58	2.70	8.35	1.60	6.23	12.40

Table 6.15 shows that on average, for a sheet to stencil area ratio of 1:1.2, the best final nests with minimum non-placement were obtained with a parameter tuning factor of 9 and a cost tuning factor of 6. Table 6.16 gives the results of regular nesting experiments with a sheet to stencil area ratio of 1:1.3.

Table 6.16 Regular nesting – Sheet to stencil area ratio 1:1.3

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	20.89	4.84	8.93	28.17	50.38	4.65	42.78	64.80
3 / 4	10.15	2.32	5.42	14.42	30.96	5.15	22.94	43.59
3 / 5	3.93	0.90	1.98	5.33	14.74	1.95	11.49	19.60
3 / 6	2.92	0.80	1.58	4.56	13.13	1.48	10.59	15.54
3 / 7	3.34	0.90	1.48	4.70	11.97	1.51	8.78	15.02
3 / 8	3.37	1.32	1.15	5.76	12.44	1.58	10.29	14.73
4 / 3	18.94	2.96	12.90	25.27	47.92	4.57	39.55	55.33
4 / 4	10.29	2.50	6.38	13.80	30.97	3.14	24.74	36.31
4 / 5	3.66	0.82	2.20	5.14	12.76	1.52	9.96	16.56
4 / 6	2.36	0.64	1.31	3.50	11.12	1.22	9.36	13.59
4 / 7	2.14	0.56	1.28	3.51	11.66	1.51	7.88	13.53
4 / 8	2.76	0.78	1.64	4.38	11.00	1.48	8.42	14.40
5 / 3	19.78	3.82	14.87	26.64	47.95	5.12	40.89	58.83
5 / 4	10.89	3.56	4.93	18.56	32.44	3.25	25.54	39.01
5 / 5	3.25	0.63	2.20	4.43	10.61	1.58	8.38	13.82
5 / 6	1.78	0.55	0.93	2.70	9.17	1.43	6.84	13.23
5 / 7	1.82	0.64	0.99	2.87	9.86	2.03	5.37	13.52
5 / 8	1.80	0.58	0.51	3.00	10.15	1.40	7.15	13.10
6 / 3	20.16	3.91	12.74	27.34	48.02	4.13	37.18	53.65
6 / 4	9.95	2.89	4.33	17.08	32.62	3.76	26.15	38.97
6 / 5	2.37	0.68	1.25	4.20	7.70	1.45	5.02	11.04
6 / 6	1.76	0.60	0.58	2.99	7.80	1.58	5.48	10.13
6 / 7	1.34	0.54	0.31	2.41	8.86	1.38	5.92	11.49
6 / 8	1.62	0.66	0.63	2.92	8.86	1.52	5.86	11.84
7 / 3	19.06	5.33	11.15	32.58	48.09	4.44	37.65	54.74
7 / 4	10.27	3.13	5.26	17.44	33.32	2.73	28.42	37.84
7 / 5	2.49	0.71	0.85	3.53	7.76	1.76	4.31	11.62
7 / 6	1.28	0.61	0.44	2.50	6.87	1.57	4.13	9.71
7 / 7	1.45	0.72	0.19	2.98	7.44	1.48	3.83	9.68
7 / 8	1.73	0.67	0.34	2.79	9.32	1.55	5.16	12.78
8 / 3	19.71	3.97	12.16	25.14	46.06	7.21	36.43	64.52
8 / 4	9.80	2.70	5.21	15.56	31.30	3.75	25.34	38.26
8 / 5	1.96	0.78	0.65	3.46	6.66	1.89	3.51	10.02
8 / 6	1.27	0.86	0.23	2.78	6.06	1.75	2.66	9.48
8 / 7	1.43	0.74	0.23	3.00	6.82	0.91	5.23	8.66
8 / 8	1.47	0.74	0.29	2.96	8.16	1.69	4.83	11.01
9 / 3	20.33	4.29	11.70	26.68	47.70	3.67	40.34	56.73
9 / 4	9.58	2.42	3.98	14.21	30.63	4.23	23.66	38.06
9 / 5	1.89	0.80	0.51	3.30	6.68	2.09	2.16	10.11
9 / 6	1.47	0.69	0.19	2.82	6.07	2.39	1.34	9.55
9 / 7	**1.12**	0.44	0.52	1.88	6.95	1.80	3.89	11.61
9 / 8	1.29	0.52	0.50	2.14	7.97	1.88	3.46	10.10
10 / 3	19.72	3.93	11.70	29.05	46.80	5.85	35.68	57.13
10 / 4	9.91	2.92	4.78	17.03	31.56	3.66	24.64	38.92
10 / 5	1.71	1.10	0.40	3.93	5.96	2.45	2.53	10.85
10 / 6	1.54	0.55	0.16	2.28	6.06	1.84	1.93	9.37
10 / 7	1.50	0.60	0.07	2.52	6.87	1.68	4.03	9.62
10 / 8	1.56	0.59	0.18	2.72	7.86	1.62	4.48	10.03
11 / 3	18.95	4.64	11.24	28.76	48.43	3.94	41.55	57.42
11 / 4	9.74	2.80	5.29	15.65	32.27	4.69	22.55	43.94
11 / 5	2.20	1.14	0.49	4.62	7.49	2.43	3.75	11.12
11 / 6	1.14	0.54	0.10	2.08	**5.73**	1.81	2.16	9.14
11 / 7	1.72	0.69	0.68	3.41	6.58	1.67	3.91	11.00
11 / 8	1.40	0.70	0.44	3.31	8.38	1.97	4.83	11.04

Table 6.16 shows that on average, for a sheet to stencil area ratio of 1:1.3, the best final nests with minimum non-placement for square stencils were obtained with a parameter tuning factor of 11 and a cost tuning factor of 6 for square stencils. Table 6.17 gives the results of regular nesting experiments with a sheet to stencil area of 1:1.4.

Table 6.17 Regular nesting – Sheet to stencil area ratio 1:1.4

Tuning Factor Parameter / Cost	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
	μ	σ	Min	Max	μ	σ	Min	Max
3 / 3	20.89	4.87	10.87	28.85	46.04	4.62	39.36	54.65
3 / 4	9.58	2.43	6.13	15.76	29.51	2.60	24.96	34.46
3 / 5	3.56	0.77	2.39	4.87	15.17	2.04	10.89	20.08
3 / 6	3.35	0.78	2.05	4.69	12.25	1.41	9.91	15.78
3 / 7	3.25	0.81	1.76	4.78	12.43	1.35	9.85	14.26
3 / 8	3.32	0.69	2.04	5.01	12.45	1.41	9.51	15.35
4 / 3	20.32	4.95	11.35	29.26	47.52	4.43	38.43	55.88
4 / 4	11.55	2.32	7.80	16.50	29.37	3.69	22.86	37.86
4 / 5	3.46	0.92	1.53	5.39	11.76	1.83	8.56	15.39
4 / 6	2.33	0.62	1.19	3.51	10.43	1.50	7.38	13.11
4 / 7	2.56	1.00	1.05	4.72	11.32	1.36	9.69	13.63
4 / 8	2.22	0.46	1.34	2.97	11.70	1.31	10.09	14.81
5 / 3	20.83	4.48	11.13	27.19	50.27	5.60	40.21	62.28
5 / 4	10.81	2.23	7.20	15.56	29.93	3.23	23.48	34.63
5 / 5	2.47	0.72	1.24	4.18	11.09	2.16	7.15	15.46
5 / 6	2.08	0.74	0.74	3.91	9.53	1.69	5.61	13.41
5 / 7	1.72	0.56	0.74	2.68	10.09	1.33	7.86	12.29
5 / 8	1.75	0.60	0.64	3.27	10.63	1.26	8.96	13.14
6 / 3	19.55	4.01	8.68	24.28	50.61	6.50	38.01	60.59
6 / 4	9.32	2.42	4.46	13.73	28.63	2.93	23.97	34.04
6 / 5	2.39	0.91	0.80	4.15	9.10	1.77	6.65	12.71
6 / 6	1.50	0.52	0.47	2.60	7.88	1.06	4.94	9.74
6 / 7	1.56	0.72	0.52	3.14	8.60	1.40	5.92	11.20
6 / 8	1.82	0.75	0.76	3.30	9.02	1.69	5.62	13.15
7 / 3	19.80	4.81	11.18	29.90	47.49	6.52	40.26	61.70
7 / 4	9.09	2.45	5.31	16.56	29.45	3.50	23.92	36.11
7 / 5	2.37	0.95	0.97	4.26	7.33	1.94	3.86	10.77
7 / 6	1.61	0.54	0.24	2.58	6.61	1.25	3.80	8.58
7 / 7	1.26	0.46	0.30	2.43	8.05	1.72	3.93	12.34
7 / 8	1.49	0.64	0.68	2.61	8.75	1.56	5.79	11.35
8 / 3	21.55	3.98	15.28	29.97	47.63	5.01	36.48	54.79
8 / 4	9.81	2.50	2.76	12.74	29.36	3.90	22.79	38.28
8 / 5	2.32	0.79	0.87	3.46	7.53	1.51	4.28	10.82
8 / 6	1.36	0.51	0.66	2.70	7.09	1.50	4.13	9.93
8 / 7	1.34	0.49	0.46	2.41	7.47	1.60	4.34	11.44
8 / 8	1.60	0.60	0.60	2.53	7.50	1.47	4.64	10.21
9 / 3	20.40	4.95	12.24	31.93	47.80	5.84	35.04	60.88
9 / 4	9.83	2.33	6.08	14.38	29.10	3.96	23.08	36.59
9 / 5	1.68	0.59	0.81	2.75	**5.68**	1.65	2.06	9.38
9 / 6	1.42	0.64	0.22	2.38	6.28	1.60	3.09	9.18
9 / 7	1.53	0.74	0.70	3.81	6.92	1.40	4.35	9.25
9 / 8	1.51	0.71	0.17	3.19	7.96	1.60	5.57	11.26
10 / 3	20.24	3.46	14.14	26.13	49.04	4.66	40.15	58.21
10 / 4	9.92	2.04	6.75	13.53	28.69	2.76	23.95	34.89
10 / 5	2.47	0.79	0.78	3.61	7.22	2.09	4.08	10.18
10 / 6	1.20	0.73	0.02	2.59	6.04	1.52	2.65	8.39
10 / 7	1.37	0.54	0.44	2.68	7.16	1.44	4.56	9.81
10 / 8	**1.16**	0.74	0.07	2.58	7.50	1.76	4.75	10.91
11 / 3	20.15	4.25	11.13	27.42	47.01	5.22	33.94	58.53
11 / 4	10.40	2.54	6.54	15.45	28.48	3.81	17.99	35.43
11 / 5	1.89	0.76	0.71	3.84	6.45	2.32	2.51	13.43
11 / 6	1.34	0.65	0.22	2.33	5.98	1.63	3.30	9.84
11 / 7	1.40	0.58	0.06	2.25	6.93	1.47	3.51	9.45
11 / 8	1.36	0.75	0.62	3.19	8.08	1.78	4.47	11.53

Table 6.17 shows that on average, for a sheet to stencil area ratio of 1:1.4, the best final nests with minimum non-placement for square stencils were obtained with a parameter tuning factor of 9 and a cost tuning factor of 5. For this part of the experimentation, which nested square stencils, no optimal final nests, having zero overlap and zero non-placement, were found. Figure 6.18 summarizes averages of final costs for increasingly smaller sheet sizes with square stencils.

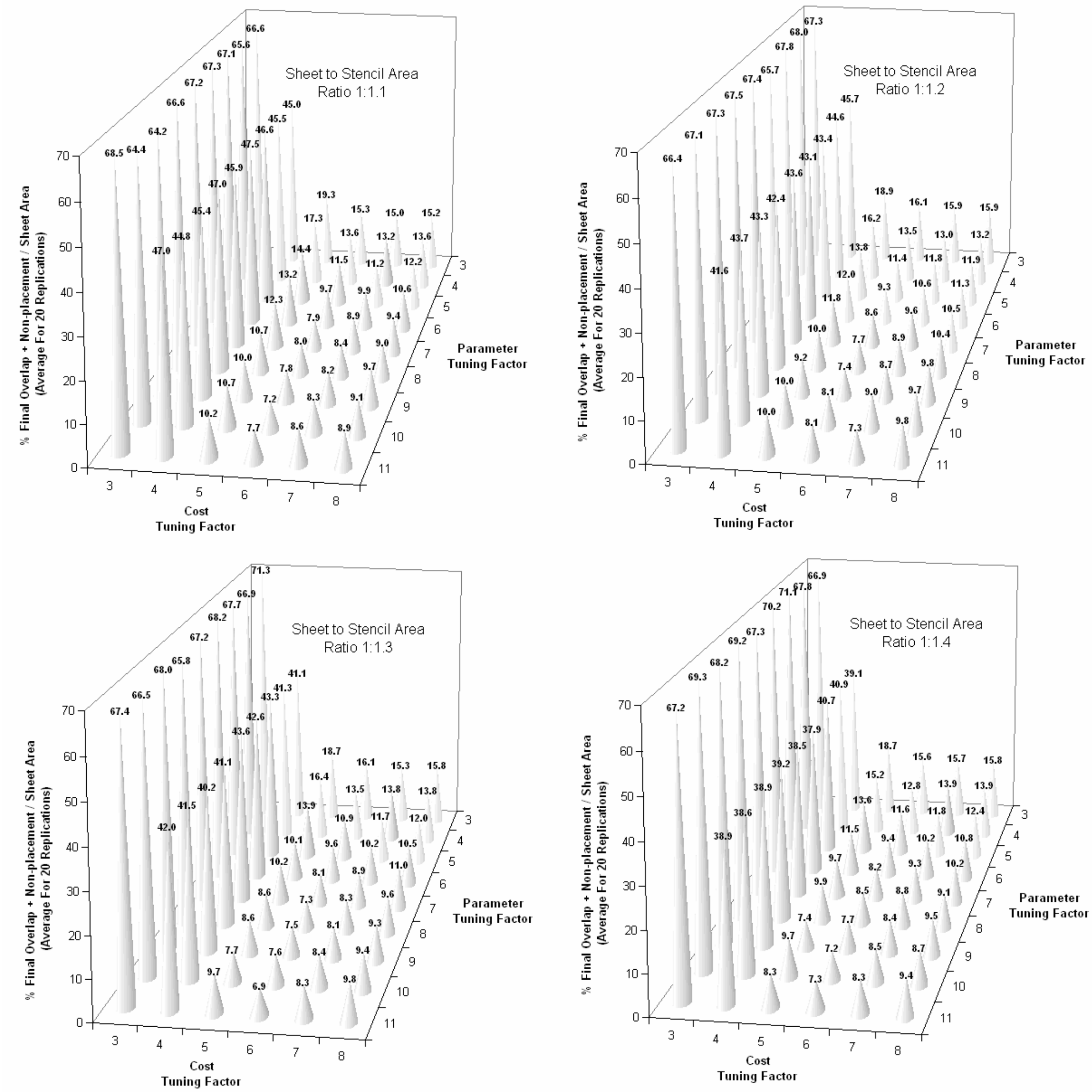


Figure 6.18 Overview regular nesting results – Reduced sheets.

Figure 6.18 shows that the worst final nests for all sheet arrangements were found for a cost tuning factor of 3, that a cost tuning factor of 4 also consistently gave poor results, and that (for the tuning factor values used) the overall average solution quality for sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3, and 1:1.4, respectively, were 26.08%, 25.84%, 25.39%, and 25.13%. It can be said that for the regular nesting problems solved with two relaxed sheet boundaries, the smaller the sheet area the better the overall average solution quality of final nests. The minimum overall average solution quality was found for a sheet to stencil area ratio of 1:1.4. However, the minimum average final cost of 6.9% was found for a ratio of 1:1.3. Figure 6.19 depicts the final nest of squares with minimum final cost, which was found for a sheet to stencil area ratio of 1:1.3 and with a parameter and cost tuning factor combination of 9 and 6.

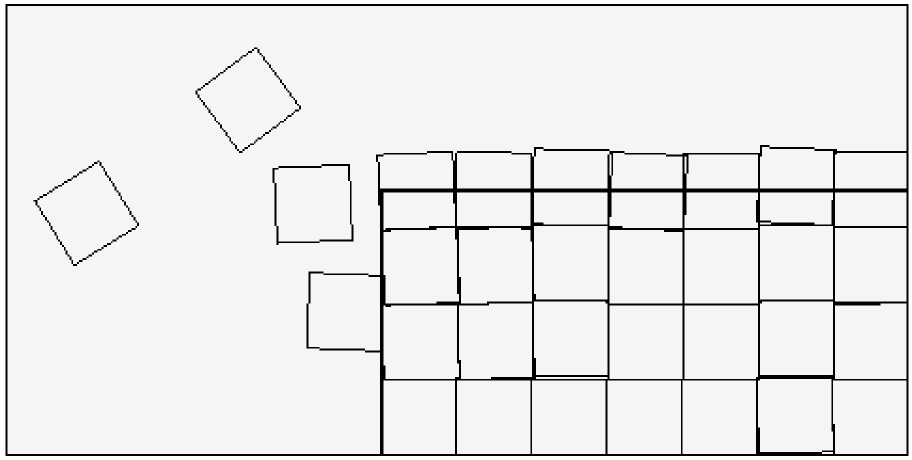


Figure 6.19 Best nest – Regular nesting – Sheet to stencil area ratio 1:1.3.

The final nest of Figure 6.19 has an overlap area equal to 0.43% and non-placement area equal to 1.34% of the inner sheet area. In Figure 6.19 the centroids of stencils which intersect with inside the inner sheet could be considered for use as nodes in the dredger routing model. Despite that the regular nesting problems should have been easier to solve than the irregular ones, none of the 4,320 regular nesting problems solved for this part of the experimentation resulted in final nests with zero non-placement. A reason for the lack of improvement in solution quality for regular nesting problems can be that the total number of iterations remained constant at 99,999.

The dredge cut nesting model modifies nesting solutions by sequentially disturbing the position of stencils. Therefore for the nesting problems with 14 irregular stencils the position of each stencil was perturbed around 7,142 times over 99,999 iterations, while for the regular nesting problems each of the 32 square stencils was perturbed around 3,124 times over 99,999 iterations. The fact that for irregular nesting problems positions of each stencil were perturbed approximately twice more than in regular nesting problems is thought to have played a part in the observed lack of improved solution quality. However, Chen *et al*. (1999) states that because of the "*excellent*" ability of the adaptive simulated annealing to self adapt the performance of the algorithm is not critically influenced by chosen values the for total number of iterations. Chen *et al*. (1999) reports this has been observed for a variety of problems solved with adaptive simulated annealing.

Another way of searching for better solutions could have been to explore more combinations of parameter and cost tuning factors. However, as explained with Table 6.2, this option is limited for the problems solved here if the solution process is to remain one of adaptive simulated annealing, that is if simulated quenching is to be avoided.

Tables 6.18 presents an overview of results for all nesting experiments discussed so far, giving minimum average final costs for 20 replications expressed as percentages of sheet areas used. Table 6.18 also gives the number of times final nests were found with zero overlap and zero non-placement and with zero non-placement only (the acronym GM stands for global minimum) out of a total of 1,080 replications carried out for each experiment.

Table 6.18 Overview irregular and regular nesting results

Sheet Type	0 Relaxed Boundaries		1 Relaxed Boundary		2 Relaxed Boundaries				3 Relaxed Boundaries	
Nest Type	Irregular		Irregular		Irregular		Regular		Irregular	
Sheet : Stencil Area	Min Average	GM / 1080	Min Average	GM / 1080	Min Average	GM / 1080	Min Average	GM / 1080	Min Average	GM / 1080
Overlap + Non-placement										
1:1	1.68	39	3.76	15	4.95	0	-	-	6.38	2
1:1.1	-	-	-	-	6.85	0	7.24	0	-	-
1:1.2	-	-	-	-	7.94	0	7.31	0	-	-
1:1.3	-	-	-	-	6.85	0	6.87	0	-	-
1:1.4	-	-	-	-	6.24	0	7.24	0	-	-
Non-placement only										
1:1	0.84	43	2.02	15	2.87	1	-	-	3.75	2
1:1.1	-	-	-	-	5.34	0	5.87	0	-	-
1:1.2	-	-	-	-	6.29	0	6.10	0	-	-
1:1.3	-	-	-	-	5.24	0	5.73	0	-	-
1:1.4	-	-	-	-	4.51	0	5.68	0	-	-

Note: All figures are percentages of (inner) sheet areas used.

Table 6.18 shows that when, for a sheet to stencil area ratio of 1:1, sheet boundaries were relaxed, overlap and non-placement costs of final nests increased. Table 6.18 also shows that reductions in sheet area for constant total stencil area beyond a ratio of 1:1.1 did not cause final costs to vary as much in comparison, for irregular as well as regular nesting. The main problem with results obtained so far, as shown in Table 6.18, is that when sheet areas were reduced for the irregular and regular nesting problem solved, no final nests with zero non-placement – the most important decision variable for dredging – were found. This is a problem because non-zero non placement for a given sheet equates to leaving parts of a dredging area undredged. It should be noted that so far all overlap and non-placement cost penalties were set to unity.

As mentioned after Table 6.13, to find more final nests with zero non-placement more weight can be added to non-placement cost than to overlap cost in the objective function of the dredge cut nesting model. To find out if this was true this was done for the next set of regular nesting experiments. Details of cost penalties used and results obtained are presented and discussed next.

6.6 Dredge Cut Nesting – Cost Penalty Increase for Square Stencils

For irregular nesting experiments with 0, 1, 2, and 3 relaxed inner sheet boundaries all cost penalty factors and exponents in the objective function (see Equation 5.4) were set to unity. For irregular and regular nesting problems with 2 relaxed sheet boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3, and 1:1.4 escape cost penalties were zeroed, but cost penalties for overlap and non-placement were kept equal to unity. The experimental results discussed so far showed that for these values of cost penalties, solution quality, in terms of finding final nests with zero non-placement, deteriorated as the complexity of nesting problems increased, even when the irregular stencil set was substituted with a regular set of identical squares.

To solve leather nesting problems Yuping *et al*. (2005) selected values greater than unity for 5 out of 6 cost penalties used. However, these values could not be applied in the same way to dredge cut nesting problems as two-dimensional dredge cut nesting differs fundamentally from conventional two-dimensional stock cutting problems (see Section 5.2 and Table 5.1). Therefore, although increased cost penalties used in this part of the experimentation have values similar to those used in Yuping *et al*. (2005), they have been applied differently to decision variables. Table 6.19 gives cost penalty values used in Yuping *et al*. (2005) and those used here for regular nesting on sheets with two relaxed boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3, and 1:1.4.

Table 6.19 Revised cost penalties – Leather and Dredge Cut Nesting

Decision Variable	Leather Nesting (Yuping *et al*., 2005)		Dredge Cut Nesting	
	Factor	Exponent	Factor	Exponent
Escape	50	2	0	0
Overlap	50	2	4	1
Non-placement	4	1	50	2

Table 6.19 shows that penalties for escape cost were kept zero here for dredge cut nesting problems, since escape is not relevant to dredging: The centroids of unit dredge cuts which are completely outside the inner sheet are not used as nodes in the dredger routing model. For leather nesting problems Yuping *et al*. (2005) subjects overlap cost to much greater penalties than non-placement cost. This relationship was inversed since for dredge cut nesting minimizing non-placement is more important than minimizing overlap.

Table 6.20 reports the mean, μ, standard deviation, σ, and minimum and maximum of values of non-placement area obtained for 20 replications carried out for each pair of tuning factors with the revised overlap and non-placement penalties for regular nesting with reduced inner sheet areas.

Table 6.20 Effect of revised cost penalties on non-placement – Regular nesting

Sheet to Stencil Area Ratio	Par. / Cost Tuning Factor	Original Penalties[1]				Revised Penalties[2]			
		μ	σ	Min	Max	μ	σ	Min	Max
1:1.1	10 / 6	17,035	4,606	9,092	29,056	897	1,965	1	7,796
1:1.2	9 / 6	16,254	3,993	7,937	25,353	94	143	4	654
1:1.3	11 / 6	14,117	4,449	5,314	22,515	73	144	1	499
1:1.4	9 / 5	12,969	3,780	4,699	21,439	20	19	0	66

Notes: 1) Overlap and non-placement penalties unity 2) Overlap penalty factor = 4; Overlap penalty exponent = 1; Non-placement penalty factor = 50; Non-placement penalty exponent = 2.

In Table 6.20 values for non-placement have not been expressed as percentages of inner sheet areas because this would have given averages of zero to two numbers after the decimal point for results obtained with the revised cost penalties. The combinations of parameter and cost tuning factors shown in Table 6.20 are values for which the minimum average non-placement was achieved when overlap and non-placement cost penalties equal to unity were used.

Table 6.20 shows that the use of the revised cost penalties resulted in drastic reductions of non-placement – the most important decision variable for dredging – of final nests for all inner sheet to stencil area ratios used. With the revised cost penalties one final nest was found with zero non-placement for a sheet to stencil area ratio of 1:1.4. Having reduced non-placement of final nests for regular nesting problems, the next step was to asses the suitability of final nests for use in determining node grids for the dredger routing model. Figure 6.20 shows final nests with minimum non-placement, found with original and revised cost penalties together with relevant cost and parameter tuning factor combinations. It should be noted that the minimum non-placement final nest 3 of Figure 6.20 was arrived at with a cost and parameter tuning factor combination of 9 and 7, whilst the minimum average non-placement for a sheet to stencil area ratio 1:1.3, given in Table 6.20, was found with a combination of 11 and 6.

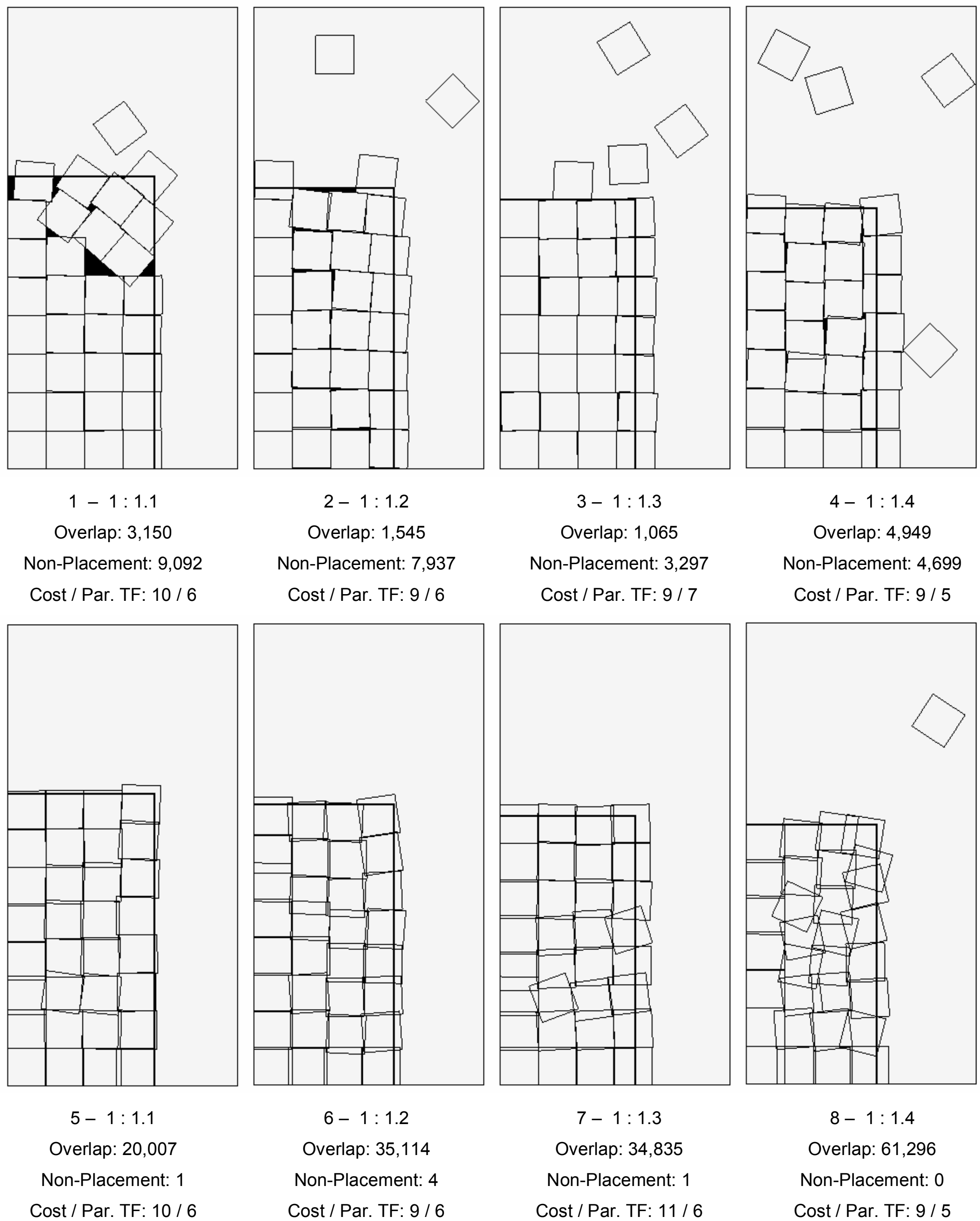


Figure 6.20 Minimum non-placement – Original (1-4) and revised (5-8) cost penalties.

In Figure 6.20 the inner sheet areas are 290,322; 266,450; 246,402; and 228,488 for sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3, and 1:1.4, respectively. Final nests 1 to 4 were arrived at with overlap and non-placement penalties set to unity, and final nests 5 to 8 were found with the revised cost penalties for dredge cut nesting given in Table 6.19. In Figure 6.20 only one of the final nests depicted, number 8, satisfies the constraint of the dredge cut nesting model that non-placement must be zero. However, final nest 8 has to be rejected for use in determining nodes for the dredger routing model because of excessive overlap: If the square stencils in final nest 8 have sides which are equal to the maximum effective cutting width of a cutter suction dredger, then inefficient use of the dredger can be expected if the centroids of the stencils were used as nodes in a dredger routing problem.

In Figure 6.20, final nests 5, 6, and 7 can be considered for use in determining nodes for the dredger routing model. It is thought that the non-placement of these layouts can be neglected and that coverage of the inner sheet is adequate to ensure complete excavation of the inner sheet if it were a dredging area. Final nests 1 and 2 have to be rejected outright for use in determining nodes for the dredger routing model as the displayed arrangements of stencils intersecting the inner sheet are too irregular. In addition, the main areas of non-placement of final nests 1 and 2 (marked in solid black) are considered too large to be neglected.

In Figure 6.20, final nests 3 and 4 can also be considered for use in determining nodes for the dredger routing model because they have regular arrangements of stencils on the inner sheet. However, it is more difficult to justify relaxing the dredge cut nesting model's constraint of zero non-placement for final nests 3 and 4 than it is for final nests 5, 6, and 7. Selecting final nest 3 or 4 increases the risk of leaving areas undredged. Therefore, out of all the final nests depicted in Figure 6.20, final nest 5 would have to be considered most appropriate for determining nodes for use in the dredger routing model as non-placement is near zero and the overlap is the smallest of final nests 5 to 8.

The fact that a final nest arrangement with considerable overlap has to be settled for indicates that varying overlap and non-placement cost penalties required more research, but this was not possible due to time constraints. As argued for final nest 8, increased overlap reduces the efficiency of the cutter suction dredger employed because sub-optimal use is made of its maximum effective cutting width. It should be noted that not all of the stencil overlap given in Figure 6.20 is situated inside the inner sheet, but this does not change the essence of the argument presented regarding inefficient dredger use.

Adaptive simulated annealing theory states that the algorithm comes with a statistical promise of being able to find global optima for complex combinatorial problems with multi-dimensional parameter spaces. However, the results discussed so far (taking over 5,000 hours to complete on the computers used) indicate it was difficult to tune algorithm and cost penalty settings for the dredge cut nesting model, such that globally optimum final nests – having zero overlap and zero non-placement – were easily found for the irregular and regular nesting problems solved here.

The tuning problem for dredge cut nesting was largely solved by changing the approach to the selection of stencil sets used for nesting, which is discussed in more detail in Section 6.9. First, results of the application of the dredger routing model to two additional square grid routing problems are presented and discussed.

6.7 Dredger Routing – 64 Node Square Grid Problem

Using Equations 5.19 and 5.20 for the 64 node routing problem with a square grid spacing of 50, the minimum route length is 3,150 and the minimum sum of turning angles is 1,250 degrees. Figure 6.21 depicts a random route used as an initial solution at the start of the routing optimization process. The route length and the sum of turning angles of the route depicted in Figure 6.21, respectively, are 12,915.00 and 7,181.54 degrees.

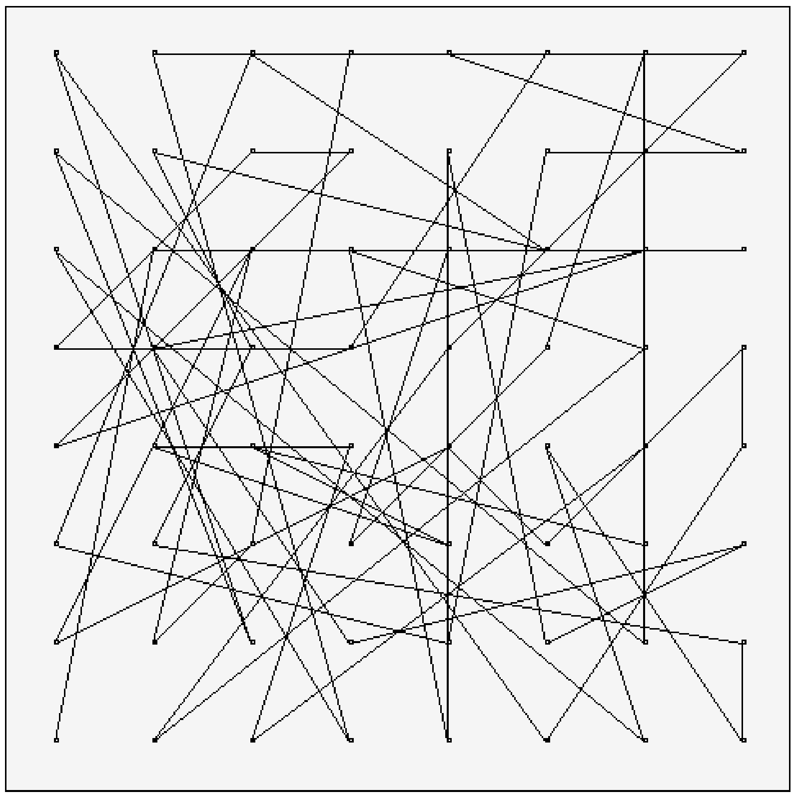


Figure 6.21 Random initial route – Regular routing – 64 Nodes.

The initial route depicted in Figure 6.21 has no links, while for the 64 node square grid routing problem solved here, the maximum link length is 350 and the optimum number of maximum length links is 8 (see Equations 5.21 an 5.22). Table 6.21 reports the mean, μ, standard deviation, σ, and minimum and maximum of route attributes for 20 final routes arrived at for local search, LS, values of 1 and 8 in the application of the dredger routing model to the 64 node square grid routing problem solved here. Values given in Table 6.21 are for 20 replications.

Table 6.21 Regular Routing – 64 Nodes

	LS	μ	σ	Min	Max
Route Length (m)	1	3,180.25	58.88	3,150.00	3,374.26
	8	3,174.05	48.42	3,150.00	3,320.71
Sum Turning Angles	1	1,351.84º	165.38º	1,260.00º	1,980.00º
	8	1,318.50º	128.20º	1,260.00º	1,800.00º
Average Link Length (m)	1	256.80	30.34	214.72	316.67
	8	298.06	45.19	220.83	350.00

For a local search of 1, the dredger routing model determined final routes with optimum route lengths and sums of turning angles 9 out of 20 times and for a local search of 8 it did so 13 out of 20 times. As observed for the 32 node routing problem, these results correspond with Koulamas *et al*. (1994), which, for a simulated annealing-based solution approach, reported finding improved final tours of symmetric travelling salesperson problems when local search was increased from 1 to 8.

None of the 9 final routes found with a local search of 1 which had optimal route lengths and sums of turning angles also had 8 maximum length links. Therefore, for a local search of 1 no optimal dredger routes were found for the 64 node square grid routing problem. Of the 13 final routes found with a local search of 8 which had optimal route lengths and sums of turning angles, 6 also had 8 maximum length links, giving them an optimal average link length of 350. Figure 6.22 depicts the two types of optimal dredger routes found for the 64 node square grid routing problem.

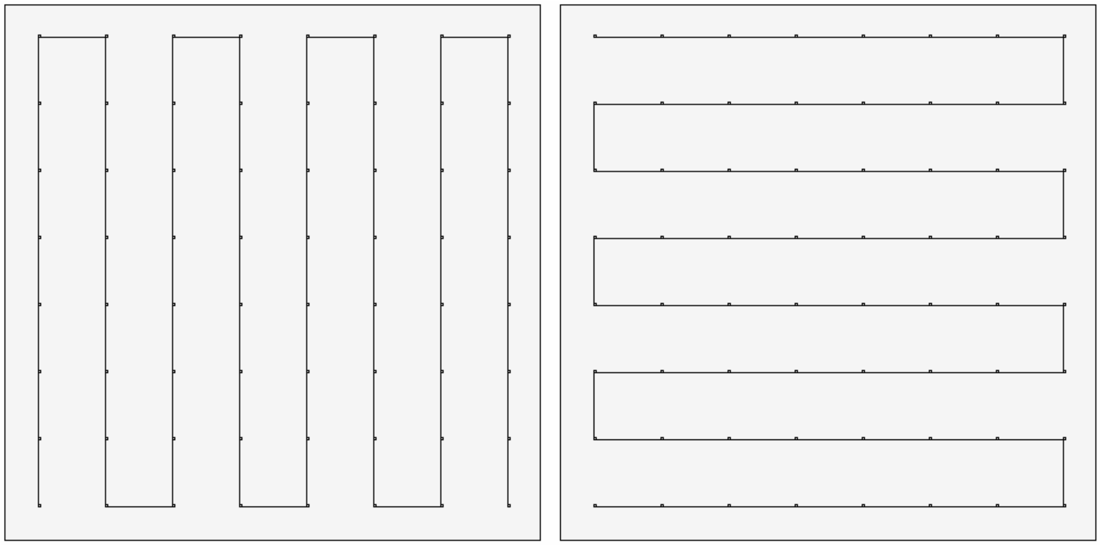


Figure 6.22 Optimal dredger routes – Regular routing – 64 Nodes.

In summary, the success rate of finding optimal dredger routes for the 64 node square grid routing problem was 0% for a local search of 1 and 30% for a local search of 8. For the 32 node square grid routing problems these success rates, respectively, were 80% and 85%. For an increase in the number of nodes from 32 to 64, a 55% drop in the success rate of finding optimal dredger routes with a local search of 8 was observed.

As a result of this 55% drop it was decided to increase values of local search for the 256 node square grid routing problem beyond the upper limit of 8 as suggested in Koulamas *et al*. (1994). To optimize the 256 node square grid routing problem local searches of 1, 8, 16, 32, 64, and 128 were used, and the results of these experiments are presented next.

6.8 Dredger Routing – 256 Node Square Grid Problem

Using Equations 5.19 and 5.20 for the 256 node routing problem, with a square grid spacing of 50, the minimum route length is 12,750 and the minimum sum of turning angles is 2,700 degrees. Figure 6.23 depicts a random route used as an initial solution at the start of the routing optimization process. The route length and the sum of turning angles of the route depicted in Figure 6.23, respectively, are 109,873.78 and 28,161.02 degrees.

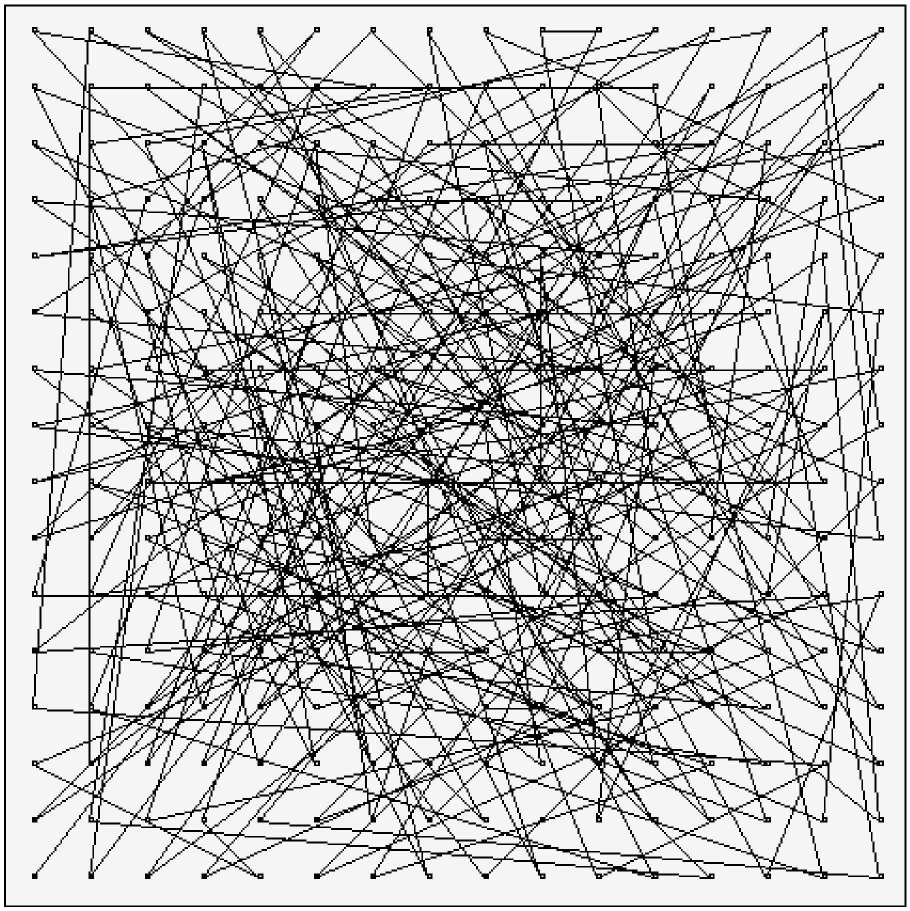


Figure 6.23 Random initial route – Regular routing – 256 Nodes.

The initial route depicted in Figure 6.23 has no links, while for the 256 node square grid routing problem solved here, the maximum link length is 750 and the optimum number of maximum length links is 16 (see Equations 5.21 an 5.22). Table 6.22 reports the mean, μ, standard deviation, σ, and minimum and maximum of route attributes for 20 final routes arrived at for local search, LS, values of 1, 8, 16, 32, 64 and 128 in the application of the dredger routing model to the 256 node square grid routing problem solved here. Values given in Table 6.22 are for 20 replications.

Table 6.22 Regular Routing – 256 Nodes

	LS	μ	σ	Min	Max
Route Length (m)	1	17,646.10	571.71	16,749.23	18,793.22
	8	13,532.23	272.93	13,082.51	13,912.11
	16	12,949.52	163.59	12,791.42	13,436.73
	32	12,789.21	66.24	12,750.00	12,982.51
	64	12,787.25	53.12	12,750.00	12,911.80
	128	12,757.50	24.47	12,750.00	12,850.00
Sum Turning Angles	1	14,059.02º	833.97º	12,559.35º	15,956.97º
	8	5,893.68º	861.55º	3,960.00º	7,596.87º
	16	3,771.10º	565.64º	2,790.00º	5,062.62º
	32	2,935.84º	242.33º	2,700.00º	3,420.00º
	64	2,775.69º	122.97º	2,700.00º	3,150.00º
	128	2,704.50º	20.12º	2,700.00º	2,790.00º
Average Link Length (m)	1	165.42	13.99	143.42	188.90
	8	313.19	50.99	223.79	455.77
	16	430.44	61.64	329.73	566.07
	32	518.29	67.96	422.41	677.78
	64	659.45	85.22	480.77	750.00
	128	725.68	43.16	580.95	750.00

Table 6.21 shows that when local search was increased beyond the upper limit of 8 suggested in Koulamas *et al*. (1994) average solution quality continued to improve. Figure 6.24 illustrates the effect of increased local search on average solution quality.

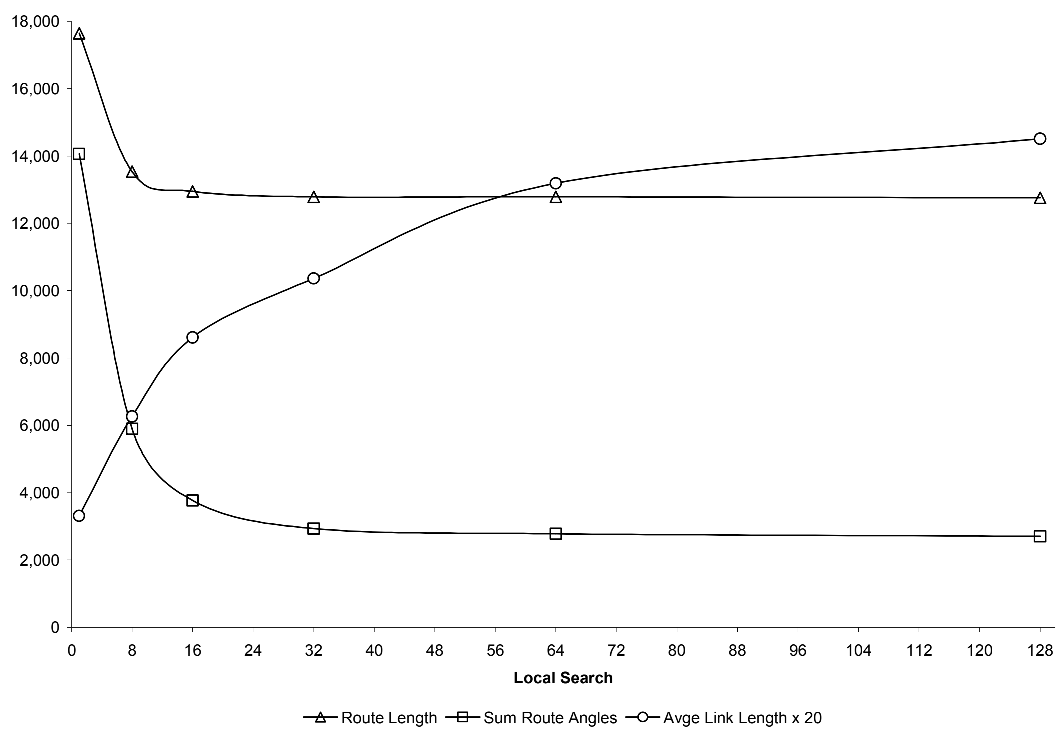


Figure 6.24 Solution quality for increased local search – Regular routing – 256 Nodes.

Figure 6.22 can explain why Koulamas *et al*. (1994) advises against using values of local search greater than 8 when optimizing symmetric travelling salesperson problems with simulated annealing where tours are modified with a 2-Opt edge exchange mechanism. When considering route length only the greatest improvement in solution quality was observed when local search was increased from 1 to 8. For the 256 node square grid routing problem, Table 6.23 gives average improvements in route lengths, sums of turning angles and average link lengths expressed as a percentage of overall average improvements observed between a local search of 1 and 128.

Table 6.23 Local search solution quality gains – 256 Nodes – Regular routing

LS	1 – 8	8 – 16	16 – 32	32 – 64	64 – 128
Route Length	84.15%	11.92%	3.28%	0.04%	0.61%
Sum of Turning Angles	71.91%	18.69%	7.36%	1.41%	0.63%
Average Link Length	26.37%	20.93%	15.68%	25.20%	11.82%

Table 6.23 shows that for the 256 node regular routing problem optimized here the greatest improvement in the sum of turning angles was also observed for an increase of local search from 1 to 8. The same cannot be said of improvements found for the average link length of final routes. To observe an improvement in average link length exceeding improvements of 84.15% and 71.92% found for average route length and sum of turning angles with a local search of 8, a local search of 64 was required. An increase in local search from 64 to 128 resulted in a further 11.82% improvement in average link length, although, as explained in Section 5.3, caution should be taken when reviewing values of average link length alone. Figure 6.25 depicts the two types of optimal dredger route found for the 256 node square grid routing problem.

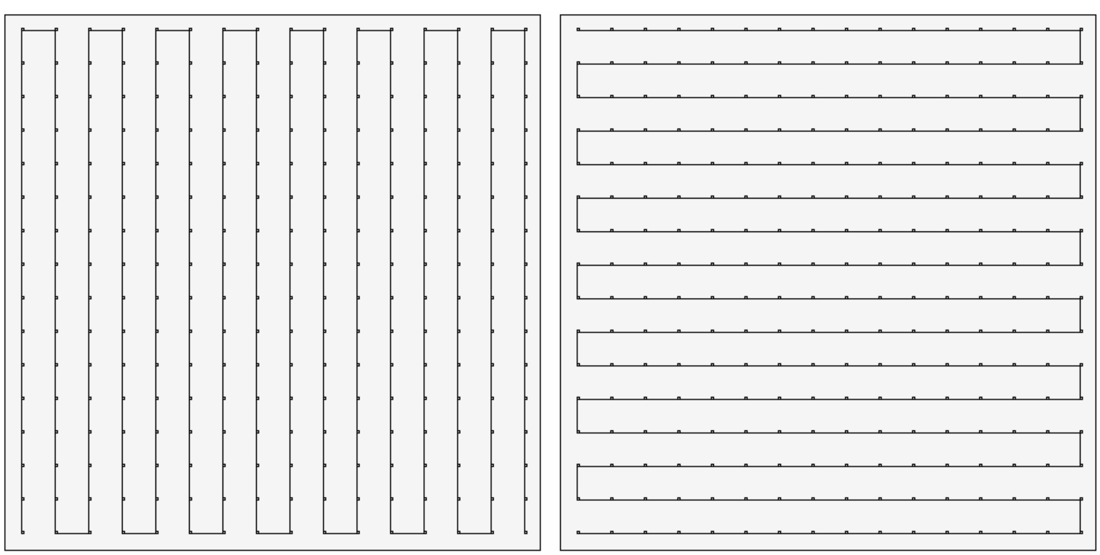


Figure 6.25 Optimal dredger routes – Regular routing – 256 Nodes.

For different values of local search, LS, Table 6.24 gives the number of instances in which optimal routes with optimal length and sums of turning angles were found and those in which optimal dredger routes with the optimal number of maximum length links were found.

Table 6.24 Local search optimal routes found – Regular routing – 256 Nodes

LS	1	8	16	32	64	128
Optimal Length & Angle Sum	0	0	0	6	9	17
Optimal Dredger Routes	0	0	0	0	6	12

Table 6.24 reinforces the argument for using values of local search greater than 8 in combination with the adaptive simulated annealing algorithm, especially when optimal dredger routes are searched for. Figure 6.26 shows the effect increasing local search had on average solution times for the 256 node square grid routing problem.

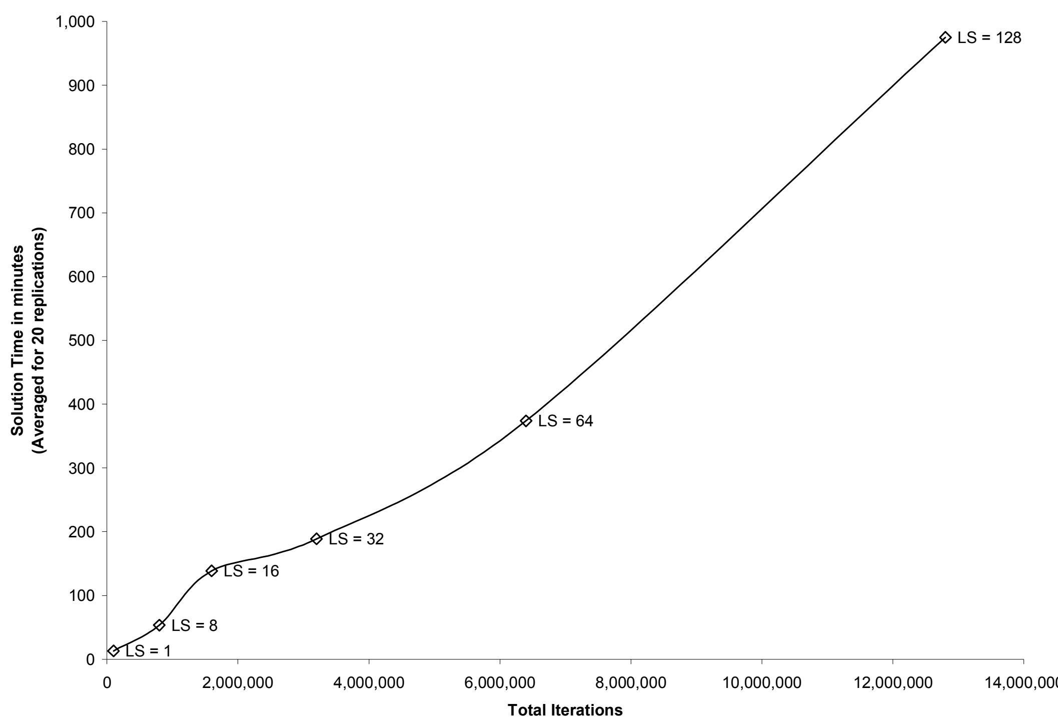


Figure 6.26 Solution times for increased local search – 256 Nodes – Regular routing.

It is thought the change in trend in Figure 6.26 after 2,000,000 iterations in part resulted from a freeing up random access computer memory for experiments with a local search of 32, 64, and 128. Results of the engineering application of the dredge cut nesting problem are presented and discussed next.

6.9 Dredge Cut Nesting – Engineering Application

As mentioned in Section 5.4.4.1, preliminary experiments were carried out for the engineering application of the dredge cut nesting model. Figure 6.27 depicts six preliminary final nests which played an important part in the selection of the stencil set used in the engineering application of the dredge cut nesting model.

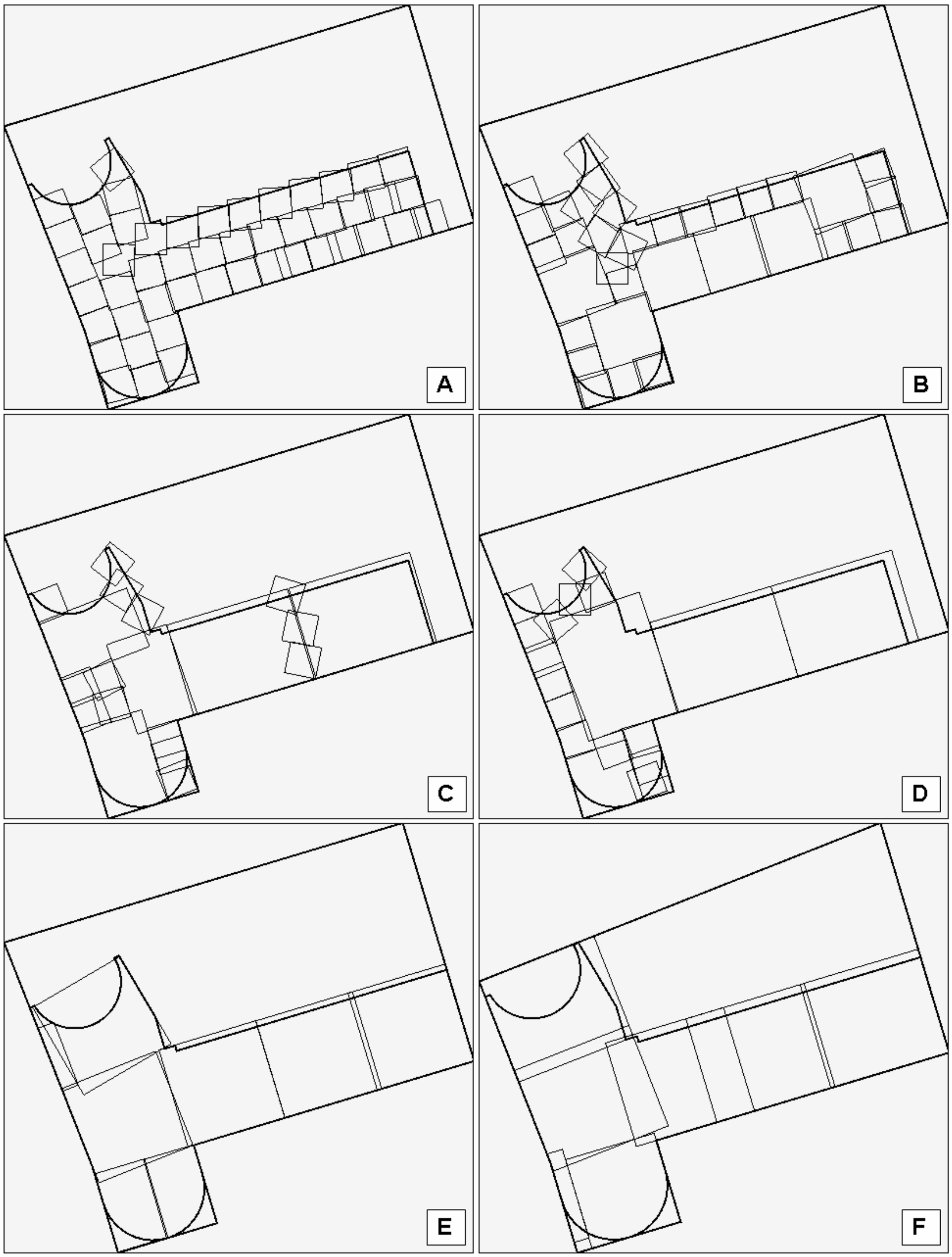


Figure 6.27 Preliminary nesting results – Engineering application.

Appendix E gives the main dredge cut nesting model settings with which the final nests depicted in Figure 6.27 were found. All final nests depicted in Figure 6.27 were found using the revised cost penalties given in Table 6.19. To reiterate, in Section 6.6 it was shown that the use of revised overlap and non-placement cost penalties drastically reduced the non-placement – the most important decision variable for dredging – exhibited by final nests of regular nesting problems where the stencil set consisted of 32 identical squares. The observed reduction of non-placement, for the first time, allowed the centroids of square stencils of final solutions to nesting problems to be considered as nodes of a dredger routing problem.

Previously, in Figure 6.20 a final dredge cut nest with negligible non-placement and overlap of 6.89% of the inner sheet area was chosen for possible determination of nodes for a routing problem. However, the non-negligible overlap of the chosen final nest still posed a problem: If the side length of square stencils is equal to the selected effective cutting width of a cutter suction dredger, non-zero overlap in the final nest will lead to inefficient use of the cutter suction dredger when the centroids of the overlapping stencils are used as routing nodes. In short, overlap of stencils causes a cutter suction dredger to dredge at widths which are less than the selected effective cutting width it can achieve.

Figure 6.27 shows how the problem of overlapping dredge cuts was largely solved by using rectangular stencils which are large conglomerates of square unit dredge cuts. These larger rectangular stencils, referred to as super stencils here, have sides which are exact multiples of a fraction of an effective cutting width of a cutter suction dredger. Fractions are still required to be able to match all dimensions of irregular dredging areas. The final nests A to F in Figure 6.27 show a gradual reduction in the total number of stencils used, as smaller stencils are grouped into larger super stencils. Table 6.25 gives the amounts of overlap and non-placement of final nests A to F expressed as a percentage of the inner sheet area (which was constant), the sheet to stencil area ratios and numbers for the amount of smallest, medium and super stencils nested.

Table 6.25 Overlap and non-placement of preliminary nests – Engineering application

Nest Layout:	A	B	C	D	E	F
Overlap	7.55%	10.66%	14.67%	7.89%	4.48%	17.67%
Non-placement	2.22%	0.03%	0.06%	0.17%	0.46%	0.02%
Smallest Stencils	49	28	14	14	1	1
Medium Stencils	0	6	3	1	2	1
Super Stencils	0	0	2	3	5	5
Sheet : Stencil Area Ratio	1:1.112	1:1.180	1:1.270	1:1.225	1:1.154	1:1.338

Table 6.25 shows that nesting greater numbers of larger super stencils does not necessarily reduce the total amount of overlap of final nests in comparison to when a greater number of smaller stencils are used. However, many of the centroids of unit dredge cuts of overlapping super stencils are discarded and the valid centroids of the remaining unit dredge cuts of super stencils give optimum use of a dredger's effective cutting width as the overlap between these unit cuts is zero by default. Table 6.25 also shows that the nesting problem with the least amount of smallest and medium stencils and the greatest sheet to stencil area ratio had the minimum final non-placement cost.

Figure 6.27 also shows another development which helped regularize final nest layouts: A reduction in the number of relaxed inner sheet boundaries. Of the total number of 14 boundary lines (12 straight lines and 2 arcs) which can be said to make up the sheet representing the dredging area, final nest layouts A to D have 10 relaxed boundaries across which stencils can escape. For final nest layout E the number of relaxed inner sheet boundaries was reduced to 9, and for final nest F it was reduced to 8. Reducing the number of relaxed inner sheet boundaries resulted in better alignment of stencils, thereby, in addition to the use of super stencils, further regularizing final nest layouts. The more regular final nest layouts arrived at are, the more regular the associated dredger routing problem will be, and the more efficient use is made of the selected effective cutting width of the cutter suction dredger employed.

The stencil set used for the engineering application of the dredge cut nesting model (see Figure 5.13) resulted from experience gained in the preliminary experiments. Further to final nests E and F of Figure 6.27, mostly near-square super stencils were opted for. Near-square super stencils were chosen because super stencils of rectangular shape, such as those of final nests C and D in Figure 6.27, were found to rotate less frequently by comparison. In this case, selecting near-square super stencils required one long narrow stencil to be included to ensure complete coverage in the most southern part of the dredging area, as shown, for example, in final nest F of Figure 6.27. Because the engineering application of the dredge cut nesting model concerns a nesting problem where the sheet and stencil set are both irregular, and where the total stencil area exceeds the inner sheet area, the parameter and cost tuning factor combination used for this part of the experimentation is the one for which the minimum average non-placement was found for irregular nesting problems with reduced inner sheets. For a sheet to stencil area ratio of 1:1.4 the minimum average non-placement was found for a parameter and cost tuning factor combination of 7 and 5 (see Table 6.11). A sheet to stencil area ratio of 1:1.4 is closest to that used in the engineering application of the dredge cut nesting model, which for the inner sheet of Figure 5.12 and stencil set of Figure 5.13 gives a ratio of 1:1.439.

Table 6.26 restates the minimum average overlap and non-placement costs found for the irregular nesting test problem with two relaxed sheet boundaries and a sheet to stencil area ratio of 1:1.4, and gives the results of the engineering application of the dredge cut nesting model. Values of overlap and non-placement given in Table 6.26 are for 20 replications and are percentages of total inner sheet area. Since escape penalties were zero for this part of the experimentation, values of escape have been excluded.

Table 6.26 Irregular dredge cut nesting – Engineering application

Nesting Problem	Sheet : Stencil Area	Overlap (% of Sheet Area)				Non-placement (% of Sheet Area)			
		μ	σ	Min	Max	μ	σ	Min	Max
Irregular Test	1:1.4	1.73	1.41	0.15	4.25	4.51	3.30	0.36	10.84
Engineering Application	1:1.44	23.18	0.85	22.45	25.75	0.23	0.17	0.11	0.75

Table 6.26 shows that, as expected, the use of super stencils in the engineering application increased average overlap, but average non-placement was reduced, which is of greater importance from a dredging perspective. Figure 6.28 depicts the final nest which was selected for the determination of nodes for the engineering application of the dredger routing model.

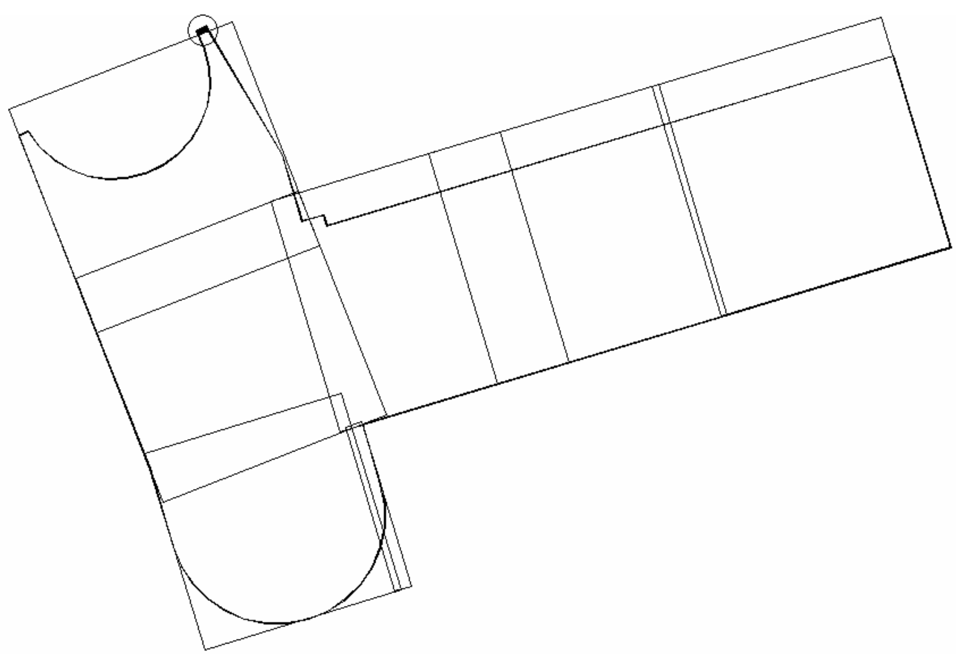


Figure 6.28 Final nest for route node selection – Engineering application.

Appendix F gives the coordinate pairs defining the positions of the stencils shown in Figure 6.28. The final nest depicted in Figure 6.28 has one particular shortcoming: An area of non-placement equal to 0.03% of the total sheet area, which is marked in solid black and encircled. The marked area of non-placement is approximately 32 metres long and 17 metres wide, and is considered negligible as long as it is excavated when the dredger arrives in its vicinity when following the route determined with the dredger routing model. Figure 6.29 shows the centroids of square unit dredge cuts with side lengths of 105 metres (the effective cut width used) which remained after eliminating centroids of surplus square unit dredge cuts according to the guidelines described in Section 5.4.4.2.

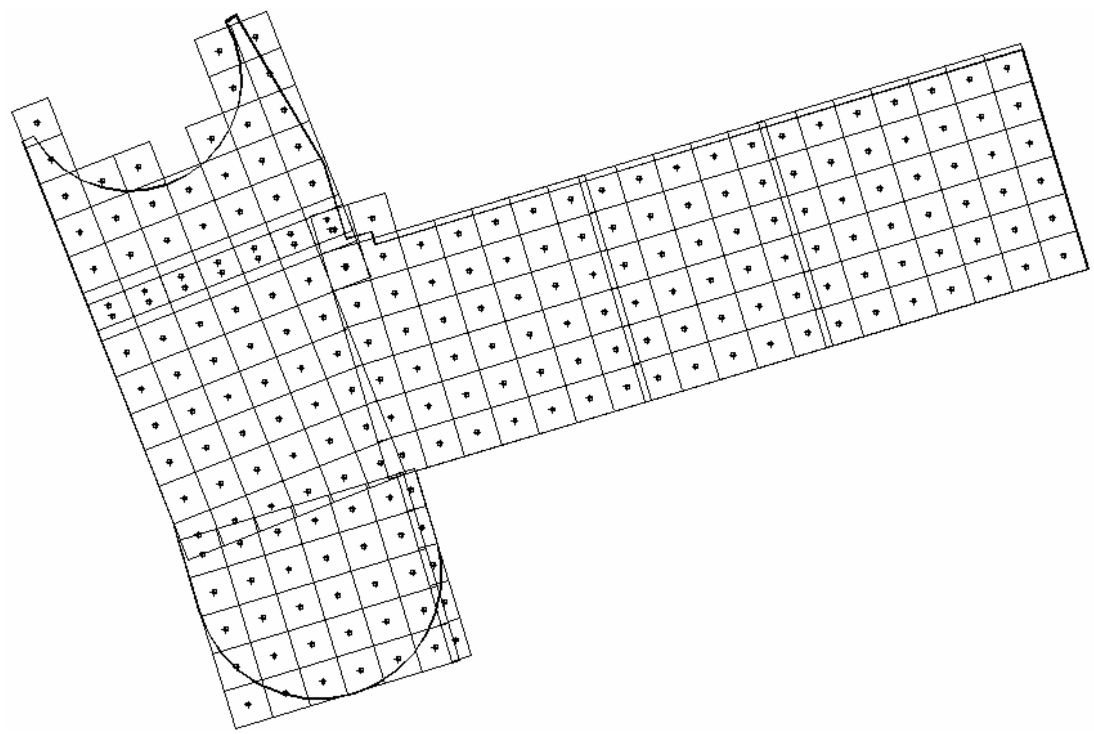


Figure 6.29 Route nodes – Engineering application.

The total number of nodes in Figure 6.29 is 228 and these are the nodes used in the engineering application of the dredger routing model. Appendix G gives the coordinate pairs defining the route nodes shown in Figure 6.29. Results of the engineering application of the dredger routing model are presented and discussed next.

6.10 Dredger Routing – Engineering Application

It should be noted that the all dredger routes depicted in this section start at the left-most route node, as encircled in Figure 6.30, which depicts a random dredger route used as an initial solution at the start of the optimization process. The route in Figure 6.30 has a total length of 245,260.93 metres, a sum of turning angles of 26,004.09 degrees and an average link length of 1,236.64 metres for a total number of 3 links.

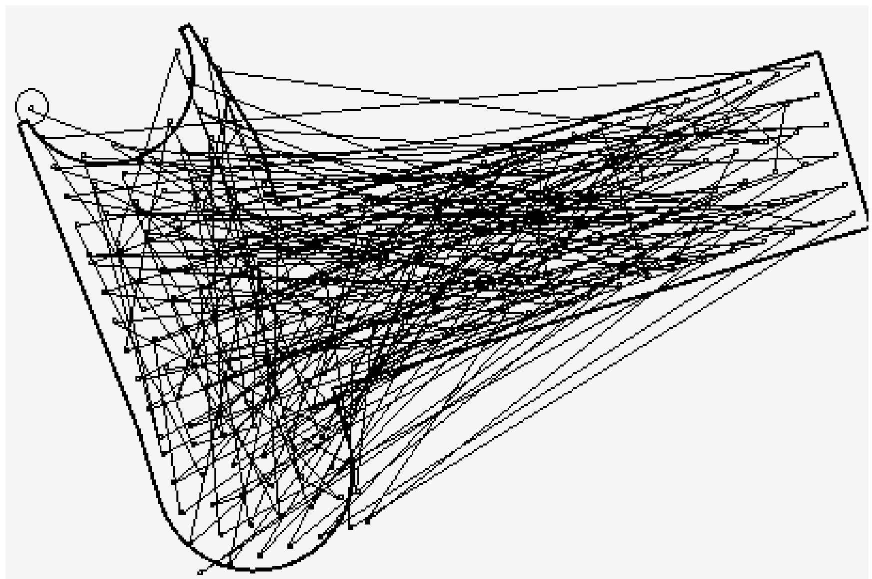


Figure 6.30 Random initial route – Engineering application – 228 Nodes.

Table 6.27 gives total route lengths, sums of turning angles and average link lengths of 10 replications carried out for the engineering application of the dredger routing model.

Table 6.27 Dredger routing – Engineering application – 228 Nodes

Replication	Total Route Length (metres)	Sum Turning Angles (degrees)	Average Link Length (metres)	Total No. of Links -
1	**22,446.22**	2,755.67	781.09	25
2	22,736.45	2,580.94	868.89	24
3	22,735.50	2,582.85	902.09	23
4	22,997.07	3,214.12	807.92	25
5	22,497.92	2,507.93	914.51	22
6	23,012.39	3,145.01	842.81	23
7	22,456.70	2,514.63	904.98	22
8	22,841.88	**2,467.66**	849.67	24
9	22,497.92	2,507.93	914.51	22
10	22,549.74	2,681.56	865.27	23
μ	22,677.18	2,695.83	865.17	23.30
σ	218.89	269.91	45.85	1.16
Min	22,446.22	2,467.66	781.09	22.00
Max	23,012.39	3,214.12	914.51	25.00

Before discussing the results in Table 6.27, the main objective of the research presented in Section 4 is reiterated: It is to contribute to improving the operational efficiency of cutter suction dredgers by providing a tool with which the optimization of two-dimensional cut planning for such dredgers can be automated. In Section 4, an optimal two-dimensional cut plan for a cutter suction dredger was defined as a plan for excavating a dredging area in which the amount of downtime resulting from non-productive dredger movements in between dredge cuts is minimal.

For a given grid of nodes defining dredge cut locations, it was further stated that finding an optimal cut plan consists of finding a route which visits each node exactly once and has a minimum total length and a minimum sum of turning angles (see Section 2.2), and has the minimum number of instances where dredging head on into previously dredged areas occurs. It was also stated that for regular grids minimizing the number of instances where dredging head on into previously excavated areas could be achieved by finding a route with the maximum number of maximum length links. To evaluate routes found for irregular grids it was stated that the route with the maximum average link length is likely to be an optimal dredger route (see Section 5.3).

Results in Table 6.27 show that the minimum total length, minimum sum of turning angles and maximum average link length were not found for the same route. Route 1 has the minimum total length (marked in bold text), while route 7 has the minimum sum of turning angles (marked in bold text), and routes 5 and 9 have the maximum average link length (the relevant rows are grey scaled). This calls for a choice to be made if one of these 4 routes is to be identified as (near) optimal. Before limiting the number of candidates from which the best route can be selected to routes 1, 7 , 5 and 9, all 10 routes are looked at in more detail. Figure 6.31 gives graphical representations of all 10 dredger routes found.

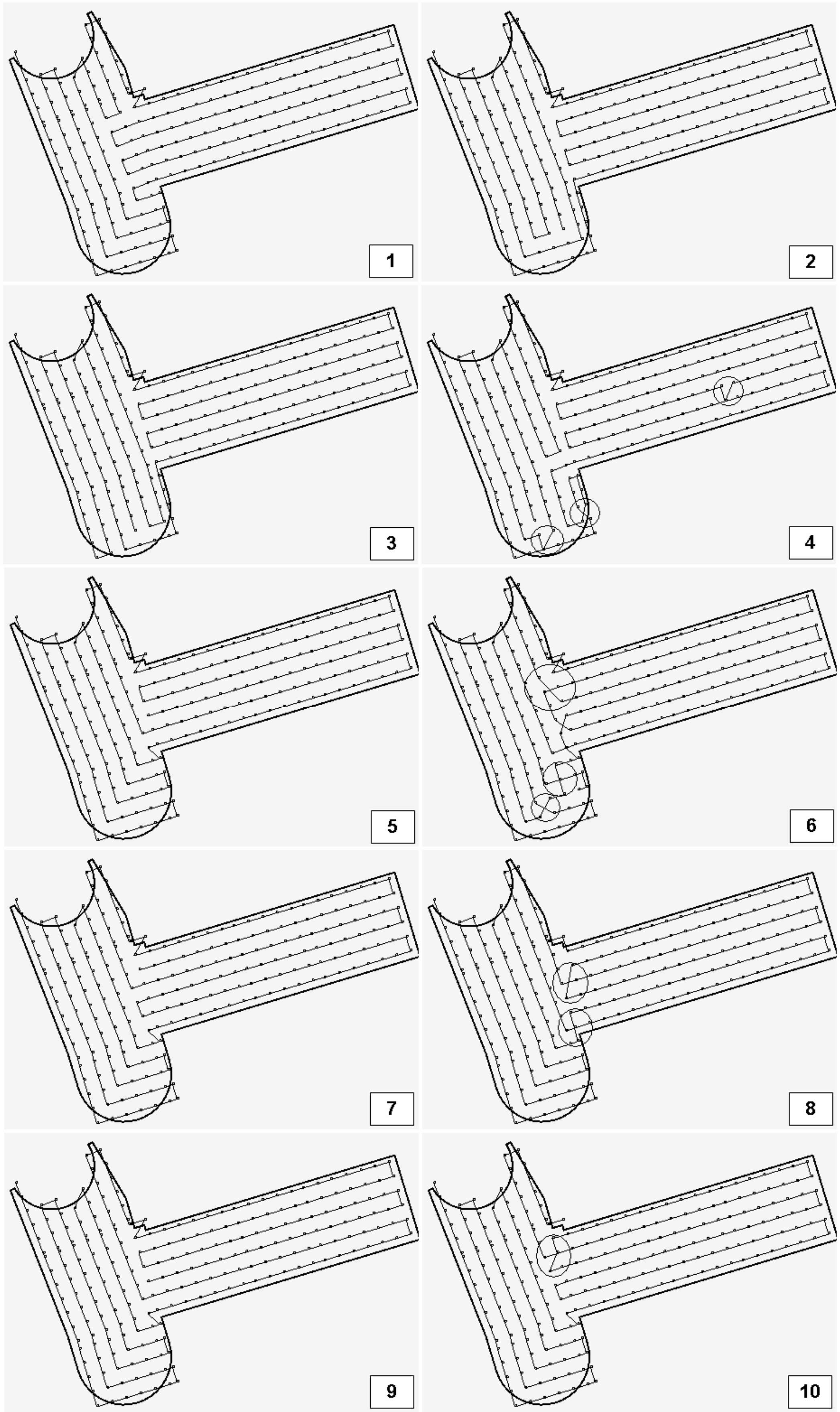


Figure 6.31 Dredger routes – Engineering application – 228 Nodes.

The first observation that can be made when assessing the routes depicted in Figure 6.31 is that routes 4, 6, 8 and 10 all recommend dredging sequences where at some point dredging is diverted to nearby dredge cut, which requires teleportation of the dredger over some distance rather than having it continue dredging uninterrupted. In Figure 6.31 the locations where dredging is interrupted in these routes are encircled. These interruptions in turn cause additional interruptions of dredging activities and renewed teleportation of the dredger when the route comes across cuts dredged as a result of earlier interruptions.

On the basis that interruptions of dredging require additional head on dredging into previously excavated areas and that other routes found show that such interruptions can be avoided, routes 4, 6, 8, and 10 are rejected as valid dredger routes. In addition, dredger routes 4, 6, 8 and 10 all have average link lengths shorter or equal to the overall average. This leaves six routes as candidates for best dredger route. Figure 6.31 also shows that routes 5 and 9 are identical, reducing the number of routes to choose from to five: Routes 1, 2, 3, 7 and 5/9. Table 6.28 ranks the remaining five routes in terms of minimum total length, minimum sum of turning angels and maximum average link length.

Table 6.28 First ranking of dredger routes – Engineering application – 228 Nodes

Dredger Route	Total Route Length	Sum Turning Angles	Average Link Length	Rank Sum
1	1	5	5	11
2	5	3	4	12
3	4	4	3	11
7	2	2	2	6
5/9	3	1	1	5

Table 6.28 shows that route 5/9 can be identified as the best dredger route on account of having the minimum sum of turning angles, the maximum average link length and the third longest total route length of all five routes considered. Route 7 is a close second, having a total route length which is 41.22 metres shorter, a sum of turning angles which is 6.70 degrees greater and an average route length which is 9.53 metres longer than route 5/9. In theory, a greater sum of turning angles equates to more dredger movements and therefore more stoppage time and a lower operational efficiency. Arguably, minute changes in the alignment of centrelines of consecutive dredge cuts can be absorbed without incurring stoppage time, but this is not considered here. Both routes 7 and 5/9 have a total of 22 links, therefore the longer average link length of 9.53 metres of route 7, means an equivalent length of dredge cut of about 210 metres (twice the effective cut width of 105 metres) is dredged more in unaligned route edges, which is reflected by the difference in the sum of turning angles of both routes.

The total length of dredger routes 5 and 9 is 51.70 metres longer or 0.23% more, and their sum of turning angles is 40.26 degrees or 1.63% more than the relevant minima found for all routes. The average link length of routes 5 and 9 is 914.51 metres, the maximum found for 22 links. Figure 6.32 depicts the dredger route representative of routes 5 and 9, where a number of practical shortcomings of the route are encircled and marked with arrows. In what follows, the assumption that dredging production is equal for all cut widths dredged, as stated in Section 4, is discarded.

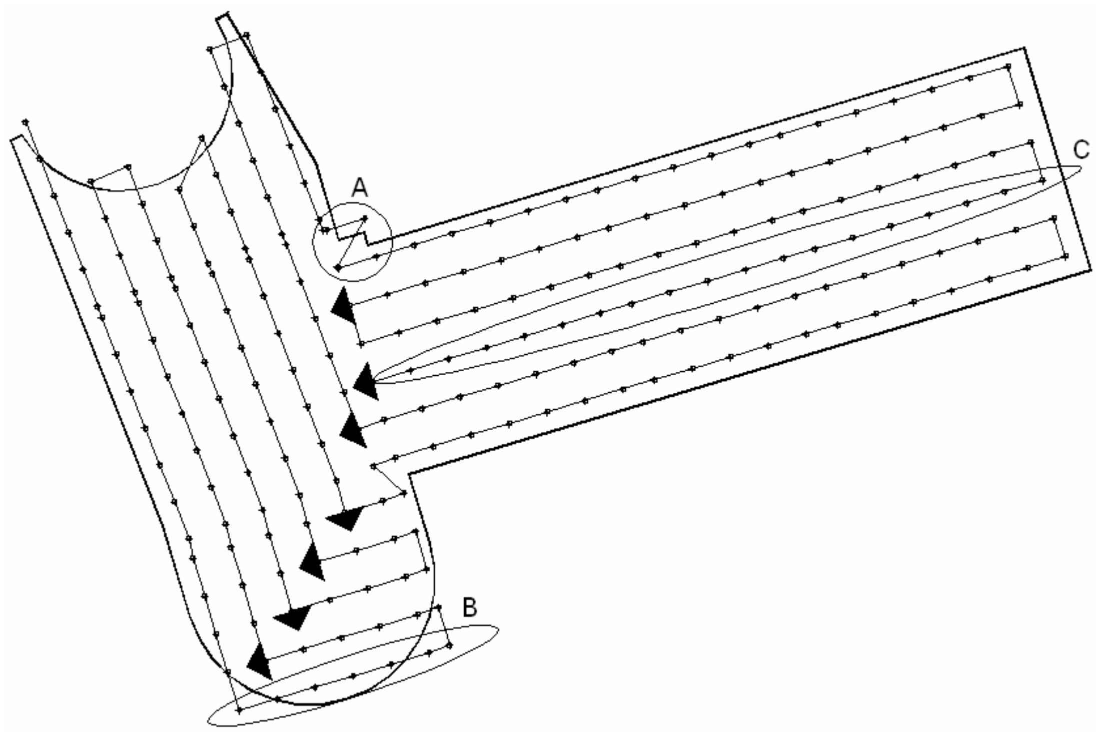


Figure 6.32 Best dredger route – Engineering application – 228 Nodes.

In Figure 6.32, the encircled route edges marked with A, as will be shown in more detail in Figure 6.33, present an impractical dredging sequence. However, this impractical sequence is shared with routes 1, 2, 3 and 7. In Figure 6.32, the encircled route edges marked with B present a dredging sequence which can be difficult to execute in practice if the dredge cut height is much greater than the diameter of the cutterhead of the cutter suction dredger used. The difficulty lies in the capability of cutter suction dredgers to break into undredged ground. To reach final excavation levels from the top of undredged ground there is a limit on the maximum slope which cutter suction dredgers can achieve: As they advance dredging depths are gradually increased over a minimum length of cut, where the total length required to reach final depths depends on the overall height of excavation. This issue is further complicated by the fact that the encircled link marked with B in Figure 6.32 intersects a limit of the dredging area diagonally.

If link B in Figure 6.32 were to be dredged and the overall excavation height is much greater than the diameter of the cutterhead used, then not only will the cutter suction dredger have to start dredging from some distance outside the dredge limit for gradual deepening, possibly resulting in over-dredging beyond permitted tolerances, but it will also have to gradually widen the dredged cut once inside the dredging area. A combined gradual widening and deepening of a dredge cut is not beyond the capabilities of modern cutter suction dredgers with experienced crews, but it should be avoided where possible. Figure 6.33 shows why the encircled route edges marked with A in Figure 6.32 present an impractical dredging sequence.

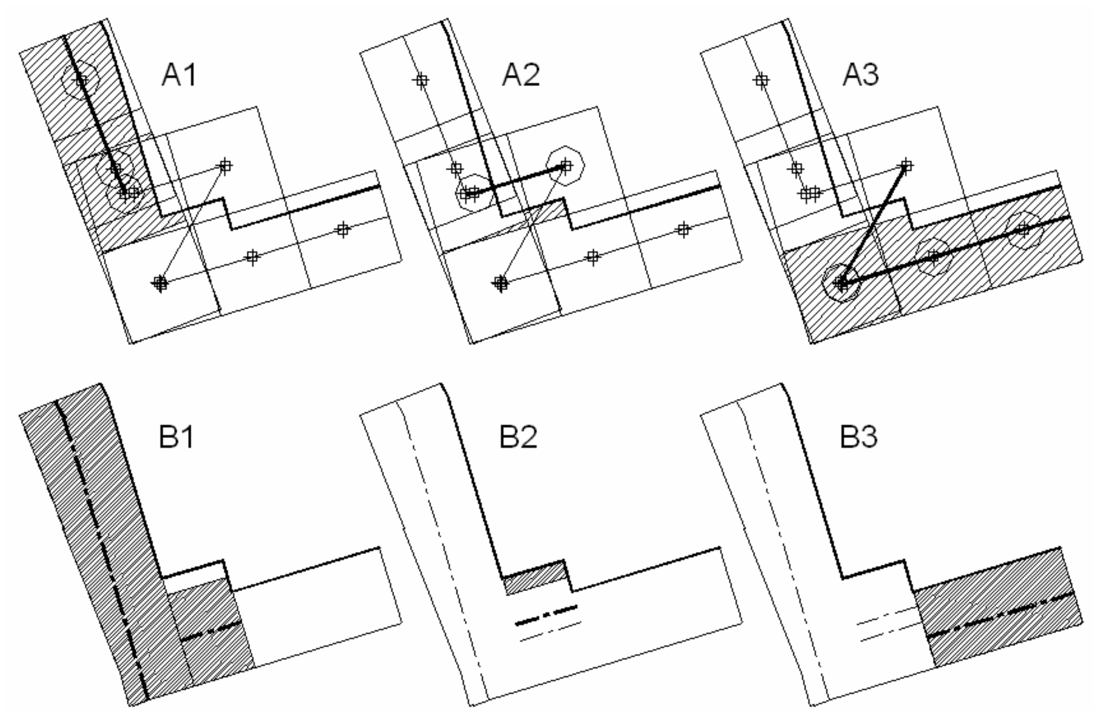


Figure 6.33 Dredger route detail – Engineering application.

In Figure 6.33, the figures A1, A2 and A3 represent the dredger's progress along the recommended dredger route. Progress is indicated by encircling the centroids of square unit dredge cuts and the corresponding areas excavated are hatched. As with the impracticality of the link marked with B in Figure 6.32, when the dredge cut height is greater than the diameter of the cutterhead of the cutter suction dredger used, the bottom-most part of the L-shaped hatched area of A1 (presenting a sudden widening of dredge cut) and the long and narrow hatched area of A2 in Figure 6.33 can be difficult to dredge in practice. In addition, if it were possible to dredge the long and narrow hatched area of A2 separately, dredging production in practice would be lower in comparison to that achieved in wider dredge cuts, although the impact on average dredging production over the entire route would have to be evaluated separately.

The sequence B1-B2-B3 in Figure 6.33, with relevant cut centrelines shown, would be more practical to dredge with a cutter suction dredger than the sequence A1-A2-A3. It can be argued that dredging sequence A1-A2-A3 only requires two dredger movements in practice, whereas B1-B2-B3 would need three dredger movements, albeit two of them being relatively minor (the movements to and from the bold centreline in figure B2). The encircled route edges marked with C in Figure 6.32 make up the last link of the best dredger route and on this last link the dredger will dredge with previously excavated areas on either side. Having previously excavated areas on either side of a dredge cut generally results in lower dredging production than when experienced on one side only.

Figure 6.32 also shows black arrows on 7 route nodes. These arrows indicate locations in the best dredger route where the cutter suction dredger will dredge head on into previously dredged areas. As stated after Figure 5.3, for the same equivalent length of cut, dredging head on into previously excavated areas results in lower dredging production than when done sideways. Therefore in Figure 6.32, in practice lower dredging productions can not only be expected at the 7 locations with arrows, but also in the whole of the last link marked with C. Figure 6.34 shows locations (marked with solid circles) where dredging head on into previously excavated areas occurs, and shows the last route links (emboldened) where dredging will occur with previously excavated areas on either side of dredge cuts for routes 1, 2, 3, and 7.

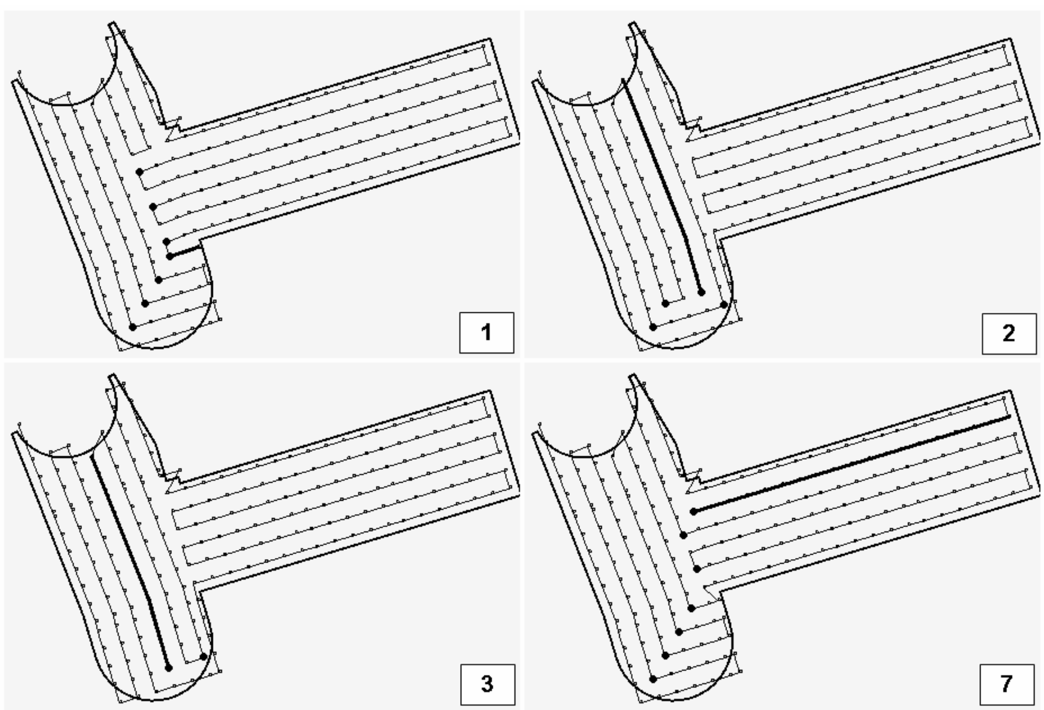


Figure 6.34 Dredging into previously excavated areas – Engineering application.

Figure 6.35 shows that dredger routes 1, 2, 3 and 7, respectively, have 7, 4, 2 and 7 locations where dredging head on into previously excavated areas will occur. Of all valid dredger routes found, route 1 in Figure 6.34 has the shortest length where dredging sideways into previously excavated areas on either side of the dredge cut will occur.

The practical issues raised in Figures 6.31 to 6.34 have resulted from applying the dredger routing problem to an irregular dredger routing problem. Except for dredging head on into previously excavated areas, none of these practical issues surfaced in global optimum dredger routes found for the regular routing problems solved here. The additional practical issues indicate that not all valid dredger routes found with the dredge cut nesting and dredger routing models should necessarily be implemented in practice, and could indicate that none of the valid dredger routes found in the engineering application of the dredger routing model are indeed global optima.

To find out, in view of the additional practical issues raised, if route 5/9 can still be considered as the best dredger route, a revised ranking of dredger routes 1, 2, 3, 7 and 5/9 seems in order. Table 6.29 adds to the previous ranking of routes in terms of minimum instances of head on dredging into previously excavated areas and total length of dredge cuts to be dredged with previously excavated areas on either side.

Table 6.29 Second ranking of dredger routes – Engineering application – 228 Nodes

Dredger Route	Previous Rank Sum	Head on dredging into previously excavated areas	Total length of dredge cuts with previously excavated areas on either side	New Rank Sum
1	11	3	1	15
2	12	2	3	17
3	11	1	2	14
7	6	3	4	13
5/9	5	3	4	12

Table 6.29 shows that route 5/9 can still be regarded as the best dredger route found, but the overall supporting argument is weaker as reflected in the smaller differences in new rank sums. Route 7 remains the most competitive, with only 6.70 degrees of additional turning angle than route 5/9, 210 metres dredged less in aligned dredge cuts, but with a total route length which is 41.22 metres shorter. To get a clearer picture, different settings of cost penalty factors for the dredger routing model could be explored, reflecting preferences for the type of optimum dredger route sought after. However, improving the accuracy of the dredger routing model is preferable.

Traditionally, overall execution times of dredging projects are calculated applying estimated operational efficiencies and estimated production rates of dredgers to the total estimated volume to be dredged, which includes volumes of over-dredging. Since the dredging volume of the modelled dredging project is constant and unavoidable and unforeseeable stoppage time is not considered here, the dredger route which leads to the shortest overall execution time is the one which has the best combination of sum of turning angles and the highest overall average dredging production. Other than being biased towards avoiding instances in which dredging occurs head on into previously dredged areas through the application of an edge length reduction factor, the objective function of the dredger routing model does not take into account estimated dredging production rates.

Route 7 therefore could be a better dredger route than route 5/9 if the time lost due to the additional 6.70 degrees of turning the dredger and the 210 metres dredged less in aligned dredge cuts is offset by the time gained by having to dredge a total route length which is 41.22 metres shorter. To arrive at such a conclusion the ranking systems used in this section need to be replaced by a post-optimization assessment of routes found, which takes into account the overall average dredging production rate of each route. To arrive at overall average dredging production rates with which estimates can be made of the net total time (excluding unavoidable and unforeseeable stoppage time) it takes the dredger to excavate the total project volume, each dredger route can be split up into sections of similar dredging conditions to which different dredging production rates apply.

Alternatively, the decision variables of the objective function of the dredger routing model can be revised by, for example, including a stepwise reduction applied to dredging production rates estimated for volumes to be dredged in particular unit dredge cuts related to each node. For example, a fixed percentage reduction in the relevant dredging production rate for each adverse factor. Adverse factors can be those already highlighted here: Entering head on into a previously excavated area; reduced cut width; having previously excavated areas on either side of dredge cuts; and can include dredging production variation resulting from the direction of dredging, in particular with respect to the dredging of side slopes of dredging areas. If the dredger touting model is revised as such, then the assumption of homogeneity of material in the dredging area can be discarded.

It is important to note that nothing so far has been said about varying heights of excavation in dredging areas. Rarely do dredging areas have a level pre-dredging bathymetry/topography, and not all dredging projects have a single final depth of excavation. Next to inhomogeneity of soils to be dredged, differences in total excavation height in different parts of dredging areas also affect dredging production rates. The excavation of a dredging area in stages at different heights of excavation before arriving at final depths is a three-dimensional cut plan optimization problem.

Lastly, to illustrate that the inclusion of average link length as a decision variable in the objective function of the dredger routing model can be useful, a manually modified dredger route is presented. A review of all the valid dredger routes suggested making minor modifications to dredger route 3 of Figure 6.34, following which the dredger route depicted in Figure 6.35 was arrived at.

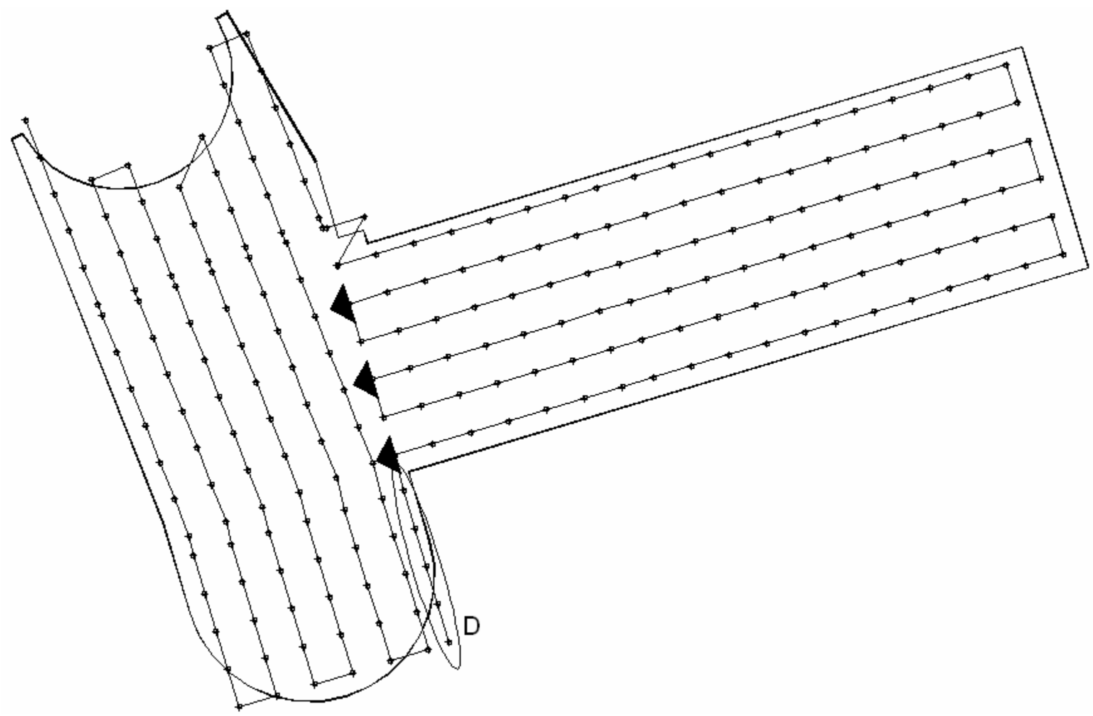


Figure 6.35 Manually modified dredger route – Engineering application – 228 Nodes.

The modified dredger route in Figure 6.35 has three locations where the cutter suction dredger will dredge head on into previously dredged areas, four less than routes 7 and 5/9. In addition, nowhere in the modified dredger route will dredging have to be carried out with previously excavated areas on either side of dredge cuts. Table 6.30 compares the main characteristics of dredger route 5/9 and the manually modified dredger route shown in Figure 6.35.

Table 6.30 Modified dredger route – Engineering application – 228 Nodes

Dredger Route	Total Route Length (metres)	Sum Turning Angles (degrees)	Average Link Length (metres)	Total No. of Links -	Total No. of Locations leading into Excavated Areas -	Total Route Length in between Excavated Areas (metres)
5/9	22,497.92	2,507.93	914.51	22	7	1,746.99
Modified	22,782.92	2,582.27	943.05	22	3	0

Table 6.30 shows that the modified dredger route is 285 metres longer than route 5/9. In addition, the modified dredger route has a sum of turning angles which is 74.34 degrees more than that of the best dredger route. However, the average link length of the modified dredger route is 28.54 metres longer than that of the best dredger route. It is possible that the new maximum average link length found is of interest for a dredging project carried out with a fully automated dredger with minimal manning, since the longer links mean greater time intervals in between dredger movements, which require additional human and plant resources. Finding the modified route with the dredger routing model would require the inclusion of average link length as a separate decision variable in the model's objective function.

However, to find out which of the two dredger routes given in Table 6.30 leads to the shortest overall execution time of the modelled dredging project, an assessment which includes dredging production rates is again required. In view of this, the part of the route depicted in Figure 6.35 marked with D should be noted. This route section, when dredged as recommended in Figure 6.35, has a dredge cut width of less than 52.5 metres, the width of the stencil used in the application of the dredge cut nesting model (see Figures 5.13 and 6.28).

Aside from the issue of cut centrelines intersecting dredging area boundaries diagonally, the best dredger route in Figure 6.32 recommends covering route section D of Figure 6.35 with dredge cut widths of 105 metres, the effective cutting width used in the dredger routing model, at which higher rates of dredger production can be expected than for a cut of less than half that width. This suggests that using stencils with dimensions which are a relatively small fraction of the effective cut width should be approached with caution.

Instead, it could have been better to use a selection of stencils based on a wider effective cut width, of up to 112 metres wide for the dredger considered here. The relevant part of the dredging area is 664 metres long and about 580 metres wide (see Figure 5.12), and therefore a single super stencil consisting of 6 cuts of 110.67 metres wide and 664 metres long each would have ensured complete coverage. This casts some doubt on whether it was necessary to automate the dredge cut nesting process. It is possible that a human expert may find equally good or better nests of suitable dredge cuts with complete coverage of the dredging area than found with the dredge cut nesting model, and in less time

In summary, results of the engineering application of the dredge cut nesting and dredger routing models developed and used here showed that, for a given cutter suction dredger and dredging project, a two-dimensional cut planning problem was optimized and therefore the hypothesis of this research can be accepted. However, the two-dimensional cut plans found, although executable in practice, had a number of shortcomings related to practical dredging issues. In addition, to arrive at more conclusive results for determining cut plans which lead to the minimum execution time of modelled dredging projects the dredger routing model also needs to take into account dredging production rates, a model feature which, for the research presented here, was not included. Table 6.31 summarizes the main results of the experimentation carried out as part of the work presented here.

Table 6.31 Main results summary

No.	Experiment	Objective	Main Result	Section
1	Irregular nesting – taken from Yuping *et al*. (2005)	Validation of Dredge Cut Nesting Model	Model validated	6.1
2	Regular routing – 32 nodes derived from Yuping *et al*. (2005)	Validation of Dredger Routing Model for local search of 1 and 8	Model validated for local search 1 and 8	6.2
3	Irregular nesting – with increasing number of relaxed sheet boundaries and sheet to stencil area ratios of 1:1	To conclude if allowing escape of stencils leads to better final dredge cut nesting solutions	Solution quality deteriorated	6.3
4	Irregular nesting – with 2 of relaxed sheet boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4	To conclude if providing an excess of stencil area leads to better final dredge cut nesting solutions	Solution quality deteriorated	6.4
5	Regular nesting – with 2 of relaxed sheet boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4	To conclude if irregular or regular stencils lead to better final dredge cut nesting solutions	Solution quality deteriorated	6.5
6	Regular nesting – with 2 of relaxed sheet boundaries and sheet to stencil area ratios of 1:1.1, 1:1.2, 1:1.3 and 1:1.4 with revised cost penalties	To conclude if revised cost penalties lead to better final dredge cut nesting solutions	Solution quality vastly improved	6.6
7	Regular routing – 64 nodes	To conclude if for local search of 1 and 8 optimal dredger routes can be found	Optimal solutions found for local search 8 only	6.7
8	Regular routing – 256 nodes	To conclude if for local search of 1, 8, 16, 32, 64 and 128 optimal dredger routes can be found	Optimal solutions found for local search 64 and 128 only	6.8
9	Irregular nesting – engineering application	To conclude if the Dredge Cut Nesting Model can optimize a real-world problem	Confirmed, but facilitated by the use of super stencils	6.9
10	Irregular routing – engineering application 228 nodes	To conclude if the Dredger Routing Model can optimize a real-world problem	Confirmed, but inconclusive if global minima were found	6.10

7 Conclusion

Two adaptive simulated annealing-based models were developed and used to solve nesting and routing problems in search of optimal two-dimensional cut plans for cutter suction dredgers to provide a tool for improving operational efficiencies of such dredgers. This research pioneered the modelling of two-dimensional cut planning for cutter suction dredgers as a combination of a modified stock cutting problem and a modified travelling salesperson problem. This research was first to use adaptive simulated annealing to optimize stock cutting problems where feasible solutions can exhibit escape of stencils beyond sheets, and was first to use adaptive simulated annealing with increased local search to optimize asymmetric routing problems with turning costs.

The research explored how the performance of the nesting model in finding optimal final nest layouts with zero non-placement, signifying complete sheet coverage, is influenced by selection of stencil set, sheet arrangement, and objective function cost penalty factors. For nesting problems with an irregular stencil set, a rectangular sheet and all cost penalty factors set to unity, model performance deteriorated significantly when stencils were allowed to escape from sheets. Model performance deteriorated further when escape cost penalty factors were set to zero and total stencil area was made to exceed sheet areas which had two boundaries across which stencil escape was permitted. For the same sheet arrangements and penalty factor settings, no significant improvement in model performance was observed when a regular stencil set of identical squares was nested.

For regular as well as irregular nesting problems, where escape of stencils is permitted and total stencil area exceeds sheet area, the use of revised cost penalty factors for non-placement and overlap of stencils resulted in significant improvement in the performance of the nesting model. The revised cost penalties increased non-placement cost in comparison to the cost of stencil overlap. Further improvement of model performance was observed when the total number of stencils nested was kept under 10 for irregular nesting problems. With revised cost penalty factors and an irregular set of 7 stencils acceptable final nest layouts were found in an engineering application of the dredge cut nesting model. The average non-placement of final nest layouts found, was 0.23% of the total sheet area with a standard deviation of 0.17%.

For square grid routing problems of rectangular shape, the research also explored how the performance of the dredger routing model in finding optimal dredger routes, signifying routes with optimal numbers of maximum length links, is influenced by problem size and increased local search. Using a 2-Opt route edge exchange mechanism to modify routes, the model's performance was found to deteriorate significantly when problem size was increased from 32 to 64, and then to 256 nodes for a local search of 1. For a square grid routing problem with 256 nodes model performance improved significantly when local search was increased gradually from 1 to 128, with results showing that 60% of routes found were global optima.

For an irregular grid routing problem with 228 nodes, derived from a final dredge cut nest layout with 0.03% non-placement found in the engineering application of the dredge cut nesting model, the dredger routing model's performance with a local search of 128 was found to be far from optimal. To cover an irregular dredging area of 2,172,151 square metres, 10 dredger routes were found with an average route length of 22,677.18 metres with a standard deviation of 218.89 metres, and an average sum of turning angles of 2,695.83 degrees with a standard deviation of 269.91 degrees. The minimum route length and minimum sum of turning angles, were not found for the same route. After detailed analysis of all 10 routes found, 4 routes were rejected as valid solutions because they recommended unnecessary dredger teleportation.

Of the remaining 6 valid routes, 2 identical routes were considered to be the best dredger route found on the basis of having the second smallest sum of turning angles and the overall maximum average link length. However, the detailed analysis of the remaining 6 valid dredger routes showed it was not possible to conclude if the route identified as the best dredger route found would also lead to the shortest overall execution time of the modelled dredging project. To make conclude this dredging production rates need to be taken into account. Either as part of a post-optimization appraisal of dredger routes found or through a revision of the decision variables of the objective function of the dredger routing model so that the overall average dredging production rate for routing problems can be optimized.

8 Future Work

A number of aspects which can lead to improved model performance in general are mentioned first. For the dredge cut nesting model, further testing with values of parameter and cost tuning factors in the between the integer values used here may lead to improved results. Of particular interest to the nesting problems solved here should be rational values of the cost tuning factor in between 3 and 5. For a cost tuning factor of 3 solution quality was poor for all nesting test problems solved, while for increases to values of 4 and 5 the greatest improvements in solution quality, and best solution qualities were found.

For both the dredge cut nesting and dredger routing problem, further testing with different settings for penalty cost factors of the objective functions is required. For the dredge cut nesting model only one revised set of cost penalty factors was used, inspired by a set used in Yuping *et al*. (2005). No other variations in cost penalty factors for the dredge cut nesting model were researched here.

For the dredger routing model all cost penalty factors were set to unity for all routing problems solved. Results of the engineering application of the dredger routing model showed that additional evaluation of optimized routes was necessary to identify the best route. The dredger routing model in its present form allows for varying cost penalty factors for total route length and sum of turning angles. By setting penalty factors for sum of turning angles to zero it could also be interesting to test the model's performance, using increased local search, on travelling salesperson problems taken from test libraries.

In its present form the dredger routing model is also capable of using a variable Opt edge exchange mechanism to modify routes, up to the maximum number of edges possible for a given problem. Helsgaun (2000) uses a heuristic solution approach with a variable Opt edge exchange mechanism for up to 5 route edges to successfully solve large travelling salesperson problems. The variable Opt edge exchange mechanism of the dredger routing model was not used in the research here because the additional feature of being able to select other edges nearby the edge under consideration was not fully verified.

A 5-Opt edge exchange in Helsgaun (2000) is limited to the nearest five neighbours of the edge considered. For an adaptive simulated annealing solution approach each edge can be associated with a parameter temperature which influences which neighbouring edges are considered for inclusion in a variable Opt edge exchange. The variable Opt exchange mechanism itself can also be subjected to annealing and re-annealing. At present the 2-Opt edge exchange used selects two edges randomly. The use of parameter temperatures for a variable Opt exchange mechanism in the dredger routing model is expected to improve the efficiency of increased local search. Clearly, for a local search of 128, as used in this research, not all of the 128 2-Opt edge exchanges considered at a particular point in the solution process would have been useful, signifying a waste of computer resources.

In addition, as done in Helsgaun (2000), previous route modifications can be remembered so that they are not carried out again or accepted for a specified number of iterations. After inclusion of an annealing and re-annealing system for a variable Opt edge exchange mechanism it would be interesting to see how the model performed when tested against travelling salesperson problems taken from test libraries.

Secondly, a number of issues related to increasing the complexity of the models developed here and improving their accuracy are worth considering. As already mentioned in the concluding section, dredging production rates need to be included as a decision variable in the dredger routing model to search for dredger routes of which it can be said that they will lead to the overall shortest execution time of the modelled dredging projects.

Then the use of centroids of square unit dredge cuts (which make up super stencils nested with the dredge cut nesting model) as nodes for a dredger routing problem needs to be reviewed. Such a review would centre on considering the use of centroids of the parts of square unit dredge cuts which are inside the dredging area instead of using centroids of unmodified square unit dredge cuts. The use of unadjusted nodes for a dredger routing problem results in what could be seen as false links edges because the dredger routing model is biased towards adding edges which are aligned, but these edges inaccurately define locations of cut volumes. Using centroids of partial unit dredge cuts will improve the models of dredging projects.

As part of increasing the accuracy of the dredge cut nesting model and the dredger routing model, they can both be extended to three spatial dimensions. This is of particular interest to the dredge cut nesting model. As mentioned briefly in the previous section, cutter suction dredging production rates vary with excavation height. Differences in excavation height are present in most dredging projects, especially capital dredging projects. In addition, in view of spillage, it is sometime preferable to divide a dredging project into two stages, one of so-called bulk dredging and one of final dredging. To maximize dredger production it is necessary to model a dredging project in three dimensions instead of two. How a three-dimensional dredge cut model is to function exactly is not yet clear, but the model should aim at finding nests of three-dimensional cut units which maximize overall average dredging production.

For multi-staged dredging projects the dredger routing model must also be able to cope with routing in a third spatial dimension. In view of this, accretion rates in areas dredged to final depth can be taken into account, in a fashion similar to, for example, lawn mowing problems where rates of vegetation re-growth are taken into account. In addition, to eliminate the assumption of unlimited access to dredging areas, the dredger routing model can be extended to include user specified time windows in which certain parts of dredging areas are inaccessible or, for instance, require early completion. Such an extension may have to include allowing for revisiting nodes in the routing problems solved. Lastly, results of the models developed here or extended versions thereof can be used to determine construction schedules. The determination of a construction schedule related to an optimized dredger route can include the optimization of transport of dredged materials from cut to fill areas with solution approaches presented in Mayer *et al*. (1981), Ford (1984) and Henderson *et al*. (2003).

Appendix A – Mass Haul Diagram – Worked Example

The mass-haul diagram and experienced engineering judgement, together with deterministic methods, have been the key factors in planning and estimating earth moving operations (Jayawardane *et al*. 1994), in particular on road and railway projects. The following paragraphs explain how a mass-haul diagram can be made with the aid of data from a hypothetical road/railway project. A mass-haul diagram is plotted after the earthwork quantities have been computed between cross-sections at chainages along the longitudinal profile of a road/railway project for which the grades are known. Cut is taken as negative and fill as positive to evaluate cumulative volumes from the beginning to the end of the project. The ordinates on the mass-haul diagram show these volumes in cubic metres. The horizontal base line, plotted to the same scale as the longitudinal profile, gives the points at which the cumulative volumes are obtained with total positive volumes plotted above the base line and total negatives below it. Numerical data of a hypothetical road/railway project taken from a textbook example (Bannister, 1984) is given in Table A.1.

Table A.1 Hypothetical data for a mass-haul diagram.

Chainage along Longitudinal Profile	Centre Height w.r.t. grade (m)	Volumes Cut (m^3)	Volumes Fill (m^3)	Shrinkage Constant	Corrected Volume	Cumulative Volume
1000	-1.22					0
1040	0		230		-230	-230
1100	1.52	480		0.90	+430	+200
1200	3.96	2560		0.90	+2300	+2500
1300	4.12	4560		0.90	+4100	+6600
1400	2.74	3940		0.90	+3550	+10150
1500	0	950		0.90	+850	+11000
1600	-3.05		1350		-1350	+9650
1700	-4.27		4010		-4010	+5640
1780	-4.72		4600		-4600	+1040
1820	-4.72		BRIDGE			+1040
1900	-3.51		4130		-4130	-3090
2000	-1.22		2370		-2370	-5460
2035	0		60		-60	-5520
2100	1.98	510		0.90	+460	-5060
2200	3.96	3180		0.90	+2860	-2200
2300	3.66	4055		0.90	+3650	+1450
2400	2.44	3860		0.90	+3470	+4920
2500	0.61	1320		0.90	+1190	+6110
2530	0	100		0.90	+90	+6200
2600	-1.07		350		-350	+5850
2700	-1.52		1230		-1230	+4620
2800	0		420		-420	+4200
2900	1.68	1080		0.89	+960	+5160
3000	3.66	3730		0.89	+3320	+8480

Assuming that the earthworks have been balanced before chainage 1000 and all cut is directly usable as fill, the data from Table A.1 can be used to plot a mass-haul diagram as illustrated in Figure A.1.

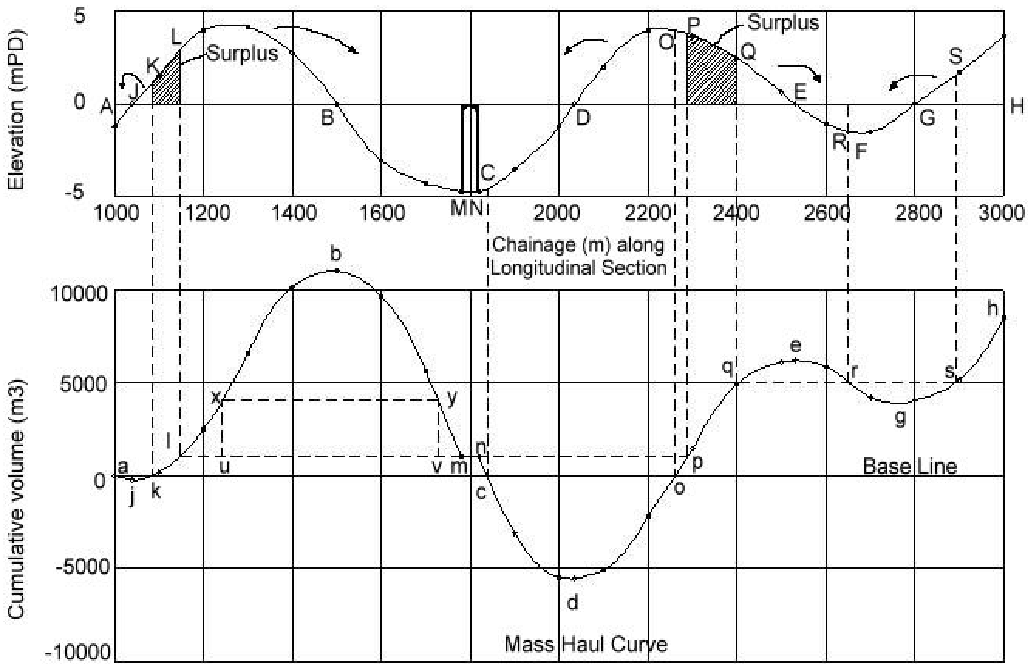


Figure A.1 Mass haul diagram plotted with hypothetical data.

Where the mass-haul curve crosses the base line a change in the sign of the cumulative volume is observed. Also, the cumulative volume between any two consecutive points between which the mass-haul curve crosses the base line is zero. Between such points the cut and fill balance each other. In addition, the following conclusions can be drawn from the mass-haul diagram depicted in Figure A.1:

- A rising mass-haul curve indicates cut, a maximum point marking an end of cut.
- A falling mass-haul curve indicates fill, a minimum point marking an end of fill.
- The differences between the ordinates of two points represents the volume of cut or fill between those points as long as there is no minimum or maximum point situated in between them.
- Any horizontal line which intersects the mass-haul curve at two or more points, for instance the dashed line between the points *l* and *m* in Figure A.1, is known as a balancing line as the volume of cut and fill balance. In other words, there is no difference in cumulative volume between the points *l* and *m*. The length of a balancing line is equal to the transport distance between the points at which it intersects the mass-haul curve. The points of intersection are also known as balancing points.
- When the mass-haul curve is above a balancing line, material must be moved to the right, for instance in the case of the part of the mass-haul curve denoted by *lbm* in Figure A.1,

and when it is below such a line the material must be moved to the left, as with the curve section *rgs*. In Figure A.1 the direction of movement is indicated by arrows.

The base line of the mass-haul diagram is in itself a possible balancing line, although not necessarily the most economical one. If the base line in Figure A.1 is used as the balancing line there will be a surplus of 8490 m^3 at the end of the project. Surplus can be considered for use elsewhere, for instance on another earthwork project not too far away.

If there is no other use for surplus then, of course, it will remain, but by selecting the balancing line(s) differently a more, or perhaps even the most economical way to carry out the works can be determined based on minimal transport distances. For this purpose any number of balancing lines may be drawn on the mass-haul curve, and it is not necessary for them to be continuous. For instance, in Figure A.1 the balancing lines *lm* and *np* are separated by the bridge and *np* and *qrs* are not connected at all.

Earthwork that is excluded by balancing lines is not balanced. In Figure A.1 it can be seen that this is the case between the points *K* and *L*, and *P* and *Q*. As the mass-haul curve rises in the sections corresponding to both these sets of points it is known that the imbalance in each case is a surplus. Where imbalances amount to a shortage the selection of one or more borrow pits will be required to complete the project.

It is common that the most economical solution for carrying out the earthwork is obtained by selecting balancing lines which are not continuous. Balancing lines that are too long would result in excessive and uneconomical transport distances. The cost of transporting excavated material depends to some extent on the distance it must be carried. Usually, in the Bill of Quantities for a project, a unit price for excavation will include the transport of the excavated material over a limited distance. This limited distance is known as *free haul*. When the material has to be transported over a distance greater than the free haul, the extra distance is referred to as *overhaul*. In some contracts overhaul is provided for.

The term *haul* itself is defined as the total of the products of the separate volumes of cut and the distances over which they are transported to areas containing volumes of fill, transport distances being measured between the centroids of the cut and the fill volumes.

Where free haul is given it can be plotted on the mass-haul diagram allowing overhaul to be estimated. In the mass-haul diagram depicted in Figure A.1 a free haul of 500 metres is represented by the balancing lines *xy* and *np*. The balancing lines *qr* and *rs* each represent transport distances equal to roughly half the free haul distance.

The areas delimited by the parts of the mass-haul curve cut off by the balancing lines and the balancing lines themselves, for instance the area *lbm* cut off by the balancing line *xy*, are equal to the haul in the relevant section as they represent the product of volume and distance. One square on the mass-haul diagram shown in Figure A.1 represents $5{,}000 \times 200 = 1 \times 10^6\ m^4$ of haul. If, for payment purposes, a unit is defined as 1 cubic metre transported over a distance of 100 metres then 1 square is equal to 10,000 units. The area giving haul within the free haul distance is then *uxbyvu*. The area giving haul exceeding the free haul distance is the area *lbml* minus the area *uxbyvu*, which in Figure A.1 is about 200,000 m^4 or 0.2 squares. By multiplying this net area expressed in squares by 10,000 the amount of haul exceeding the free haul distance is expressed in units, in this case 2,000 units. The volume for which additional transport costs are paid is given by the dashed line *xu*, i.e. 3,000 m^3, the additional distance over which this volume has to be transported being approximately 67 metres.

A balancing exercise will have taken into account the free haul distance and will most likely include borrowing for some sections and running waste from others. In addition, the balancing of earthworks has to include the consideration that it is preferable to transport excavated material downhill as this will require less effort. In cases where transporting material over long distances along steep uphill sections between cut and fill costs more than wasting excavated material followed by excavating once again from a borrow pit, the latter option may be chosen.

In summary, when haul costs are directly proportional to transport distance, a mass-haul diagram has two useful properties which aid in determining the minimum amount of haul and therefore lead to the most economic allocation of cut-and-fill (Mayer *et al.,* 1981), and these are:

- Equal amounts of cut-and-fill can be indicated by horizontal balancing lines between two or more balancing points intersecting the mass-haul curve, and;

- Quantities of haul can be minimized by distributing cut-and-fill between, rather than across, balancing points.

Appendix B – Daily Report Cutter Suction Dredger "Cyrus"

DAILY REPORT	**CYRUS**	Tuesday	Report No.:	307
			Date:	02-Feb-99
			Calendar Wk	6
Location:	Laem Chabang Port Proj. Ph. 2, Stage 1		Dredger Wk	45

Cutterpos. at	7.00 a.m.	CL	E	704473	Start	E	CUT:	21-14
			N	1443440		N		

Cutter Depth	Full Width	Original Bed Level	OutSurvey Bed Level	Chainage Start	Chainage Stop	Advance	Quantity Dredged		Type of Material % Dredged
-17.80	112	-14.36	-16.00	-4	-8	4	735		Silt
-17.80	105	-12.35	-16.00	1455	1393	62	23,762	*	Sand
-15.00	105	-12.35	-14.47	1393	1388	5	1,113	*	Clay
									Soft Rock
Estimated m³/hr		1,403		Estimated Volume m³			25,609		Hard Rock

From Hrs.	To Hrs.	Dredging Mins.	Delays Mins.	Description of Delay	Total Daily Delays Reason	Cd.	Mins.
7.00	8.15	75			Holiday	1	
8.15	9.00		45	FL.pipe line 2 leaking pipe out 4 pipe in c.c 18 new f.p 16 used f.p + 3 lock	Change Cutter	2	
9.00	11.20	140			Change Teeth	3	85
11.20	11.35		15	Stop dredging cut 21 change to cut 14 shift st.b anchor c.c 2 new f.p	Shift Anchors	4	155
11.35	13.55	140			Shift Dredger	5	
13.55	14.10		15	C.C 1 used f.p	Shift Float Line	6	45
14.10	16.10	120			Obstructions	7	
16.10	17.50		100	Welding fl.pipe line check p.1, p.2 and p.3 shift both side anchors	Shift Spider	8	
17.50	19.45	115			Shift Dumping	9	
19.45	20.00		15	C.C 4 new f.p +6 used f.p +2 lock	No Barge	10	
20.00	22.00	120			Reclamation	11	
22.00	22.15		15	C.C 2 new f.p +4 used f.p + 1 lock	Spud Change	12	
22.15	0.05	110			Sounding	13	
0.05	0.25		20	C.C 8 new f.p +6 used f.p	Deck Dredger	14	60
0.25	2.40	135			Deck Spider	15	
2.40	3.00		20	C.C 8 new f.p	Engine Room	16	
3.00	4.45	105			Weather	17	
4.45	6.25		100	Shift st.b anchor not holding c.c 13 new f.p +5 used f.p+ 2 lock	Misc.	18	
6.25	7.00	35			Sch. Maint.	19	
				Note real prod. By -16.52 total 29,227 m3 1,602 m3/hr 531 m3 f.p		Tot	345
Total this page		1095	345	Dredging Hrs. 18.25			

CUTTER		
Type of cutter		38 DS D.13
Teeth Used	38 D	55
Lock		8
m³/Pickpoint		466

FUEL	
Comsuption Ltr.	38,400
Ltr/hr	1,600
Ltr/m3	1.50

EFFICIENCY	
Tot. Avail/Dred.Hrs	76%
Operational/Dred.Hrs	76%
Total Non Avail. Hrs.	0.00

PIPELINE	Dia.	Length
Float. Pipe	900	544
Sinker Pipe	900	300
Shore Pipe	850	540
Shore Pipe		
Tot.Pipeline		1384

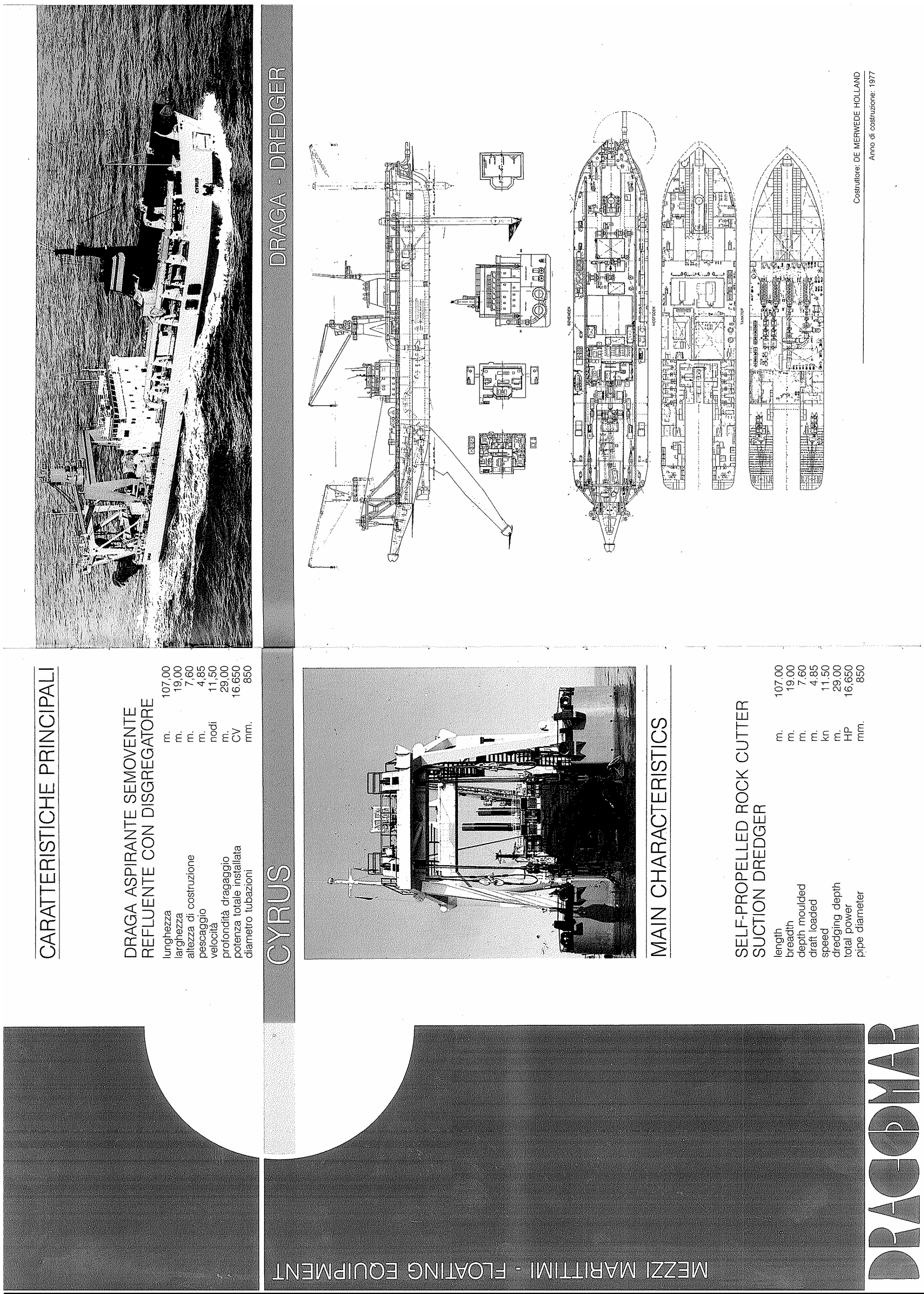
PUMP	Outlet Press.	Impellor Dia.	b
No. 1	1.4 kg	1960	4
No. 2	6.2 kg	2060	4
No. 3	12.5 kg	2060	4
Est. Velocity	4.5		
Est. Conc.	1.20		

Engine Running Hrs	
Engine No. 1	
Engine No. 2	24
Engine No. 3	24
Engine No. 4	24

Dredged m³
Up to date
10,295,334
This month
54,953

Dredge Master: **C.H. van der Tier** **Superintendent:** **M. de Ruyter**

Appendix C – Main Characteristics Cutter Suction Dredger “Cyrus”



Appendix D – Sheets Engineering Application Dredge Cut Nesting

Inner Sheet (Dredging Area)

Point ID	x	y	Point ID	x	y
1	166	1575	27	1111	1299
2	567	556	28	1101	1333
3	639	318	29	1033	1313
4	658	269	30	973	1516
5	684	224	31	738	1912
6	717	184	32	710	1895
7	756	149	33	731	1853
8	799	121	34	745	1808
9	847	100	35	752	1762
10	897	87	36	751	1715
11	949	81	37	743	1668
12	1001	84	38	728	1624
13	1052	95	39	706	1582
14	1101	114	40	678	1544
15	1146	140	41	644	1511
16	1186	173	42	605	1484
17	1221	212	43	563	1463
18	1249	256	44	518	1450
19	1270	304	45	472	1443
20	1284	354	46	425	1444
21	1289	406	47	378	1452
22	1286	458	48	334	1467
23	1275	509	49	292	1489
24	1220	692	50	254	1517
25	3025	1233	51	221	1551
26	2848	1820	52	194	1589

Outer Sheet (Dredging Area + Escape Regions)

Point ID	x	y	Point ID	x	y
1	128	1671	5	1220	692
2	567	556	6	3025	1233
3	734	0	7	2601	2647
4	1370	191			

Appendix E – Preliminary Engineering Application Dredge Cut Nesting

Model Parameter	Nest A	Nest B	Nest C	Nest D	Nest E	Nest F
Escape Penalty Factor	0	0	0	0	0	0
Overlap Penalty Factor	4	4	4	4	4	4
Non-placement Penalty Factor	50	50	50	50	50	50
Escape Penalty Exponent	0	0	0	0	0	0
Overlap Penalty Exponent	1	1	1	1	1	1
Non-placement Penalty Exponent	2	2	2	2	2	2
Sampled States	100	5	5	5	5	5
Parameter Tuning Factor	10	10	9	9	11	11
Cost Tuning Factor	6	6	5	5	5	5
Parameter Temp Re-anneal	150 AS / 1,500 GS	100 AS / 1,000 GS	60 AS / 600 GS	60 AS / 600 GS	100 AS / 1,000 GS	100 AS / 1,000 GS
Cost Temp Re-anneal	1,500 GS	1,000 GS	600 GS	600 GS	1,000 GS	1,000 GS
Parameter Temp Anneal	1,500 GS	1,000 GS	600 GS	600 GS	1,000 GS	1,000 GS
Cost Temp Anneal	1,500 AS	1,000 AS	600 AS	600 AS	1,000 AS	1,000 AS
Total Iterations	149,999	99,999	59,999	59,999	99,999	99,999

Notes: AS = Accepted States; GS = Generated States

Appendix F – Final Nest Layout Engineering Application

```
X-COORD, Y-COORD

1367.994, 197.0486
1217.104, 699.6717
1166.842, 684.5908
1317.726, 181.9639

735.2547, 1.743868
1338.07, 184.5971
1280.023, 375.9636
1155.218, 787.4132
743.7764, 662.6107
552.3987, 604.5595

403.9372, 970.5125
1088.2, 1238.755
1051.704, 1331.855
1015.209, 1424.952
819.9573, 1923.022
321.8846, 1727.768
228.7901, 1691.273
135.6947, 1654.776

1633.972, 816.7046
2337.902, 1028.039
2309.15, 1123.81
2280.397, 1219.581
2126.565, 1731.968
1614.18, 1578.138
1518.41, 1549.385
1422.637, 1520.632

338.2393, 1137.135
607.7052, 453.3772
700.7349, 490.0388
793.7618, 526.6989
1291.462, 722.8442
1095.323, 1220.543
1058.661, 1313.572
1022, 1406.603

2107.662, 1725.667
2320.183, 1022.084
2415.91, 1050.998
2511.637, 1079.912
3023.764, 1234.604
2869.077, 1746.739
2840.161, 1842.463
2811.248, 1938.189

937.1989, 1374.438
1148.963, 670.6453
1244.716, 699.4578
1340.47, 728.2677
1852.759, 882.4111
1698.615, 1394.694
1669.803, 1490.451
1640.993, 1586.204
```

Appendix G – Route Nodes Engineering Application

Point ID	x	y	Point ID	x	y	Point ID	x	y	Point ID	x	y
1	204	1625	58	810	299	115	1194	1068	172	2011	1204
2	242	1527	59	815	930	116	1207	642	173	2021	1536
3	280	1430	60	829	1757	117	1212	421	174	2041	1103
4	319	1332	61	833	598	118	1224	967	175	2051	1435
5	357	1234	62	836	1164	119	1234	1299	176	2071	1003
6	378	1468	63	840	198	120	1237	541	177	2081	1335
7	395	1136	64	846	1426	121	1242	320	178	2112	1234
8	406	1108	65	849	530	122	1254	867	179	2122	1566
9	416	1370	66	853	832	123	1264	1199	180	2142	1134
10	445	1010	67	867	1659	124	1267	441	181	2152	1465
11	455	1272	68	871	98	125	1273	220	182	2172	1033
12	476	1506	69	874	1066	126	1285	766	183	2182	1365
13	483	912	70	880	430	127	1294	1098	184	2203	1590
14	493	1175	71	884	1328	128	1298	340	185	2212	1264
15	504	1146	72	892	735	129	1325	998	186	2234	1490
16	514	1408	73	895	1300	130	1328	240	187	2242	1164
17	522	814	74	905	1562	131	1334	1329	188	2264	1389
18	543	1048	75	910	329	132	1355	897	189	2273	1063
19	553	1311	76	913	968	133	1365	1229	190	2295	1289
20	560	717	77	930	637	134	1385	797	191	2304	1620
21	581	951	78	933	1202	135	1395	1128	192	2325	1188
22	591	1213	79	941	229	136	1425	1028	193	2334	1520
23	599	619	80	944	1464	137	1435	1360	194	2355	1087
24	602	1185	81	950	561	138	1455	927	195	2365	1419
25	612	1447	82	951	871	139	1465	1259	196	2395	1319
26	620	853	83	971	128	140	1486	827	197	2404	1651
27	637	521	84	972	1105	141	1496	1159	198	2425	1218
28	640	1087	85	980	460	142	1526	1058	199	2435	1550
29	648	469	86	982	1366	143	1536	1390	200	2456	1118
30	650	1349	87	990	773	144	1556	958	201	2465	1450
31	658	755	88	992	1339	145	1566	1289	202	2496	1349
32	671	1583	89	1003	1339	146	1586	857	203	2505	1681
33	679	989	90	1010	1007	147	1596	1189	204	2526	1249
34	679	369	91	1011	360	148	1626	1088	205	2535	1581
35	689	1251	92	1028	675	149	1636	1420	206	2556	1148
36	693	1817	93	1031	1241	150	1657	988	207	2566	1480
37	697	658	94	1033	1239	151	1666	1320	208	2596	1380
38	699	1223	95	1041	259	152	1687	887	209	2606	1712
39	709	268	96	1049	909	153	1697	1219	210	2626	1279
40	710	1485	97	1050	591	154	1719	1445	211	2636	1611
41	717	891	98	1063	1138	155	1727	1119	212	2657	1179
42	731	1719	99	1072	159	156	1750	1345	213	2666	1510
43	735	560	100	1081	491	157	1757	1018	214	2697	1410
44	738	1125	101	1087	812	158	1780	1244	215	2706	1742
45	740	168	102	1093	1038	159	1787	918	216	2727	1309
46	748	1387	103	1103	1370	160	1810	1144	217	2736	1641
47	749	500	104	1111	390	161	1820	1475	218	2757	1209
48	756	794	105	1124	937	162	1840	1043	219	2767	1541
49	769	1621	106	1126	714	163	1850	1375	220	2797	1440
50	770	67	107	1133	1269	164	1870	942	221	2807	1772
51	776	1028	108	1142	290	165	1880	1274	222	2827	1340
52	779	399	109	1151	621	166	1910	1174	223	2837	1672
53	786	1290	110	1154	837	167	1920	1506	224	2858	1239
54	790	1855	111	1164	1168	168	1941	1073	225	2867	1571
55	794	696	112	1172	189	169	1951	1405	226	2898	1471
56	797	1262	113	1181	521	170	1971	973	227	2928	1370
57	807	1523	114	1184	736	171	1981	1305	228	2958	1270

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